International Journal of Applied Mathematical Research, 2 (4) (2013) 418-422
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Exact solution of the Biswas-Milovic equation by Adomian decomposition method

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Abstract

This paper studies the exact solution of the Biswas-Milovic equation with Kerr law nonlinearity by the aid of adomian decomposition method. Biswas-Milovic equation is a generalized version of the familiar nonlinear Schrodinger's equation describing the propagation of solitons through optical fibers for trans-continental and trans-oceanic distances.

Keywords: *Exact solutions, Biswas-Milovic equation, Adomian decomposition method*

1 Introduction

Recent years have witnessed an explosion of research activities in the field of soliton propagation in nonlinear optical media. These activities are motivated by the fact that optical solitons, both temporal and spatial variety, do have practical relevance in the latest communication technology based on generation and transportation of localized optical pulses or solitons. The dynamics of the propagation of solitons is studied by the aid of nonlinear Schrodinger's equation. An improved model to describe this dynamics is Biswas-Milovic model first appeared in 2010 [1] with generalization parameter, which dictate the factor of departure from perfection. Thus Biswas-Milovic equation is of special interest in the fiber optics community.

The study of the integrability aspects of nonlinear evolution equations has a lot of advances in the past couple of decades. There are various mathematical techniques that have been developed during this time frame to carry out the integration of these equations. Some of these commonly studied techniques are Inverse Scattering Transform [5], bilinear transformation[4], the tanh-sech method[6, 7], adomian decomposition method [3], the tanh-coth method[8], homogeneous balance method[9], Exp-function method [10], and many others.

The Adomian decomposition method was introduced and developed by George Adomian in [11, 12] and is well addressed in the literature. A reliable modification of the Adomian decomposition method developed by Wazwaz and presented in [3]. A considerable amount of research work has been invested recently in applying this method to a wide class of linear and nonlinear equations for detail see [13, 14, 15, 16, 17, 18] and the references therein.

In this paper the Adomian decomposition method will determine exact solutions to a Biswas-Milovic equation (BME) with Kerr law nonlinearity. In Section 2, we described this method for finding exact solution for nonlinear PDEs. In Section 3, we illustrated this method in detail with the Biswas-Milovic equation. In Section 4, we gave some conclusions.

2 Adomian decomposition method for nonlinear PDEs

We first consider the nonlinear partial differential equation given in an operator form

$$L_x u(x,y) + L_y u(x,y) + R(u(x,y)) + F(u(x,y)) = g(x,y), \tag{1}$$

where L_x is the highest order differential in x, L_y is the highest order differential in y, R contains the remaining linear terms of lower derivatives, $F(u(x,y))$ is an analytic nonlinear term, and $g(x,y)$ is an inhomogeneous or forcing term. the decision as to which operator L_x or L_y should be used to solve the problem depends mainly on two bases: (i) The operator of lowest order should be selected to minimize the size of computational work. (ii) The selected operator of lowest order should be of best known conditions to accelerate the evaluation of the components of the solution.For more detail see[3]. Assume that L_y meet these two conditions, therefore we set

$$L_y u(x,y) = g(x,y) - L_x u(x,y) - R(u(x,y)) - F(u(x,y)). \tag{2}$$

Applying L_y^{-1} to both sides of (2.2) gives

$$u(x,y) = \Phi_0 - L_y^{-1} g(x,y) - L_y^{-1} L_x u(x,y) - L_y^{-1} R(u(x,y)) - L_y^{-1} F(u(x,y)), \tag{3}$$

where

$$\Phi_0 = \begin{cases} u(x,0) & L = \frac{\partial}{\partial y}, \\ u(x,0) + y u_y(x,0) & L = \frac{\partial^2}{\partial y^2}, \\ u(x,0) + y u_y(x,0) + \frac{1}{2!} y^2 u_{yy}(x,0) & L = \frac{\partial^3}{\partial y^3}, \\ u(x,0) + y u_y(x,0) + \frac{1}{2!} y^2 u_{yy}(x,0) + \frac{1}{3!} y^3 u_{yyy}(x,0) & L = \frac{\partial^4}{\partial y^4}, \end{cases}$$

Take the solution $u(x,y)$ in a series form

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y), \tag{4}$$

and the nonlinear term $F(u(x,y))$ by

$$F(u(x,y)) = \sum_{n=0}^{\infty} A_n, \tag{5}$$

where A_n are Adomian polynomials that can be generated for all forms of nonlinearity and can be evaluated by using the following expression

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[F\left(\sum_{i=0}^{n} \lambda^i u_i \right) \right]_{\lambda=0}, n = 0,1,2 \tag{6}$$

Based on these assumptions, Eq. (3) become

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x,y) = \Phi_0 - L_y^{-1} g(x,y) - L_y^{-1} L_x \left(\sum_{n=0}^{\infty} u_n(x,y) \right) \\ -L_y^{-1} R \left(\sum_{n=0}^{\infty} u_n(x,y) \right) - L_y^{-1} \left(\sum_{n=0}^{\infty} A_n \right). \end{aligned} \tag{7}$$

The components $u_n(x,y)$, $n \geq 0$ of the solution $u(x,y)$ can be recursively determined by using the relation

$$\begin{aligned} &u_0(x,y) = \Phi_0 - L_y^{-1} g(x,y), \\ &u_{k+1}(x,y) = -L_y^{-1} L_x u_k - L_y^{-1} R(u_k) - L_y^{-1}(A_k), \quad k \geq 0. \end{aligned} \tag{8}$$

Next find the components of $\sum_{n=0}^{\infty} u_n(x,y)$ by

$$\begin{aligned} &u_0(x,y) = \Phi_0 - L_y^{-1} g(x,y), \\ &u_1(x,y) = -L_y^{-1} L_x u_0(x,y) - L_y^{-1} R\left(u_0(x,y)\right) - L_y^{-1} A_0, \\ &u_2(x,y) = -L_y^{-1} L_x u_1(x,y) - L_y^{-1} R\left(u_1(x,y)\right) - L_y^{-1} A_1, \\ &u_3(x,y) = -L_y^{-1} L_x u_2(x,y) - L_y^{-1} R\left(u_2(x,y)\right) - L_y^{-1} A_2, \\ &u_4(x,y) = -L_y^{-1} L_x u_3(x,y) - L_y^{-1} R\left(u_3(x,y)\right) - L_y^{-1} A_3, \\ &. \\ &. \\ &. \end{aligned}$$

where each component can be determined by using the preceding component. Having the calculated the components $u_n(x,y)$, $n \geq 0$, the solution in a series form is readily obtained.

3 Exact solutions for the Biswas-Milovic equation

In this section we obtain exact solution of the Biswas-Milovic equations, which are nonlinear PDEs by using the decomposition method described in Section 2. The BM equation is given by[1].

$$i(q^m)_t + a(q^m)_{xx} + bF(|q|^2)q^m = 0 \tag{9}$$

where q is a complex valued function, while x and t are the two independent variables. The coefficients a and b are constants where $ab > 0$, and parameter $m \geq 1$.
In this paper we will study the case $m = 1$ of Biswas-Milovic equation

$$i(q)_t + a(q)_{xx} + bF(|q|^2)q = 0 \tag{10}$$

Eq. (10) is a nonlinear PDE that is not integrable, in general. The non-integrability is not necessarily related to the nonlinear term in it. Also, in (9), F is a real-valued algebraic function and it is necessary to have smoothness of the complex function

$$F(|q|^2)q : C \mapsto C.$$

Considering the complex plane C as a two-dimensional linear space R^2, the function $F(|q|^2)q$ is k times continuously differentiable, so [1]

$$F(|q|^2)q \in \cup_{m,n=1}^{\infty} C^k\left((-n, n) \times (-m, m); R^2\right).$$

In order to seek exact solutions of Eq. (10), we assume that $q(x, 0) = e^{ix}$ and in this case the Kerr law of nonlinearity appears in nonlinear optics [2] is

$$F(s) = s$$

so Eq. (10) becomes

$$iq_t + aq_{xx} + b|q|^2 q = 0, \quad q(x, 0) = e^{ix} \tag{11}$$

Multiplying Eq.(11) by i, we may express this equation in an operator form as follows

$$L_t q(x, t) = iaq_{xx} + ib|q|^2 q. \tag{12}$$

where L_t is defined by $L_t = \frac{\partial}{\partial t}$ and the inverse operator L_t^{-1} is identified by

$$L_t^{-1}(\cdot) = \int_0^t (\cdot)dt$$

Applying L_t^{-1} to both sides of (12) and using the initial condition we obtain

$$q(x, t) = e^{ix} + iaL_t^{-1} q_{xx} + ibL_t^{-1}|q|^2 q, \tag{13}$$

where $F(q(x, t)) = |q|^2 q$ is nonlinear term.
Substituting

$$q(x, t) = \sum_{n=0}^{\infty} q_n(x, t) \tag{14}$$

and nonlinear term

$$|q|^2 q = \sum_{n=0}^{\infty} A_n \tag{15}$$

into (13) gives

$$\sum_{n=0}^{\infty} q_n(x, t) = e^{ix} + iaL_t^{-1}\left(\left(\sum_{n=0}^{\infty} q_n(x, t)\right)_{xx}\right) + ibL_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \tag{16}$$

Adomian's analysis introduces the recursive relation

$$\begin{aligned} &q_0(x, t) = e^{ix}, \\ &q_{k+1}(x, t) = iaL_t^{-1}(q_k)_{xx} + ibL_t^{-1}(A_k), k \geq 0. \end{aligned} \tag{17}$$

since q is a complex function so we can write

$$|q|^2 = q\bar{q} \tag{18}$$

where $\bar{q}$ is the conjugate of q. this means that (15) can be written as

$$q^2\bar{q} = \sum_{n=0}^{\infty} A_n \tag{19}$$

By using formal technique to find adomian polynomial used in [3] we find that $F(q)$ has the following polynomial representation

$$\begin{aligned} &A_0 = q_0^2\bar{q}_0, \\ &A_1 = 2q_0q_1\bar{q}_0 + q_0^2\bar{q}_1, \\ &A_2 = 2q_0q_2\bar{q}_0 + q_1^2\bar{q}_0 + 2q_0q_1\bar{q}_1 + q_0^2\bar{q}_2, \\ &A_3 = 2q_0q_3\bar{q}_0 + 2q_1q_2\bar{q}_0 + 2q_0q_2\bar{q}_1 + q_1^2\bar{q}_1 + 2q_0q_1\bar{q}_2 + q_0^2\bar{q}_3 \end{aligned} \tag{20}$$

that in turn gives the first few components by

$$\begin{aligned} &q_0(x,t) = e^{ix}, \\ &q_1(x,t) = iaL_t^{-1}\left(q_{0_{xx}}\right) + ibL_t^{-1}(A_0), \\ &q_2(x,t) = iaL_t^{-1}\left(q_{1_{xx}}\right) + ibL_t^{-1}(A_1), \\ &q_3(x,t) = iaL_t^{-1}\left(q_{2_{xx}}\right) + ibL_t^{-1}(A_2), \end{aligned} \tag{21}$$

we obtain

$$\begin{aligned} &q_0(x,t) = e^{ix}, A_0 = e^{ix}, \\ &q_1(x,t) = iaL_t^{-1}\left(-e^{ix}\right) + ibL_t^{-1}(e^{ix}) = it(b-a)e^{ix}, \\ &q_2(x,t) = iaL_t^{-1}\left(it(a-b)e^{ix}\right) + ibL_t^{-1}(it(b-a)e^{ix}) = \tfrac{(it)^2}{2!}(b-a)^2e^{ix} \\ &q_3(x,t) = iaL_t^{-1}\left(-\tfrac{(it)^2}{2!}(b-a)^2e^{ix}\right) + ibL_t^{-1}\left(\tfrac{(it)^2}{2!}(b-a)^2e^{ix}\right) = \tfrac{(it)^3}{3!}(b-a)^3e^{ix} \end{aligned} \tag{22}$$

Accordingly, the series solution is given by

$$q(x,t) = \sum_{n=0}^{\infty} q_n(x,t) = q_1 + q_2 + q_3 + ...$$

$$q(x,t) = e^{ix} + it(b-a)e^{ix} + \frac{(it)^2}{2!}(b-a)^2e^{ix} + \frac{(it)^3}{3!}(b-a)^3e^{ix} + ... \tag{23}$$

$$q(x,t) = e^{ix}\left(1 + it(b-a) + \frac{(it)^2}{2!}(b-a)^2 + \frac{(it)^3}{3!}(b-a)^3 + ...\right), \tag{24}$$

that gives exact solution of (11) in closed form

$$q(x,t) = e^{i(x+(b-a)t)}. \tag{25}$$

4 Conclusion

The Adomian decomposition method is successfully used to establish new exact solution. The performance of this method is found to be reliable and effective and can give more solutions, which may be important for the explanation of some new practical physical problems.

Acknowledgements

The authors would like to thank the anonymous referees for their valuable comments on this work. The second author is supported in part by NSF of China (11071266) and in part by the Natural Science Foundation Project of CQ CSTC(2010BB9218).

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