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Augmented Eccentric Connectivity Index of Some Thorn Graph

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Abstract

The augmented eccentric connectivity index is defined as the summation of the quotients of the product of adjacent vertex degrees and eccentricity of the concerned vertex of a graph which is a generalization of eccentric connectivity index. In this paper we present explicit expressions for the values of augmented eccentric connectivity indices of some particular thorn graphs like thorn path, thorn cycle and thorn star and then consider a particular thorn star, dendrimers.

Keywords: *Eccentric Connectivity Index, Thorn Graph, Dendrimers.*

1 Introduction

Let G be any simple connected graph with vertex set $V(G)$ and edge set $E(G)$ and n be the number of vertices. Let the distance between any two vertices of $V(G)$, is equal to the length of the shortest path connecting them. Also for a given vertex of $V(G)$ its eccentricity is the largest distance from that vertex to any other vertices of G. The radius and diameter of the graph are the smallest and largest eccentricity among all the vertices of G respectively [1]. The eccentric connectivity index of a graph G was proposed by Sharma, Goswami and Madan [4] and is defined as

$$\xi^c(G) = \sum_{i=1}^{n} d(v_i)\varepsilon(v_i)$$

where $d(v_i)$ is the degree i.e. number of first neighbor of v_i of $V(G)$ and $\varepsilon(v_i)$ is the eccentricity of the vertex v_i of $V(G)$. This topological index is subject to a large number of chemical [18] and mathematical [9] studies. A generalization of eccentric connectivity index, known as augmented eccentric connectivity index of a graph G was proposed by Dureja and Madan [5] and is defined as

$$\xi^{ac}(G) = \sum_{i=1}^{n} \frac{M(v_i)}{\varepsilon(v_i)}$$

where $M(v_i)$ denotes the product of degrees of all neighbors of vertex v_i. From above definition it is clear that, as the degrees are taken over the neighborhoods and then multiplied, so the contribution of a vertex to this index is non-local and again since the reciprocal of eccentricity is considered for a vertex so the contribution of a vertex is also non-linear. The study of mathematical as well as chemical properties of this index has started recently (see [2], [3]).

Gutman [8] first proposed the concept of thorn graphs and different applications have been studied by many others. Let G be the parent graph on n vertices $v_1, v_2, ..., v_n$ and $(p_1, p_2, ..., p_n)$ be an n-tuple on positive integers. Then the thorn graph $G^* = G^*(p_1, p_2, ..., p_n)$ formed by attaching $p_i\ (\geq 1)$, $i=1,2,...n$ new vertices of degree one to each vertex v_i of G. Several study of different topological indices of general and some particular thorn graphs and trees like wiener number [10,12], terminal Wiener index [14], modified Wiener index [15], altered Wiener Index [16], Hosoya polynomial [11], Zagreb polynomial [13], Schultz Index [7] and eccentric connectivity index [9] has already been considered.

In this paper we derive some explicit expressions of augmented eccentric connectivity indices of some particular thorn graphs like thorn path, thorn cycle, thorn star and dendrimers either in terms of other topological indices like Zagreb indices and multiplicative Zagreb Indices or recursively express bigger thorn graph in terms of smaller thorn graphs.

2 Main Results

2.1 Thorn Cycle

Let C_n is an n vertex cycle with vertices labeled as v_i and C_n^* is the thorn cycle obtained from C_n where each v_i consists of p_i thorn named as v_{ij} ($i=1,2,…,n$ and $j=1,2,…, p_i$). Thus C_n^* consists of n vertices of degree (p_i+2) and T vertices v_{ij} of degree one, where T is the total number of thorns. The augmented eccentric connectivity index of thorn cycle C_n^* is obtained as follows:

Theorem 2.1: *The augmented eccentric connectivity index of thorn cycle C_n^* is given by*

$$\xi^{ac}(C_n^*) = \begin{cases} \dfrac{2}{n+4}\left[M_1(C_n^*) - 3T - 4n\right] + \dfrac{2}{n+2}\sum_{i=1}^{n} M(v_i)\,, \textit{when n is even} \\ \dfrac{2}{n+3}\left[M_1(C_n^*) - 3T - 4n\right] + \dfrac{2}{n+1}\sum_{i=1}^{n} M(v_i)\,, \textit{when n is odd} \end{cases} \qquad (1)$$

where $M(v_i) = (p_j + 2)(p_k + 2), (i, j) \,\&\, (i,k) \in E(C_n)$.

Proof: To calculate augmented eccentric connectivity index of C_n^* we consider the following two cases:

Case I: When *n* is an odd number. Then $\varepsilon(v_i) = \dfrac{n+1}{2}$, $M(v_i) = (p_j + 2)(p_k + 2)$ where $(i, j) \,\&\, (i,k) \in E(C_n)$. Also $\varepsilon(v_{ij}) = \dfrac{(n+3)}{2}$ and $M(v_{ij}) = p_i + 2$ for *j=1,2,…,p*$_i$ and *i=1,2,….n*.

Thus, $\xi^{ac}(C_n^*) = \sum_{i-1}^{n} \dfrac{M(v_i)}{\varepsilon(v_i)} + \sum_{i=1}^{n}\sum_{j=1}^{p_i} \dfrac{M(v_{ij})}{\varepsilon(v_{ij})}$

$$= \frac{2}{n+1}\sum_{i-1}^{n} M(v_i) + \frac{2}{n+3}\sum_{i=1}^{n} p_i(p_i + 2)$$

$$= \frac{2}{n+1}\sum_{i-1}^{n} M(v_i) + \frac{4}{n+3}\sum_{i=1}^{n} p_i + \frac{2}{n+3}\sum_{i=1}^{n} p_i^2$$

Again since, $M_1(C_n^*) = \sum_{i=1}^{n} p_i^2 + 5T + 4n$, thus from above we get the desired result.

Case II: When *n* is an even number. Then, similarly $\varepsilon(v_i) = \dfrac{n+2}{2}$, $M(v_i) = (p_j + 2)(p_k + 2)$ where $(i, j) \,\&\, (i,k) \in E(C_n)$. Also $\varepsilon(v_{ij}) = \dfrac{(n+4)}{2}$ and $M(v_{ij}) = p_i + 2$ for *j=1,2,…,p*$_i$ and *i=1,2,….n*. So proceeding similarly we get the desired result (1). □

Corollary 2.1: *For t-thorny ring* $C_{n.t}$ *(having n ring as parent and t-2 thorns at each vertex) [10] the augmented eccentric connectivity index is given by*

$$\xi^{ac}(C_{n,t}) = \begin{cases} \dfrac{4nt}{n+4}\left[\left(\dfrac{n+2}{n+1}\right)t - 1\right] \textit{when n is even} \\ \dfrac{4nt}{n+4}\left[\left(\dfrac{n+2}{n+1}\right)t - 1\right] \textit{when n is odd} \end{cases} \qquad (2)$$

2.2 Thorn Path

Let P_m* be a thorn path obtained by attaching (*a-2)* pendent vertices to each non pendent vertices of a path P_m. Now to calculate augmented eccentric connectivity index of a thorn path first we consider the augmented eccentric connectivity index of a path P_m.

Theorem 2.3: [17] *If* $H_m = \sum_{i=1}^{m} \frac{1}{i}$, *the augmented eccentric connectivity index of a path* P_m *with m vertices is given by*

$$\xi^{ac}(P_m) = \begin{cases} 8(H_{m-1} - H_{\frac{m-2}{2}}) - 4\left(\frac{1}{m-1} + \frac{1}{m-2}\right), \text{when } m \text{ is even} \\ 8(H_{m-1} - H_{\frac{m-3}{2}}) - 4\left(\frac{3}{m-1} + \frac{1}{m-2}\right), \text{when } m \text{ is odd} \end{cases} \quad (3)$$

The above result can also be written as

$$\xi^{ac}(P_m) = \begin{cases} 8\sum_{i=0}^{n-1} \frac{1}{n+2+i} + 4\left(\frac{1}{2n+2} + \frac{1}{2n+3}\right), \text{when } m = 2n+4 \text{ that is even} \\ 8\sum_{i=1}^{n-1} \frac{1}{n+1+i} + 4\left(\frac{1}{2n+1} + \frac{3}{2n+2}\right), \text{when } m = 2n+3 \text{ that is odd} \end{cases} \quad (4)$$

Now using these results we find the augmented eccentric connectivity index of thorn path P_m^* in terms of the augmented eccentric connectivity index of path P_m.

Theorem 2.4: *The augmented eccentric connectivity index of the thorn path* P_m^* *is given by*

$$\xi^{ac}(P_m^*) = \begin{cases} \frac{1}{2}a(a-1)\xi^{ac}(P_m) - \frac{4a(a-2)}{m}, \text{when } m \text{ is even} \\ \frac{1}{2}a(a-1)\xi^{ac}(P_m) - \frac{4a(a-2)}{m+1}, \text{when } m \text{ is odd} \end{cases} \quad (5)$$

Proof: To poof this result we consider the following two cases.

Case: I Let P_m^* be a thorn path obtained from a path P_m on even number of vertices, so that $m = 2n+4$. Also let the thorn tree P_{2n+4}^* is obtained by attaching (*a-2)* pendent vertices to the non pendent vertices of P_{2n+4}. Let the vertices of P_{2n+4} labeled as $v'_{n+1}, v'_n, v'_{n-1}, \ldots v'_2, v'_1, v'_0, v_0, v_1, v_2, \ldots, v_{n-1}, v_n, v_{n+1}$ where v'_0 and v_0 are centers of the parent path so that for P_{2n+4}^* we have $M(v_i) = a^2 = M(v'_i)$ for *i=0,1,2,…,n-1* and $M(v_j) = a = M(v'_j)$ for *j=n,n+1*. Again for P_{2n+4}^* we have $\varepsilon(v_i) = n+2+i = \varepsilon(v'_i)$ for *i=0,1,2,…,n+1*.

Again, let the newly attached pendent vertices to the vertices v_i and v'_i of P_{2n+4}, are denoted by l_{ij} and l'_{ij} for *i=0,1,2,…,n* and *j=1,2,…,(a-2).*Thus

$\varepsilon(l_{ij})=\varepsilon(v_i)+1=\varepsilon(v'_i)+1=\varepsilon(l'_{ij})$ for *i=0,1,2,…,n* and *j=1,2,…,(a-2)*. Similarly $M(l_{ij})=a=M(l'_{ij})$ for *i=0,1,2,…,n* and *j=1,2,…,(a-2)*.

Thus the augmented eccentric connectivity index of P^*_{2n+4} is given by

$$\xi^{ac}(P^*_{2n+4})=\xi_1^{ac}(P^*_{2n+4})+\xi_2^{ac}(P^*_{2n+4})$$

Where, $\xi_1^{ac}(P^*_{2n+4})=\sum_{i-0}^{n+1}\frac{M(v_i)}{\varepsilon(v_i)}+\sum_{i-0}^{n+1}\frac{M(v'_i)}{\varepsilon(v'_i)}$

and $\xi_2^{ac}(P^*_{2n+4})=\sum_{i-0}^{n}\sum_{j=1}^{a-2}\frac{M(l_{ij})}{\varepsilon(l_{ij})}+\sum_{i-0}^{n}\sum_{j=1}^{a-2}\frac{M(l'_{ij})}{\varepsilon(l'_{ij})}$

Now $\xi_1^{ac}(P^*_{2n+4})=2\left[\sum_{i-0}^{n-1}\frac{M(v_i)}{\varepsilon(v_i)}+\sum_{j=n}^{n+1}\frac{M(v_j)}{\varepsilon(v_j)}\right]=2\left[\sum_{i-0}^{n-1}\frac{a^2}{n+2+i}+\sum_{j=n}^{n+1}\frac{a}{(n+2+j)}\right]$

$$=2a^2\left\{\frac{1}{8}\xi^{ac}(P_{2n+4})-\frac{1}{2}\left(\frac{1}{2n+2}+\frac{1}{2n+3}\right)\right\}+2a\left\{\frac{1}{2n+2}+\frac{1}{2n+3}\right\}$$

$$=\frac{a^2}{4}\xi^{ac}(P_{2n+4})+a(a-2)\left(\frac{1}{2n+2}+\frac{1}{2n+3}\right)$$

Again, $\xi_2^{ac}(P^*_{2n+4})=\sum_{i-0}^{n}\sum_{j=1}^{a-2}\frac{a}{\varepsilon(v_i)+1}+\sum_{i-0}^{n}\sum_{j=1}^{a-2}\frac{a}{\varepsilon(v'_i)+1}=2a(a-2)\sum_{i=0}^{n}\frac{1}{(n+3+i)}$

$$=a(a-2)\left[\frac{1}{4}\xi^{ac}(P_{2n+3})+\left(\frac{1}{2n+2}+\frac{1}{2n+3}\right)-\frac{2}{n+2}\right]$$

Thus from (3) $\xi^{ac}(P^*_{2n+4})=\frac{1}{2}a(a-1)\xi^{ac}(P_{2n+4})-\frac{2a(a-2)}{n+2}$

and hence the desired result follows.

Case: II Let P^*_m be a thorn path obtained from a path P_m on odd number of vertices, so that $m=2n+3$. Also let the thorn tree P^*_{2n+3} is obtained by attaching (*a-2*) pendent vertices to the non pendent vertices of P_{2n+3}. Let the vertices of P_{2n+3} labeled as $v'_{n+1},v'_n,v'_{n-1},....v'_2,v'_1,v_0,v_{1,}v_2,...,v_{n-1},v_n,v_{n+1}$, where v_0 is the center of the parent path so that for P^*_{2n+3} we have $M(v_0)=a^2, M(v_i)=a^2=M(v'_i)$ for *i=1,2,…,n-1* and $M(v_j)=a=M(v'_j)$ for *j=n,n+1*. Again for P^*_{2n+3} we have $\varepsilon(v_0)=n+1, \varepsilon(v_i)=n+1+i=\varepsilon(v'_i)$ for *i=1,2,…,n+1*.

Again, let the newly attached pendent vertices to the vertices v_0, v_i and v'_i of P_{2n+3}, are denoted by l_{0j}, l_{ij} and l'_{ij} for *i=0,1,2,…,n* and *j=1,2,…,(a-2)*.Thus $\varepsilon(l_{ij})=\varepsilon(v_i)+1=\varepsilon(v'_i)+1=\varepsilon(l'_{ij})$ for *i=0,1,2,…,n* and *j=1,2,…,(a-2)*. Similarly $M(l_{ij})=a=M(l'_{ij})$ for *i=0,1,2,…,n* and *j=1,2,…,(a-2)*.

Thus the augmented eccentric connectivity index of P^*_{2n+4} is given by

$$\xi^{ac}(P^*_{2n+4}) = \xi_1^{ac}(P^*_{2n+4}) + \xi_2^{ac}(P^*_{2n+4})$$

where

$$\xi_1^{ac}(P^*_{2n+3}) = \frac{M(v_0)}{\varepsilon(v_0)} + \sum_{i-1}^{n+1}\frac{M(v_i)}{\varepsilon(v_i)} + \sum_{i-1}^{n+1}\frac{M(v_i')}{\varepsilon(v_i')} = \frac{a^2}{4}\xi^{ac}(P_{2n+4}) - a(1-a)\left(\frac{1}{2n+1}+\frac{1}{2n+2}\right)$$

and $\xi_2^{ac}(P^*_{2n+3}) = \sum_{j=0}^{a-2}\frac{M(l_{0j})}{\varepsilon(l_{0j})} + \sum_{i-1}^{n}\sum_{j=1}^{a-2}\frac{M(l_{ij})}{\varepsilon(l_{ij})} + \sum_{i-1}^{n}\sum_{j=1}^{a-2}\frac{M(l_{ij}')}{\varepsilon(l_{ij}')}$

$$= a(a-2)\left[\frac{1}{4}\xi^{ac}(P_{2n+3}) + \left(\frac{1}{2n+1}+\frac{1}{2n+2}\right) - \frac{2}{n+2}\right]$$

So proceeding similarly we get the desired result as stated above. □

Also from *Theorem 2.3* and *Theorem 2.4* the augmented eccentric connectivity index of thorn path P_m^* in terms of H_m is given by

Theorem 2.5: *The augmented eccentric connectivity index of the thorn path P_m^* is given by*

$$\xi^{ac}(P_m^*) = \begin{cases} 2a(a-1)\left[2\left(H_{m-1} - H_{\frac{m-2}{2}}\right) - \left(\frac{1}{m-1}+\frac{1}{m-2}-\frac{2}{m}\right)\right] + \frac{4a}{m}, & \text{when } m \text{ is even} \\ 2a(a-1)\left[2\left(H_{m-1} - H_{\frac{m-3}{2}}\right) - \left(\frac{3}{m-1}+\frac{1}{m-2}-\frac{2}{m+1}\right)\right] + \frac{4a}{m+1}, & \text{when } m \text{ is odd} \end{cases} \quad (6)$$

2.3 Thorn Star

The thorn star $K^*_{1,n}$ with parameters with parameters p_i, $i=1,2,\ldots n$, is the graph obtained from the star $K_{1,n}$ by adding p_i pendent vertices to the i th pendent vertex of $K_{1,n}$. Let the center of the star is denoted by c and the all the remaining pendent vertices are denoted by v_i so that $M(c) = \varepsilon(c) = 1$, $M(v_i) = n$ and $\varepsilon(v_i) = 2$.

Thus, $\xi^{ac}(K_{1,n}) = (1+\frac{n^2}{2})$.

Theorem 2.6: *The augmented eccentric connectivity index of thorn star $K^*_{1,n}$ is given by*

$$\xi^{ac}(K_{1,n}^*)=\frac{1}{2n}\sqrt{\Pi_1(K_{1,n}^*)}+\frac{1}{4}M_1(K_{1,n}^*)-\frac{1}{4}T+\frac{n}{4}(\frac{n}{3}-1) \quad (7)$$

Proof: Let the newly attached pendent vertices are denoted by v_{ij} so that in $K_{1,n}^*$,

$M(c)=\prod_{i=1}^{n}(p_i+1)$, $\varepsilon(c)=2$, $M(v_i)=n$, $\varepsilon(v_i)=3$ and $M(v_{ij})=p_i+1$, $\varepsilon(v_{ij})=4$

for $j=1,2,\ldots,p_i$ and $i=1,2,\ldots.n$. Thus the eccentric connectivity index of thorn star is given by

$$\xi^{ac}(K_{1,n}^*)=\frac{M(c)}{\varepsilon(c)}+\sum_{i-1}^{n}\frac{M(v_i)}{\varepsilon(v_i)}+\sum_{i=1}^{n}\sum_{j=1}^{p_i}\frac{M(v_{ij})}{\varepsilon(v_{ij})}$$

$$=\frac{1}{2}\prod_{i=1}^{n}(p_i+1)+\frac{n^2}{3}+\sum_{i=1}^{n}\sum_{j=1}^{p_i}\frac{p_i+1}{4}$$

$$=\frac{1}{2}\sqrt{\Pi_1(K_{1,n}^*)}+\frac{n^2}{3}+\frac{T}{4}+\sum_{i=1}^{n}p_i^2$$

where the first multiplicative Zagreb index of G is defined as [6],

$\prod_1(G)=\prod_{i\in V(G)}d(v_i)^2$. Now since the first Zagreb index of $K_{1,n}^*$ is given by

$M_1(K_{1,n}^*)=\sum_{i=1}^{n}p_i^2+2T+n^2+n$ so the desired result follows from above. □

Corollary 2.2: *The augmented eccentric connectivity index of $S_{k,t}$ is given by*

$$\xi^{ac}(S_{k,t})=\frac{4nt}{n+1}\left[\frac{(t-2)(n+2)}{n+3}+1\right] \quad (8)$$

where, the thorn star $S_{k,t}$ (as in [10]) is obtained from a star with (k+1) vertices by joining (t-1) pendent vertices to each v_i (i=1,2,..,k).

2.4 Dendrimers

Let the monocentric regular dendrimers is denoted by $T_{d,k}$ so that the centre v_0 and the all the other non-pendent vertices is of degree d and the distance of every pendent vertices from the centre v_0 is k that is k denotes the diameter of $T_{d,k}$. Obviously the number of vertices of $T_{d,k}$ is $1+\frac{d[(d-1)^k-1]}{d-2}$ so that $T_{d,k}$ consists of $d(d-1)^k$ number of thorns. Thus we can also say that there is total $d(d-1)^{p-1}$ number of vertices whose distance from the centre v_0 is p.

Theorem 2.7: *The augmented eccentric connectivity index of regular monocentric dendrimers is recursively expressed as*

$$\xi^{ac}(T_{d,k})=\frac{1}{d-1}\xi^{ac}(T_{d,k-1})+\frac{d^2(d-1)^{k-2}(2kd-d+1)}{2k(2k-1)}-\frac{d^d(2k-1)}{k(k-1)(d-1)}+$$

$$\frac{d(d^d-d)(d-1)^{k-4}(2kd-3d+1)}{2(k-1)(2k-3)} \quad (9)$$

where d is the degree of the non pendent vertices and k is the diameter of the tree.

Proof: Let the vertices whose distance from the centre is p be denoted by v_{pj}. Clearly the eccentricity of the central vertex v_o is k i.e. $\varepsilon(v_0)=k$ and $\varepsilon(v_{ij})=k+i$ where v_{ij} are the vertices whose distance from v_0 is i. Also $M(v_0)=d^d$ for $i=1,2,\ldots,k-2$ and $M(v_{ij})=d^d$ for $i=1,2,\ldots,k-2$ and $M(v_{ij})=d$ for $i=k-1$ & $k-2$, $j=1,2,\ldots,d(d-1)^{i-1}$.

Thus the augmented eccentric connectivity index of $T_{d,k}$ is given by

$$\xi^{ac}(T_{d,k})=\frac{M(v_0)}{\varepsilon(v_0)}+\sum_{i=1}^{k-2}\sum_{j=1}^{d(d-1)^{i-1}}\frac{M(v_{ij})}{\varepsilon(v_{ij})}+\sum_{i=k-1}^{k}\sum_{j=1}^{d(d-1)^{i-1}}\frac{M(v_{ij})}{\varepsilon(v_{ij})}$$

$$=\frac{d^d}{k}+\sum_{i=1}^{k-2}\frac{d^d d(d-1)^{i-1}}{k+i}+\sum_{j=1}^{d(d-1)^{k-2}}\frac{d}{2k-1}+\sum_{j=1}^{d(d-1)^{k-1}}\frac{d}{2k}$$

$$=\frac{d^d}{k}+\frac{d^2(d-1)^{k-2}}{2k-1}+\frac{d^2(d-1)^{k-1}}{2k}+\sum_{i=1}^{k-2}\frac{d^{d+1}(d-1)^{i-1}}{k+i}$$

From above it is clear that,

$$\xi^{ac}(T_{d,k-1})=\frac{d^d}{k-1}+\frac{d^2(d-1)^{k-3}}{2k-3}+\frac{d^2(d-1)^{k-2}}{2k-2}+\sum_{i=1}^{k-3}\frac{d^{d+1}(d-1)^{i-1}}{k-1+i}$$

Now let, $S_1=\sum_{i=1}^{k-2}\frac{d^{d+1}(d-1)^{i-1}}{k+i}$ and $S_2=\sum_{i=1}^{k-3}\frac{d^{d+1}(d-1)^{i-1}}{k-1+i}$ so that

$$S_1=\frac{1}{d-1}S_2+d^{d+1}\left[\frac{(d-1)^{k-4}}{2k-3}+\frac{(d-1)^{k-3}}{2k-2}-\frac{(d-1)^{-1}}{k}\right]$$

So we have

$$S_1=\frac{1}{d-1}\xi_2^{ac}(T_{d,k-1})+d(d^d-1)\left[\frac{(d-1)^{k-4}}{2k-3}+\frac{(d-1)^{k-3}}{2k-2}\right]+\frac{d^d(k+kd-d)}{k(d-1)(k-1)}$$

Then from above we get

$$\xi^{ac}(T_{d,k})=S_1+\frac{d^d}{k}+\frac{d^2(d-1)^{k-2}}{2k-1}+\frac{d^2(d-1)^{k-1}}{2k}+\frac{d^2(d-1)^{k-1}}{2k}$$

$$=\frac{1}{d-1}\xi_2^{ac}(T_{d,k-1})+d(d^d-1)\left[\frac{(d-1)^{k-4}}{2k-3}+\frac{(d-1)^{k-3}}{2k-2}\right]+\frac{d^d(k+kd-d)}{k(d-1)(k-1)}+\frac{d^d}{k}+\frac{d^2(d-1)^{k-2}}{2k-1}$$

Now after some calculations we get the desired result. □

3 Conclusion

Since the augmented eccentric connectivity index is based on both non-local and non-linear contributions of vertices, so it difficult to calculate some explicit relations of this index for generalized thorn graphs (like [9]). In this paper we have presented some explicit mathematical expressions of the augmented eccentric connectivity index of some particular thorn graphs. In *Theorem 1* the augmented eccentric connectivity index of a thorn cycle is expressed as first Zagreb index of thorn cycle and sum of product of degree of all neighbors of vertices of C_n. Also in *Theorem 4* first the augmented eccentric connectivity index of thorn path is expressed in terms of its parent path and then in *Theorem 5* it is expressed in terms of H_m. From *Theorem 6* it is clear that there is a nonlinear relation between the augmented eccentric connectivity index, first Zagreb index and first multiplicative Zagreb index of thorn stars. Also using the recursive relation *(9)*, for a particular value of d, we can obtain a series of augmented eccentric connectivity index for monocentric dendrimers. For example when $d=3$, for $k=2,3,4,\ldots$ we get the series $\{21,38.85,64.09,\ldots.\}$. However, our results still leave much to be desired. These results can be extended to generalized thorn graph where some p_i are equal to zero.

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