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Research Paper,

# On transformation formulae of ordinary hypergeometric series

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**Abstract**

In this paper, making use of some well-known summation formulae and generating relations due to Qureshi, Khan and Pathan, an attempt has been made to establish some transformation formulae of ordinary hyper geometric series which are seemed to be new and in different form. We have also given some special cases.

***Keywords***: *Summation Formulae, Ordinary Hyper geometric Series, Transformation Formulae, Generating Relation*.

## 1. Introduction

The hypergeometric function and its generalizations, summation theorems and transformation formulae have been presented in many textbooks [1], [5], [6], and [11] where references to the extensive literature on the subject may be found. Mathematicians working in the area of ordinary and basic hypergeometric series were interested for transformation formulae among various generalised hypergeometric functions and they succeeded in their goal. The celebrated Bailey [2] transform was extensively used to obtain transformation formulae of ordinary hypergeometric series and basic hypergeometric series with help of known summation formulae. The technique provided by Bailey [2] and Slater [3], [4] motivated a number of mathematicians namely Andrews [7], [8], Verma and Jain [9], [10], U.B.Singh [12], Agarwal [13], S.P. Singh [14], Denis et.al. [16], [17], Srivastav and Rudravarapu [18] and S. Singh [19] and they enriched the literature of ordinary and basic hypergeometric series. Motivated by the aforementioned works, making use of some well-known summation formulae and generating relations due to Qureshi et.al. [15], an attempt has been made to establish some transformation formulae of ordinary hypergeometric series on the same track as many authors established numerous transformation formulae of ordinary hypergeometric series and basic hypergeometric series by using Bailey transform and certain known summation formulae. Here, we have used generating relations instead of Bailey transform to establish transformation formulae of ordinary hypergeometric series which are believed to be new. In view of the importance and usefulness of the generating relations, we have extended the idea of generating relations for obtaining transformation formulae of ordinary hypergeometric series.The transformation formulae of hypergeometric series play a pivotal role in the investigation of various useful properties and can also be used as a new platform for further study.

## 2. Definitions and notations

The following notations and definitions shall be used throughout this paper
For ‘ $a$ ’ real or complex and ‘ $n$ ’ be a positive integer, we define

$$(a)_0 = 1$$

$$(a)_n = a(a+1)(a+2)...(a+n-1), \quad n = 1,2,3,...$$

If ‘ $a$ ’ is a negative integer $-m$ , then

$$(a)_n = (-m)_n \quad if \quad m \geq n$$

$$(a)_0 = 0 \quad if \quad m < n$$

Now, we define a generalized hypergeometric function,

$${}_rF_s\left[\begin{matrix} a_1, a_2, ..., a_r; z \\ b_1, b_2, ..., b_s; \end{matrix}\right] = {}_rF_s\left[\begin{matrix} (a_r); z \\ (b_s); \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n ... (a_r)_n z^n}{(b_1)_n (b_2)_n ... (b_s)_n (1)_n} = \sum_{n=0}^{\infty} \frac{((a_r))_n z^n}{((b_s))_n (1)_n} \tag{1}$$

Where there are always $r$ of $a$ parameters and $s$ of the $b$ parameters. The meaning of ( $a$ ) and ( $b$ ) are sequences of parameters $a_1, a_2, a_3, ..., a_r$ *and* $b_1, b_2, b_3, ..., b_s$ respectively.

The series (1) is convergent if

i) $Rl\left(\sum_{v=1}^{s} b_v - \sum_{v=1}^{r} a_v\right) > 0$ when $z = 1$

ii) $Rl\left(\sum_{v=1}^{s} b_v - \sum_{v=1}^{r} a_v\right) > -1$ when $z = -1$

iii) $r = s + 1$ when $|z| < 1$,

iv) $r > s + 1$ when $z = 0$

In, 2002, Qureshi et.al. [15] Established two hyper geometric generating relations-

$$\sum_{n=0}^{\infty} \frac{(d)_n \left(d - \frac{1}{2}\right)_n x^n}{(2d)_n \lfloor n} \sum_{m=0}^{n} \frac{A_m (-n)_m (-y)^m}{(2d+n)_m \lfloor m}$$

$$= \left(\frac{1+\sqrt{1-x}}{2}\right)^{1-2d} \sum_{n=0}^{\infty} \frac{A_n \left(d - \frac{1}{2}\right)_n}{\left(d + \frac{1}{2}\right)_n \lfloor n} \left(\frac{xy}{\left(1+\sqrt{1-x}\right)^2}\right)^n \tag{2}$$

$$\sum_{n=0}^{\infty} \frac{(d)_n \left(d + \frac{1}{2}\right)_n x^n}{(2d)_n \lfloor n} \sum_{m=0}^{n} \frac{A_m (-n)_m (-y)^m}{(2d+n)_m \lfloor m}$$

$$= \frac{1}{\sqrt{1-x}} \left(\frac{1+\sqrt{1-x}}{2}\right)^{1-2d} \sum_{n=0}^{\infty} \frac{A_n}{\lfloor n} \left(\frac{xy}{\left(1+\sqrt{1-x}\right)^2}\right)^n \tag{3}$$

Where $\{A_n\}$ be the bounded sequence of arbitrary complex numbers with any values of numerator and denominator parameters and x, y are the variables.

Further, we shall use following well known summation formulae to derive our main results-

$${}_3F_2\left[\begin{matrix} a, & b, & -n; \ 1 \\ & 1+a-b, & 1+a+n; \end{matrix}\right] = \frac{(1+a)_n \left(1+\frac{a}{2}-b\right)_n}{\left(1+\frac{a}{2}\right)_n (1+a-b)_n} \tag{4}$$

(*cf* Slater [6], App.III .9, p-243)

$${}_4F_3\left[\begin{matrix} a, & 1+\frac{a}{2}, & b, & -n; \ -1 \\ & \frac{a}{2}, & 1+a-b, & 1+a+n; \end{matrix}\right] = \frac{(1+a)_n}{(1+a-b)_n} \tag{5}$$

(*cf* Slater [6], App.III .11, p-244)

$${}_5F_4\left[\begin{matrix} a, & 1+\frac{a}{2}, & b, & c, & -n; \ 1 \\ & \frac{a}{2}, & 1+a-b, & 1+a-c, & 1+a+n; \end{matrix}\right] = \frac{(1+a)_n (1+a-b-c)_n}{(1+a-b)_n (1+a-c)_n} \tag{6}$$

(*cf* Slater [6], App.III.13 p-244)

$$ {}_7F_6\left[\begin{matrix} a, & 1+\frac{a}{2}, & b, & c, & d, & e, & -n; & 1 \\ & \frac{a}{2}, & 1+a-b, & 1+a-c, & 1+a-d, & 1+a-e, & 1+a+n; & \end{matrix}\right] $$
$$ =\frac{(1+a)_n(1+a-b-c)_n(1+a-b-d)_n(1+a-c-d)_n}{(1+a-b)_n(1+a-c)_n(1+a-d)_n(1+a-b-c-d)_n} \tag{7} $$

Provided that $1+2a=b+c+d+e-n$

(*cf* Slater [6], App.III.14, p-244)

$$ {}_3F_2\left[\begin{matrix} a, & 1+\frac{a}{2}, & -n; & -1 \\ & \frac{a}{2}, & 1+a+n; & \end{matrix}\right]=\frac{(1+a)_n}{\left(\frac{1}{2}+\frac{a}{2}\right)_n} \tag{8} $$

(*cf* Slater [6], App.III .25, p-245)

$$ {}_4F_3\left[\begin{matrix} a, & 1+\frac{a}{2}, & b, & -n; & 1 \\ & \frac{a}{2}, & 1+a-b, & 1+a+n; & \end{matrix}\right]=\frac{(1+a)_n\left(\frac{1}{2}+\frac{a}{2}-b\right)_n}{\left(\frac{1}{2}+\frac{a}{2}\right)_n(1+a-b)_n} \tag{9} $$

(*cf* Slater [6], App.III .26, p-245)

$$ {}_3F_2\left[\begin{matrix} a, & 1+\frac{a}{2}, & -n; & 1 \\ & \frac{a}{2}, & 1+a+n; & \end{matrix}\right]=(1+a)_n \tag{10} $$

(*cf* Slater [6], (2.3.4.10), p-57)

Gauss's theorem,

$$ {}_2F_1\left[\begin{matrix} a,b;1 \\ c; \end{matrix}\right]=\Gamma\left[\begin{matrix} c,c-a-b \\ c-a,c-b \end{matrix}\right] \tag{11} $$

(*cf* Slater [6], App.III.3, p-243)

Kummer's theorem,

$$ {}_2F_1\left[\begin{matrix} a, & b; & -1 \\ & 1+a-b; & \end{matrix}\right]=\Gamma\left[\begin{matrix} 1+a-b,1+\frac{a}{2} \\ 1+a,1+\frac{a}{2}-b \end{matrix}\right] \tag{12} $$

(*cf* Slater [6], App.III.5, p-243)

## 3. Main results

Our main results are as under

$$ {}_3F_2\left[\begin{matrix} \frac{a}{2}, & \frac{1}{2}+\frac{a}{2}, & 1+\frac{a}{2}-b; & x \\ & 1+\frac{a}{2}, & 1+a-b; & \end{matrix}\right]=\left(\frac{1+\sqrt{1-x}}{2}\right)^{-a}{}_3F_2\left[\begin{matrix} a, & \frac{a}{2}, & b; & \frac{-x}{\left(1+\sqrt{1-x}\right)^2} \\ & 1+\frac{a}{2}, & 1+a-b; & \end{matrix}\right] \tag{13} $$

$$ {}_2F_1\left[\begin{matrix} \frac{a}{2}, & \frac{1}{2}+\frac{a}{2}; & x \\ & 1+a-b; & \end{matrix}\right]=\left(\frac{1+\sqrt{1-x}}{2}\right)^{-a}{}_2F_1\left[\begin{matrix} a, & b; & \frac{x}{\left(1+\sqrt{1-x}\right)^2} \\ & 1+a-b; & \end{matrix}\right] \tag{14} $$

$$ {}_3F_2\left[\begin{matrix} \frac{a}{2}, & \frac{1}{2}+\frac{a}{2}, & 1+a-b-c; & x \\ & 1+a-b, & 1+a-c; & \end{matrix}\right]= $$
$$ \left(\frac{1+\sqrt{1-x}}{2}\right)^{-a}{}_3F_2\left[\begin{matrix} a, & b, & c; & \frac{-x}{\left(1+\sqrt{1-x}\right)^2} \\ & 1+a-b, & 1+a-c; & \end{matrix}\right] \tag{15} $$

$$ {}_5F_4\left[\begin{matrix} \frac{a}{2}, & \frac{1}{2}+\frac{a}{2}, & 1+a-b-c, & 1+a-b-d, & 1+a-c-d; & x \\ & 1+a-b, & 1+a-c, & 1+a-d, & 1+a-b-c-d; & \end{matrix}\right] $$

$$=\left(\frac{1+\sqrt{1-x}}{2}\right)^{-a} {}_5F_4\left[\begin{matrix} a, \quad b, \quad c, \quad d, \quad e; & \dfrac{-x}{\left(1+\sqrt{1-x}\right)^2} \\ 1+a-b, 1+a-c, 1+a-d, 1+a-e; & \end{matrix}\right] \tag{16}$$

$${}_1F_0\left[\begin{matrix} \frac{a}{2}; & x \\ -; & \end{matrix}\right] = \left(\frac{1+\sqrt{1-x}}{2}\right)^{-a} {}_1F_0\left[\begin{matrix} a; & \dfrac{x}{\left(1+\sqrt{1-x}\right)^2} \\ -; & \end{matrix}\right] \tag{17}$$

$${}_2F_1\left[\begin{matrix} \frac{a}{2}, \ \frac{1}{2}+\frac{a}{2}-b; & x \\ 1+a-b; & \end{matrix}\right] = \left(\frac{1+\sqrt{1-x}}{2}\right)^{-a} {}_2F_1\left[\begin{matrix} a, \quad b; & \dfrac{-x}{\left(1+\sqrt{1-x}\right)^2} \\ 1+a-b; & \end{matrix}\right] \tag{18}$$

$${}_2F_0\left[\begin{matrix} \frac{1}{2}+\frac{a}{2}, \ 1+\frac{a}{2}; & x \\ -; & \end{matrix}\right] = \frac{1}{\sqrt{1-x}} \left(\frac{1+\sqrt{1-x}}{2}\right)^{-a} {}_2F_1\left[\begin{matrix} a, \ 1+\frac{a}{2}; & \dfrac{-x}{\left(1+\sqrt{1-x}\right)^2} \\ \frac{a}{2}; & \end{matrix}\right] \tag{19}$$

# 4. Proof

In this section, we shall derive our main results by taking suitable values of $A_m$ , $2d$ and $y$ in generating relation (2) and (3) and appropriate summation formula accordingly.

**Proof of 3.1:**

Taking, $A_m = \dfrac{(a)_m (b)_m}{(1+a-b)_m}$, $2d = 1+a$ and $y = -1$ in (2), we have

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}+\frac{a}{2}\right)_n \left(\frac{a}{2}\right)_n x^n}{(1+a)_n \, n!} \sum_{m=0}^{n} \frac{(a)_m (b)_m (-n)_m (1)^m}{(1+a-b)_m (1+a+n)_m \, m!}$$

$$=\left[\frac{1+\sqrt{1-x}}{2}\right]^{-a} \sum_{n=0}^{\infty} \frac{(a)_n (b)_n \left(\frac{a}{2}\right)_n}{(1+a-b)_n \left(1+\frac{a}{2}\right)_n n!} \left[\frac{-x}{\left(1+\sqrt{1-x}\right)^2}\right]^n \tag{20}$$

Using (4) in the inner series on left hand side of (20); we get required result (13).

**Proof of 3.2:**

Taking, $A_m = \dfrac{(a)_m \left(1+\frac{a}{2}\right)_m (b)_m}{\left(\frac{a}{2}\right)_m (1+a-b)_m}$ , $2d = 1+a$ and $y = 1$ in (2), we have

$$\sum_{n=0}^{\infty} \frac{\left(\frac{a}{2}\right)_n \left(\frac{1}{2}+\frac{a}{2}\right)_n x^n}{(1+a)_n \, n!} \sum_{m=0}^{n} \frac{(a)_m (b)_m \left(1+\frac{a}{2}\right)_m (-n)_m (-1)^m}{\left(\frac{a}{2}\right)_m (1+a-b)_m (1+a+n)_m \, m!}$$

$$=\left[\frac{1+\sqrt{1-x}}{2}\right]^{-a} \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(1+a-b)_n \, n!} \left[\frac{x}{\left(1+\sqrt{1-x}\right)^2}\right]^n \tag{21}$$

Using (5) in the inner series on left hand side of (21); we get required result (14).

**Proof of 3.3:**

Taking, $A_m = \dfrac{(a)_m \left(1+\frac{a}{2}\right)_m (b)_m (c)_m}{\left(\frac{a}{2}\right)_m (1+a-b)_m (1+a-c)_m}$ $2d = 1+a$ and $y = -1$ in (2), we have

$$\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}+\frac{a}{2}\right)_n\left(\frac{a}{2}\right)_n x^n}{(1+a)_n\, n!}\sum_{m=0}^{n}\frac{(a)_m\left(1+\frac{a}{2}\right)_m (b)_m (c)_m (-n)_m (1)^m}{\left(\frac{a}{2}\right)_m (1+a-b)_m (1+a-c)_m (1+a+n)_m\, m!}$$

$$=\left[\frac{1+\sqrt{1-x}}{2}\right]^{-a}\sum_{n=0}^{\infty}\frac{(a)_n (b)_n (c)_n}{(1+a-b)_n (1+a-c)_n\, n!}\left[\frac{-x}{\left(1+\sqrt{1-x}\right)^2}\right]^n \tag{22}$$

Using (6) in the inner series on left hand side of (22); we get required result (15).

**Proof of 3.4:**

Taking, $A_m=\dfrac{(a)_m\left(1+\frac{a}{2}\right)_m (b)_m (c)_m (d)_m (e)_m}{\left(\frac{a}{2}\right)_m (1+a-b)_m (1+a-c)_m (1+a-d)_m (1+a-e)_m}$ $2d=1+a$

And $y=-1$ in (2.2), we have

$$\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}+\frac{a}{2}\right)_n\left(\frac{a}{2}\right)_n x^n}{(1+a)_n\, n!}\sum_{m=0}^{n}\frac{(a)_m\left(1+\frac{a}{2}\right)_m (b)_m (c)_m (d)_m (e)_m (-n)_m (1)^m}{\left(\frac{a}{2}\right)_m (1+a-b)_m (1+a-c)_m (1+a-d)_m (1+a-e)_m (1+a+n)_m\, m!}$$

$$=\left[\frac{1+\sqrt{1-x}}{2}\right]^{-a}\sum_{n=0}^{\infty}\frac{(a)_n (b)_n (c)_n (d)_n (e)_n}{(1+a-b)_n (1+a-c)_n (1+a-d)_n (1+a-e)_n\, n!}\left[\frac{-x}{\left(1+\sqrt{1-x}\right)^2}\right]^n \tag{23}$$

Using (7) in the inner series on left hand side of (23); we get required result (16).

**Proof of 3.5:**

Taking $A_m=\dfrac{(a)_m\left(1+\frac{a}{2}\right)_m}{\left(\frac{a}{2}\right)_m}$, $2d=1+a$ and $y=1$ in (2), we have

$$\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}+\frac{a}{2}\right)_n\left(\frac{a}{2}\right)_n x^n}{(1+a)_n\, n!}\sum_{m=0}^{n}\frac{(a)_m\left(1+\frac{a}{2}\right)_m (-n)_m (-1)^m}{\left(\frac{a}{2}\right)_n (1+a+n)_m\, m!}$$

$$=\left[\frac{1+\sqrt{1-x}}{2}\right]^{-a}\sum_{n=0}^{\infty}\frac{(a)_n}{n!}\left[\frac{x}{\left(1+\sqrt{1-x}\right)^2}\right]^n \tag{24}$$

Using (8) in the inner series on left hand side of (24); we get required result (17).

**Proof of 3.6:**

Taking, $A_m=\dfrac{(a)_m\left(1+\frac{a}{2}\right)_m (b)_m}{\left(\frac{a}{2}\right)_m (1+a-b)_m}$, $2d=1+a$ and $y=-1$ in (2), we have

$$\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}+\frac{a}{2}\right)_n\left(\frac{a}{2}\right)_n x^n}{(1+a)_n\, n!}\sum_{m=0}^{n}\frac{(a)_m\left(1+\frac{a}{2}\right)_m (b)_m (-n)_m (1)^m}{\left(\frac{a}{2}\right)_m (1+a-b)_m (1+a+n)_m\, m!}$$

$$=\left[\frac{1+\sqrt{1-x}}{2}\right]^{-a}\sum_{n=0}^{\infty}\frac{(a)_n (b)_n}{(1+a-b)_n\, n!}\left[\frac{-x}{\left(1+\sqrt{1-x}\right)^2}\right]^n \tag{25}$$

Using (9) in the inner series on left hand side of (25); we get required result (18).

**Proof of 3.7:**

Taking $A_m = \dfrac{(a)_m \left(1+\frac{a}{2}\right)_m}{\left(\frac{a}{2}\right)_m}$, $2d = 1+a$ and $y = -1$ in (3), we have

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}+\frac{a}{2}\right)_n \left(1+\frac{a}{2}\right)_n x^n}{(1+a)_n\, n!} \sum_{m=0}^{n} \frac{(a)_m \left(1+\frac{a}{2}\right)_m (-n)_m}{\left(\frac{a}{2}\right)_m (1+a+n)_m\, m!}$$

$$= \frac{1}{\sqrt{1-x}} \left[\frac{1+\sqrt{1-x}}{2}\right]^{-a} \sum_{m=0}^{\infty} \frac{(a)_n \left(1+\frac{a}{2}\right)_n}{\left(\frac{a}{2}\right)_n n!} \left[\frac{-x}{\left(1+\sqrt{1-x}\right)^2}\right]^n \tag{26}$$

Using (10) in the inner series on left hand side of (26); we get required result (19).

## 5. Special cases

i) If we take $x = 1$, in (13), we have

$$ {}_3F_2\left[\begin{matrix} \frac{a}{2}, \frac{1}{2}+\frac{a}{2}, 1+\frac{a}{2}-b; & 1 \\ 1+\frac{a}{2}, 1+a-b; & \end{matrix}\right] = 2^a\, {}_3F_2\left[\begin{matrix} a, \frac{a}{2}, b; & -1 \\ 1+\frac{a}{2}, 1+a-b; & \end{matrix}\right]$$

ii) If we take, $x=1$ in (14), we have

$$ {}_2F_1\left[\begin{matrix} \frac{a}{2}, \frac{1}{2}+\frac{a}{2}; & 1 \\ 1+a-b; & \end{matrix}\right] = \left(\frac{1+\sqrt{1-x}}{2}\right)^{-a} {}_2F_1\left[\begin{matrix} a, b; & 1 \\ 1+a-b; & \end{matrix}\right] \tag{27}$$

Using Gauss theorem (11) in the right hand side of (27), we have

$$ {}_2F_1\left[\begin{matrix} \frac{a}{2}, \frac{1}{2}+\frac{a}{2}; & 1 \\ 1+a-b; & \end{matrix}\right] = 2^a\, \Gamma\left[\begin{matrix} 1+a-b, 1-2b \\ 1-b, 1+a-2b \end{matrix}\right]$$

Provided that $Rl\ (1+a-b) > 0, Rl(1-2b) > 0, Rl(1-b) > 0$ and, $Rl(1+a-2b) > 0$,

iii) If we take, $x=1$, $b=1+\frac{a}{2}$ and $c=-n$ in (15), we have

$$ {}_2F_1\left[\begin{matrix} \frac{1}{2}+\frac{a}{2}, \frac{a}{2}+n; & 1 \\ 1+a+n; & \end{matrix}\right] = 2^a\, {}_3F_2\left[\begin{matrix} a, 1+\frac{a}{2}, -n; & -1 \\ \frac{a}{2}, 1+a+n; & \end{matrix}\right] \tag{28}$$

Summing the right hand side of (28) by making use of (8), we have

$$ 2F1\left[\begin{matrix} \frac{1}{2}+\frac{a}{2}, \frac{a}{2}+n; 1 \\ 1+a+n; \end{matrix}\right] = 2^a \frac{(1+a)_n}{\left(\frac{1}{2}+\frac{a}{2}\right)_n} \tag{29}$$

iv) Using Kummer's theorem (12) after putting x=1in (18), we have

$$ {}_2F_1\left[\begin{matrix} \frac{a}{2}, \frac{1}{2}+\frac{a}{2}-b; & 1 \\ 1+a-b; & \end{matrix}\right] = 2^a\, \Gamma\left[\begin{matrix} 1+a-b, 1+\frac{a}{2} \\ 1+a, 1+\frac{a}{2}-b \end{matrix}\right]$$

v) The equation (17) can be rewritten as under:

$$ {}_1F_0\left[\begin{matrix} a; \dfrac{x}{\left(1+\sqrt{1-x}\right)^2} \\ -; \end{matrix}\right] = \left[\frac{1+\sqrt{1-x}}{2}\right]^a (1-x)^{-\frac{a}{2}}$$

On simplification, we have

$$ {}_1F_0\left[\begin{matrix} a; \dfrac{x}{\left(1+\sqrt{1-x}\right)^2} \\ -; \end{matrix}\right] = 2^{-a}\left[1+\frac{1}{\sqrt{1-x}}\right]^a $$

Several other special cases could also be deduced.

## 6. Conclusion

In this paper, we obtained some transformation formulae of ordinary hyper geometric functions having different arguments i.e. argument of hyper geometric function on left hand side is different from argument on right hand side. This is the main beauty of our transformation formulae. Also, we provided some special cases of our main results. A number of other interesting and useful results can also be recorded by applying same technique.

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