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Research Paper

Static deformation due to two-dimensional seismic sources embedded in an isotropic half-space in smooth contact with an orthotropic half-space

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Abstract

Closed form analytical expressions for displacements and stresses at any point of a two-phase medium consisting of a homogeneous, isotropic, perfectly elastic half-space in smooth contact with a homogeneous, orthotropic, perfectly elastic half-space caused by two-dimensional seismic sources located in the isotropic half-space are obtained. The method consists of first finding the integral expressions for two half-spaces in smooth contact from the corresponding expressions for an unbounded medium by applying suitable boundary conditions at the interface and then evaluating the integrals analytically. Here, we discuss the horizontal and vertical displacements for vertical dip-slip fault numerically. Numerical computations indicate that the deformation field due to a source in an isotropic half-space in smooth contact with an anisotropic half-space may differ substantially from the deformation field when both the half-spaces are isotropic.

Keywords: *Isotropic Half-Space, Orthotropic Half-Space, Static-Deformation, Smooth Contact.*

1. Introduction

The computation of the deformation and of the stresses generated by a dislocation source in an elastic infinite domain is a classical problem that has received greater attention and that has found complete analytical solution only in a few known cases. Steketee (1958a, b) [1], [2] applied the elasticity theory of dislocations. Steketee dealt with a semi-infinite, non-gravitating, isotropic and homogenous medium. Homogeneity means that the medium is uniform throughout, whereas isotropy specifies that the elastic properties of the medium are independent of direction. Maruyama (1966) [3] calculated all sets of Green's function for obtaining displacements and stresses around faults in a half space. Freund and Barnett (1976) [4] obtained two dimensional surface deformation due to dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable. Singh and Garg (1986) [5] obtained the integral expressions for the Airy stress function in an unbounded medium due to various two-dimensional seismic sources. Singh ET. al (1991) [6] followed a similar procedure to obtain closed form analytical expression for the displacements and stresses at any point of either of two homogenous, isotropic, perfectly elastic half spaces in welded contact due to two-dimensional sources.
Using the concept of orthotropic media, Singh (1986) [7], Garg and Singh (1987) [8], Pan (1989a) [9] studied the static deformation of a transversely isotropic multilayered half-space by surface loads. The problem of the static deformation of a transversely isotropic multilayered half-space by buried sources has been discussed by Pan (1989b) [10]. Static deformation of an orthotropic multilayered elastic half-space by two-dimensional surface loads has been investigated by Garg et al. (1991) [11]. Singh et al (1991) [12] obtained closed form analytical expression for displacements and stress at any point of a two phase medium consisting of a homogenous, isotropic, perfectly elastic half-space in welded contact with a homogeneous, orthotropic, perfectly elastic half-space caused by two-dimensional seismic sources located in the isotropic half-space. Rani et al (2009) [13] obtained the closed-form expressions for the elastic residual

field caused by a long dip-slip fault of finite width located in an isotropic half-space any point isotropic half-space in welded contact with orthotropic half-space. Recently, Singh et al. (2013) [14] obtained closed-form analytical expressions for displacement and stress field at any points of the two homogeneous, isotropic, perfectly elastic half-spaces in smooth contact caused by various two-dimensional sources embedded in one of the half-spaces. Up to now, there is very little literature for isotropic and orthotropic half-spaces in smooth contact.

Therefore, In the present paper, assuming the smooth contact for isotropic and orthotropic half-spaces, and following Singh et al. (1991) [12], we study the static deformation of two phase medium consisting of a homogenous, isotropic, perfectly elastic half-space in smooth contact with a homogeneous, orthotropic, perfectly elastic half-space caused by two-dimensional seismic sources located in the isotropic half-space. Two orthotropic materials namely Topaz and Barytes have been considered for numerical computations in case of vertical dip-slip fault. Numerical results show that the effect of anisotropy on the displacement field is more pronounced when the observer is in the orthotropic half-space.

2. Theory

Let the Cartesian co-ordinates be denoted by (x, y, z) with z-axis vertically upwards. Consider two homogeneous, perfectly elastic half-spaces which are in smooth contact along the plane $z = 0$. The upper half-space (z > 0) is assumed to be isotropic with stress-strain relation

$$p_{ij} = 2\mu\left[e_{ij} + \frac{\sigma}{1-2\sigma}\delta_{ij}e_{kk}\right], \quad (i,j = 1,2,3) \tag{1}$$

Where, p_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, μ is the shear modulus and σ is Poisson's ratio.

The lower half-space (z < 0) is assumed to be orthotropic with stress-strain relation.

$$\begin{bmatrix} p'_{11} \\ p'_{22} \\ p'_{33} \\ p'_{23} \\ p'_{31} \\ p'_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e'_{11} \\ e'_{22} \\ e'_{33} \\ 2e'_{23} \\ 2e'_{31} \\ 2e'_{12} \end{bmatrix} \tag{2}$$

We consider a two dimensional approximation in which displacement component u_1,u_2,u_3 are independent of x so that $\partial/\partial x \equiv 0$. Under this assumption the plane strain problem $(u_1 = 0)$ and anti-strain problem $(u_2 = 0$ and $u_3 = 0)$ are decoupled and therefore, can be solved separately. The plane strain problem for an isotropic medium can be solved in terms of Airy stress function U such that

$$p_{22} = \frac{\partial^2 U}{\partial z^2}, \quad p_{33} = \frac{\partial^2 U}{\partial y^2}, \quad p_{23} = -\frac{\partial^2 U}{\partial y \partial z} \tag{3}$$

$$\nabla^2\nabla^2 U = 0 \tag{4}$$

The plane strain problem for an orthotropic medium can be solved in terms of the Airy stress function U^* such that (Garg et al (1991))

$$p'_{22} = \frac{\partial^2 U^*}{\partial z^2}, \quad p'_{33} = \frac{\partial^2 U^*}{\partial y^2}, \quad p'_{23} = -\frac{\partial^2 U^*}{\partial y \partial z} \tag{5}$$

$$\left(a^2\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\left(b^2\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)U^* = 0 \tag{6}$$

$$a^2 + b^2 = \frac{(c_{22}c_{33} - c_{23}^2 - 2c_{23}c_{44})}{c_{33}c_{44}}, \quad a^2b^2 = \frac{c_{22}}{c_{33}} \tag{7}$$

For an isotropic medium,

$$c_{11} = c_{22} = c_{33} = \frac{2\mu(1-\sigma)}{1-2\sigma}$$

$$c_{12} = c_{13} = c_{23} = \frac{2\mu\sigma}{1-2\sigma}$$

$$c_{44} = c_{55} = c_{66} = \mu \tag{8}$$

This yields $a^2 = b^2 = 1$ and equation (6) reduces to equation (4)

Let there be a line source parallel to the x-axis passing through the point (0,0,h) of the upper half-space z >0 . As shown by Singh and Garg (1986) [5], the Airy stress function U_0 for a line source parallel to the x-axis passing through the point (0,0,h) in an unbounded, isotropic medium can be expressed in the form.

$$U_0 = \int_0^\infty [(L_0 + M_0 k|z-h|)\sin ky + (P_0 + Q_0 k|z-h|)\cos ky]k^{-1}\mathrm{e}^{-k|z-h|}dk, \tag{9}$$

Where, the source coefficients $L_0,\ M_0,\ P_0,\ Q_0$ are independent of k. Singh and Garg (1986) [5] obtained these source coefficients for various seismic sources. These are listed in Appendix II for ready reference. We use the notations of Ben-Menahem and Singh (1981) [15] for labelling various sources.

For a line source parallel to the x-axis acting at the point (0,0,h) of medium I (z > 0) which is in smooth contact with medium II (z < 0), the Airy stress function in medium I is a solution of Eq.(4) and may be taken to the form

$$U = U_0 + \int_0^\infty [(L_1 + M_1 kz)\sin ky + (P_1 + Q_1 kz)\cos ky]k^{-1}\mathrm{e}^{-kz}dk, \tag{10}$$

The Airy stress function in medium II is a solution of Eq.(6) and is of the form (assuming $a \neq b$)

$$U^* = \int_0^\infty [(L_2\mathrm{e}^{akz} + M_2\mathrm{e}^{bkz})\sin ky + (P_2\mathrm{e}^{akz} + Q_2\mathrm{e}^{bkz})\cos ky]k^{-1}dk, \tag{11}$$

The constants $L_1,\ M_1,\ L_2,\ M_2$ etc. are to be determined from the boundary conditions.

Since the half-spaces are assumed to be in smooth contact along the plane z = 0, the boundary conditions are

$$p_{33} = p'_{33}, \quad u_3 = u'_3, \quad p_{23} = 0, \quad p'_{23} = 0 \tag{12}$$

at z = 0. The displacements for the isotropic medium in terms of Airy stress function U are given by Rani et al., (1991) [16].

$$2\mu u_2 = -\frac{\partial U}{\partial y} + \frac{1}{2\alpha}\int (p_{22} + p_{33})dy, \tag{13a}$$

$$2\mu u_3 = -\frac{\partial U}{\partial z} + \frac{1}{2\alpha}\int (p_{22} + p_{33})dz, \tag{13b}$$

where , $$\alpha = \frac{1}{2(1-\sigma)} \tag{14}$$

The displacements for the orthotropic medium are given by Garg et al., (1991) [11]

$$u'_2 = \frac{1}{\Delta}\int (c_{33}p'_{22} - c_{23}p'_{33})dy, \tag{15a}$$

$$u'_3 = \frac{1}{\Delta}\int (c_{22}p'_{33} - c_{23}p'_{22})dz, \tag{15b}$$

Where, $$\Delta = c_{22}c_{33} - c_{23}^2 \tag{16}$$

From Eqs. (3), (9), (10), (13a) and (13b), we will obtain the following expressions for stresses and displacements for an isotropic half-space

$$p_{22} = \int_0^\infty \left[(L_0 - 2M_0 + M_0 k|z-h|)\mathrm{e}^{-k|z-h|} + (L_1 - 2M_1 + M_1 kz)\mathrm{e}^{-kz}\right]\sin ky\, kdk + \int_0^\infty \left[(P_0 - 2Q_0 + Q_0 k|z-h|)\mathrm{e}^{-k|z-h|} + (P_1 - 2Q_1 + Q_1 kz)\mathrm{e}^{-kz}\right]\cos ky\, kdk \tag{17}$$

$$p_{23} = \int_0^\infty \left[\pm(L_0 - M_0 + M_0 k|z-h|)\mathrm{e}^{-k|z-h|} + (L_1 - M_1 + M_1 kz)\mathrm{e}^{-kz}\right]\cos ky\, kdk + \int_0^\infty \left[\mp(P_0 - Q_0 + Q_0 k|z-h|)\mathrm{e}^{-k|z-h|} - (P_1 - Q_1 + Q_1 kz)\mathrm{e}^{-kz}\right]\sin ky\, kdk \tag{18}$$

$$p_{33} = -\int_0^\infty \left[(L_0 + M_0 k|z-h|)\mathrm{e}^{-k|z-h|} + (L_1 + M_1 kz)\mathrm{e}^{-kz}\right]\sin ky\, kdk - \int_0^\infty \left[(P_0 + Q_0 k|z-h|)\mathrm{e}^{-k|z-h|} + (P_1 + Q_1 kz)\mathrm{e}^{-kz}\right]\cos ky\, kdk \tag{19}$$

$$2\mu u_2 = \int_0^\infty \left[\left(-L_0 + \frac{M_0}{\alpha} - M_0 k|z-h|\right)\mathrm{e}^{-k|z-h|} - (L_1 - \frac{M_1}{\alpha} + M_1 kz)\mathrm{e}^{-kz}\right]\cos ky\, kdk$$
$$+ \int_0^\infty \left[\left(P_0 - \frac{Q_0}{\alpha} + Q_0 k|z-h|\right)\mathrm{e}^{-k|z-h|} + (P_1 - \frac{Q_1}{\alpha} + Q_1 kz)\mathrm{e}^{-kz}\right]\sin ky\, kdk \quad (20)$$
$$2\mu u_3 = \int_0^\infty \left[\pm\left(L_0 - M_0 + \frac{M_0}{\alpha} + M_0 k|z-h|\right)\mathrm{e}^{-k|z-h|} + (L_1 - M_1 + \frac{M_1}{\alpha} + M_1 kz)\mathrm{e}^{-kz}\right]\sin ky\, kdk$$
$$+ \int_0^\infty \left[\pm\left(P_0 - Q_0 + \frac{Q_0}{\alpha} + Q_0 k|z-h|\right)\mathrm{e}^{-k|z-h|} + (P_1 - Q_1 + \frac{Q_1}{\alpha} + Q_1 kz)\mathrm{e}^{-kz}\right]\cos ky\, kdk \quad (21)$$

In Eqs. (18) And (21), the upper sign is for z > h and the lower sign is for 0 < z < h.

Similarly, from Eqs. (5), (11), (15a), and (15b), we will obtain the following expressions for stresses and displacements for an orthotropic half-space

$$p_{22}' = \int_0^\infty [(a^2 L_2 \mathrm{e}^{akz} + b^2 M_2 \mathrm{e}^{bkz})\sin ky + (a^2 P_2 \mathrm{e}^{akz} + b^2 Q_2 \mathrm{e}^{bkz})\cos ky]k\, dk \quad (22)$$
$$p_{23}' = \int_0^\infty [-(a L_2 \mathrm{e}^{akz} + b M_2 \mathrm{e}^{bkz})\cos ky + (a P_2 \mathrm{e}^{akz} + b Q_2 \mathrm{e}^{bkz})\sin ky]k\, dk \quad (23)$$
$$p_{33}' = -\int_0^\infty [(L_2 \mathrm{e}^{akz} + M_2 \mathrm{e}^{bkz})\sin ky + (P_2 \mathrm{e}^{akz} + Q_2 \mathrm{e}^{bkz})\cos ky]k\, dk \quad (24)$$
$$u_2' = \int_0^\infty [-(r_1 L_2 \mathrm{e}^{akz} + r_2 M_2 \mathrm{e}^{bkz})\cos ky + (r_1 P_2 \mathrm{e}^{akz} + r_2 Q_2 \mathrm{e}^{bkz})\sin ky]\, dk \quad (25)$$
$$u_3' = -\int_0^\infty [(s_1 L_2 \mathrm{e}^{akz} + s_2 M_2 \mathrm{e}^{bkz})\sin ky \ + (s_1 P_2 \mathrm{e}^{akz} + s_2 Q_2 \mathrm{e}^{bkz})\cos ky]\, dk \quad (26)$$

where,

$$r_1 = \frac{c_{33}a^2 + c_{23}}{\Delta}, \qquad r_2 = \frac{c_{33}b^2 + c_{23}}{\Delta},$$
$$s_1 = \frac{c_{23}a^2 + c_{22}}{a\Delta}, \qquad s_2 = \frac{c_{23}b^2 + c_{22}}{b\Delta}, \qquad \Delta = (c_{22}c_{33} - c_{23}^2), \quad (27)$$

It is noticed from appendix for source coefficients that the coefficientsL_0, M_0, P_0, Q_0 might have different values for $z > h$ and $z < h$; let L^-, M^-, P^-, and Q^- be the values of L_0, M_0, P_0 and Q_0 respectively, valid for $z < h$. The boundary conditions (12) give the following system of equations:

$$(L^- + M^- kh)\mathrm{e}^{-kh} + L_1 = L_2 + M_2$$
$$(P^- + Q^- kh)\mathrm{e}^{-kh} + P_1 = P_2 + Q_2$$
$$-\left(L^- - M^- + \frac{M^-}{\alpha} + M^- kh\right)\mathrm{e}^{-kh} + L_1 - M_1 + \frac{M_1}{\alpha} = -2\mu(s_1 L_2 + s_2 M_2)$$
$$-\left(P^- - Q^- + \frac{Q^-}{\alpha} + Q^- kh\right)\mathrm{e}^{-kh} + P_1 - Q_1 + \frac{Q_1}{\alpha} = -2\mu(s_1 P_2 + s_2 Q_2)$$
$$-(L^- - M^- + M^- kh)\mathrm{e}^{-kh} + L_1 - M_1 = 0$$
$$-(P^- - Q^- + Q^- kh)\mathrm{e}^{-kh} + P_1 - Q_1 = 0$$
$$aL_2 + bM_2 = 0$$
$$aP_2 + bQ_2 = 0 \quad (28)$$

Solving the system for L_1, M_1, P_1, Q_1 L_2, $M_{2,}$ P_2, Q_2, we get

$$L_1 = A_1(L^- + M^- kh)\mathrm{e}^{-kh}$$
$$M_1 = [M^- - 2A_2(L^- + M^- kh)]\mathrm{e}^{-kh}$$
$$L_2 = -2B_2(L^- + M^- kh)\mathrm{e}^{-kh}$$
$$M_2 = 2B_1(L^- + M^- kh)\mathrm{e}^{-kh}$$
$$P_1 = A_1(P^- + Q^- kh)\mathrm{e}^{-kh}$$
$$Q_1 = [Q^- - 2A_2(P^- + Q^- kh)]\mathrm{e}^{-kh}$$
$$P_2 = -2B_2(P^- + Q^- kh)\mathrm{e}^{-kh}$$
$$Q_2 = 2B_1(P^- + Q^- kh)\mathrm{e}^{-kh} \quad (29)$$

where,

$$A_1 = \frac{(a-b) - 2\mu\alpha(as_2 - bs_1)}{G},$$
$$A_2 = \frac{2\mu\alpha(as_2 - bs_1)}{G},$$
$$B_1 = \frac{a}{G}, \quad B_2 = \frac{b}{G}$$
$$G = (a-b) + 2\mu\alpha(as_2 - bs_1) \tag{30}$$

Putting the values of the constants $L_1,\ M_1,\ P_1$, etc. in Eqs. (10) And (11), we get the integral expressions for the Airy stress function in the two media. These integrals can be evaluated analytically using the standard integrals given in appendix. The displacements and stresses can be obtained similarly. Using the notation ($z \neq h,\ az \neq h, bz \neq h$)

$$R^2 = y^2 + (z-h)^2, \quad T^2 = y^2 + (h - az)^2$$
$$S^2 = y^2 + (z+h)^2, \quad H^2 = y^2 + (h - bz)^2 \tag{31}$$

Finally we get, for the isotropic half-space,

$$\begin{aligned} U = {} & L_0 \tan^{-1}\left(\frac{y}{|z-h|}\right) + M_0 \frac{y|z-h|}{R^2} - P_0 \ln R + Q_0 \frac{(z-h)^2}{R^2} + L^- \left[A_1 \tan^{-1}\left(\frac{y}{z+h}\right) - 2A_2 \frac{yz}{S^2}\right] \\ & + M^- \left[\frac{(A_1 h + z)y}{S^2} - \frac{4A_2 hyz(z+h)}{S^4}\right] + P^- \left[-A_1 \ln S - 2A_2 \frac{(z+h)z}{S^2}\right] \\ & + Q^- \left[\frac{(A_1 h + z)(z+h) + 2A_2 hz}{S^2} - \frac{4A_2 hz(z+h)^2}{S^4}\right] \end{aligned} \tag{32}$$

$$\begin{aligned} p_{22} = {} & 2L_0 \frac{y|z-h|}{R^4} + 2M_0 \frac{y|z-h|}{R^4}\left[-3 + \frac{4(z-h)^2}{R^2}\right] + P_0 \frac{1}{R^2}\left[-1 + \frac{2(z-h)^2}{R^2}\right] \\ & + 2Q_0 \frac{1}{R^2}\left[1 - \frac{5(z-h)^2}{R^2} + \frac{4(z-h)^4}{R^4}\right] + 2L^- \frac{y}{S^4}\left[A_1(z+h) + 2A_2(3z+2h) - \frac{8A_2 z(z+h)^2}{S^2}\right] \\ & - 2M^- \frac{y}{S^4}\left[(4A_2 + A_1)h + (3z+2h) - \frac{4(z+h)^2\{(A_1 + 4A_2)h + z\}}{S^2} - 24A_2 \frac{hz(z+h)}{S^2}\right. \\ & \left. + 48A_2 \frac{hz(z+h)^3}{S^4}\right] \\ & - P^- \frac{1}{S^2}\left[4A_2 + A_1 - \frac{2(z+h)\{6A_2 z + (A_1 + 4A_2)(z+h)\}}{S^2} + 16A_2 \frac{z(z+h)^3}{S^4}\right] \\ & + 2Q^- \frac{1}{S^2}\left[1 - \frac{(z+h)\{3(A_1 + 4A_2)h + (5z+2h)\}}{S^2} - 6A_2 \frac{hz}{S^2}\right. \\ & \left. + \frac{4(z+h)^3\{(A_1 + 4A_2)h + z\} + 48A_2 hz(z+h)^2}{S^4} - 48A_2 \frac{hz(z+h)^4}{S^6}\right] \end{aligned} \tag{33}$$

$$\begin{aligned} p_{23} = {} & \mp L_0 \frac{1}{R^2}\left[1 - \frac{2(z-h)^2}{R^2}\right] \pm M_0 \frac{1}{R^2}\left[1 - \frac{8(z-h)^2}{R^2} + \frac{8(z-h)^4}{R^4}\right] \mp 2P_0 \frac{y|z-h|}{R^4} \pm 4Q_0 \frac{y|z-h|}{R^4}\left[1 - \frac{2(z-h)^2}{R^2}\right] \\ & - L^- \frac{1}{S^2}\left[2A_2 + A_1 - \frac{2(z+h)\{6A_2 z + (A_1 + 2A_2)(z+h)\}}{S^2} + 16A_2 \frac{z(z+h)^3}{S^4}\right] \\ & + M^- \frac{1}{S^2}\left[1 - \frac{2(z+h)\{3(A_1 + 2A_2)h + (4z+h)\}}{S^2} - 12A_2 \frac{hz}{S^2}\right. \\ & \left. + \frac{8(z+h)^3\{(A_1 + 4A_2)h + z\} + 96A_2 hz(z+h)^2}{S^4} - 96A_2 \frac{hz(z+h)^4}{S^6}\right] \\ & - 2P^- \frac{y}{S^4}\left[A_1(z+h) + 2A_2(2z+h) - \frac{8A_2 z(z+h)^2}{S^2}\right] \\ & + 2Q^- \frac{y}{S^4}\left[(2A_2 + A_1)h + (2z+h) - \frac{4(z+h)^2\{(A_1 + 2A_2)h + z\}}{S^2} - 24A_2 \frac{hz(z+h)}{S^2}\right. \\ & \left. + 48A_2 \frac{hz(z+h)^3}{S^4}\right] \end{aligned} \tag{34}$$

$$p_{33}=-2L_0\frac{y|z-h|}{R^4}+2M_0\frac{y|z-h|}{R^4}\left[1-\frac{4(z-h)^2}{R^2}\right]+P_0\frac{1}{R^2}\left[1-\frac{2(z-h)^2}{R^2}\right]+2Q_0\frac{(z-h)^2}{R^4}\left[3-\frac{4(z-h)^2}{R^2}\right]$$
$$-2L^-\frac{y}{S^4}\left[A_1(z+h)+2A_2z-\frac{8A_2z(z+h)^2}{S^2}\right]$$
$$+2M^-\frac{y}{S^4}\left[A_1h+z-\frac{4(z+h)^2(A_1h+z)}{S^2}-24A_2\frac{hz(z+h)}{S^2}+48A_2\frac{hz(z+h)^3}{S^4}\right]$$
$$+P^-\frac{1}{S^2}\left[A_1-\frac{2(z+h)\{6A_2z+A_1(z+h)\}}{S^2}+16A_2\frac{z(z+h)^3}{S^4}\right]$$
$$+2Q^-\frac{1}{S^2}\left[\frac{3(z+h)(A_1h+z)}{S^2}+6A_2\frac{hz}{S^2}-\frac{4(z+h)^3(A_1h+z)+48A_2hz(z+h)^2}{S^4}\right.$$
$$\left.+48A_2\frac{hz(z+h)^4}{S^6}\right] \tag{35}$$

$$2\mu u_2=-L_0\frac{|z-h|}{R^2}+M_0\frac{|z-h|}{R^2}\left[1+\frac{1}{\alpha}-\frac{2(z-h)^2}{R^2}\right]+P_0\frac{y}{R^2}+Q_0\frac{y}{R^2}\left[-\frac{1}{\alpha}+\frac{2(z-h)^2}{R^2}\right]$$
$$-L^-\frac{1}{S^2}\left[(A_1+\frac{2A_2}{\alpha})(z+h)+2A_2z-\frac{4A_2z(z+h)^2}{S^2}\right]$$
$$+M^-\frac{y}{S^2}\left[(A_1+\frac{2A_2}{\alpha}+\frac{1}{\alpha})h+z(1+\frac{1}{\alpha})-\frac{2(z+h)^2\{(A_1+\frac{2A_2}{\alpha})h+z\}}{S^2}-12A_2\frac{hz(z+h)}{S^2}\right.$$
$$\left.+16A_2\frac{hz(z+h)^3}{S^4}\right]+P^-\frac{y}{S^2}\left[A_1+\frac{2A_2}{\alpha}-4A_2\frac{z(z+h)}{S^2}\right]$$
$$+2Q^-\frac{y}{S^2}\left[-\frac{1}{\alpha}+\frac{2(z+h)\{(A_1+\frac{2A_2}{\alpha})h+z\}}{S^2}+4A_2\frac{hz}{S^2}-16A_2\frac{hz(z+h)^2}{S^4}\right] \tag{36}$$

$$2\mu u_3=\pm L_0\frac{y}{R^2}\pm M_0\frac{y}{R^2}\left[-1+\frac{1}{\alpha}+\frac{2(z-h)^2}{R^2}\right]+P_0\frac{|z-h|}{R^2}+Q_0\frac{|z-h|}{R^2}\left[\frac{1}{\alpha}-2+\frac{2(z-h)^2}{R^2}\right]$$
$$+L^-\frac{y}{S^2}\left[A_1-\frac{2A_2}{\alpha}+2A_2-\frac{4A_2z(z+h)}{S^2}\right]$$
$$+M^-\frac{y}{S^2}\left[-1+\frac{1}{\alpha}+\frac{2((z+h)\{(A_1-\frac{2A_2}{\alpha})h+z\}}{S^2}+4A_2\frac{h(2z+h)}{S^2}-16A_2\frac{hz(z+h)^2}{S^4}\right]$$
$$+P^-\frac{1}{S^2}\left[\left(A_1-\frac{2A_2}{\alpha}\right)(z+h)+2A_2(2z+h)-4A_2\frac{z(z+h)^2}{S^2}\right]$$
$$+Q^-\frac{1}{S^2}\left[\left(\frac{1}{\alpha}-1\right)(z+h)-z-\left(A_1-\frac{2A_2}{\alpha}+2A_2\right)h+\frac{2(z+h)^2\{(A_1+2A_2-\frac{2A_2}{\alpha})h+z\}}{S^2}\right.$$
$$\left.+12A_2\frac{hz(z+h)}{S^2}-16A_2\frac{hz(z+h)^3}{S^4}\right] \tag{37}$$

For the orthotropic medium,

$$U^*=2L^-\left[-B_2\tan^{-1}\left(\frac{y}{h-az}\right)+B_1\tan^{-1}\left(\frac{y}{h-bz}\right)\right]+2M^-\left[hy\left(\frac{-B_2}{T^2}+\frac{B_1}{H^2}\right)\right]$$
$$-2P^-[-B_2\ln T+B_1\ln H]+2Q^-\left[\frac{-B_2h(h-az)}{T^2}+\frac{B_1h(h-bz)}{H^2}\right] \tag{38}$$

$$p'_{22} = 4L^{-}y\left[\frac{-B_2a^2(h-az)}{T^4}+\frac{B_1b^2(h-bz)}{H^4}\right]+4M^{-}y\left[\frac{B_2a^2h}{T^4}-\frac{B_1b^2h}{H^4}-\frac{4B_2a^2h(h-az)^2}{T^6}+\frac{4B_1b^2h(h-bz)^2}{H^6}\right]$$
$$+2P^{-}\left[\frac{B_2a^2}{T^2}-\frac{B_1b^2}{H^2}-\frac{2B_2a^2(h-az)^2}{T^4}+\frac{2B_1b^2(h-bz)^2}{H^4}\right]$$
$$+2Q^{-}\left[\frac{6B_2ha^2(h-az)}{T^4}-\frac{6B_1hb^2(h-bz)}{H^4}-\frac{8B_2a^2h(h-az)^3}{T^6}+\frac{8B_1b^2h(h-bz)^3}{H^6}\right] \tag{39}$$

$$p'_{23} = 2L^{-}\left[-\frac{B_2a}{T^2}+\frac{B_1b}{H^2}+\frac{2B_2ah(h-az)^2}{T^4}-\frac{2B_1bh(h-bz)^2}{H^4}\right]$$
$$+2M^{-}\left[-\frac{6B_2ah(h-az)}{T^4}+\frac{6B_1bh(h-bz)}{H^4}+\frac{8B_2ah(h-az)^3}{T^6}-\frac{8B_1bh(h-bz)^3}{H^6}\right]$$
$$+4P^{-}y\left[\frac{-B_2a(h-az)}{T^4}+\frac{B_1b(h-bz)}{H^4}\right]$$
$$+4Q^{-}y\left[\frac{B_2ah}{T^4}-\frac{B_1bh}{H^4}-\frac{4B_2ah(h-az)^2}{T^6}+\frac{4B_1bh(h-bz)^2}{H^6}\right] \tag{40}$$

$$p'_{33} = 4L^{-}y\left[\frac{B_2(h-az)}{T^4}-\frac{B_1(h-bz)}{H^4}\right]-4M^{-}y\left[\frac{B_2h}{T^4}-\frac{B_1h}{H^4}-\frac{4B_2h(h-az)^2}{T^6}+\frac{4B_1h(h-bz)^2}{H^6}\right]$$
$$-2P^{-}\left[\frac{B_2}{T^2}-\frac{B_1}{H^2}-\frac{2B_2(h-az)^2}{T^4}+\frac{2B_1(h-bz)^2}{H^4}\right]$$
$$-2Q^{-}\left[\frac{6B_2h(h-az)}{T^4}-\frac{6B_1h(h-bz)}{H^4}-\frac{8B_2h(h-az)^3}{T^6}+\frac{8B_1h(h-bz)^3}{H^6}\right] \tag{41}$$

$$u'_2 = 2L^{-}\left[\frac{B_2r_1(h-az)}{T^2}-\frac{B_1r_2(h-bz)}{H^2}\right]-2M^{-}\left[\frac{B_2r_1h}{T^2}-\frac{B_1r_2h}{H^2}-\frac{2B_2r_1h(h-az)^2}{T^4}+\frac{2B_1r_2h(h-bz)^2}{H^4}\right]$$
$$-2P^{-}y\left[\frac{B_2r_1}{T^2}-\frac{B_1r_2}{H^2}\right]-2Q^{-}y\left[\frac{2B_2r_1h(h-az)}{T^4}-\frac{2B_1r_2h(h-bz)}{H^4}\right] \tag{42}$$

$$u'_3 = 2L^{-}y\left[\frac{B_2s_1}{T^2}-\frac{B_1s_2}{H^2}\right]+2M^{-}y\left[\frac{2B_2s_1h(h-az)}{T^4}-\frac{2B_1s_2h(h-bz)}{H^4}\right]+2P^{-}\left[\frac{B_2s_1(h-az)}{T^2}-\frac{B_1s_2(h-bz)}{H^2}\right]$$
$$-2Q^{-}\left[\frac{B_2s_1h}{T^2}-\frac{B_1s_2h}{H^2}-\frac{2B_2s_1h(h-az)^2}{T^4}+\frac{2B_1s_2h(h-bz)^2}{H^4}\right] \tag{43}$$

3. Particular cases

3.1. Dip- slips dislocation

The Airy stress function due to a line dip-slip fault of arbitrary dip can be expressed in terms of the fields due to a vertical dip-slip and a dip-slip at a 45° dipping fault as given below:

$$U = \mu b' ds\left[\frac{U_{(23)+(32)}}{D_{23}}\cos 2\delta+\frac{U_{(33)-(22)}}{D'_{23}}\sin 2\delta\right], \tag{44}$$

Where *b'* is the slip, δ is dip angle, ds is the width of the dislocation plane and D_{23}=D'_{23}= μbds are the moments of double couple (23) + (32) and (33)−(22) respectively. $U_{(23)+(32)}$ And $U_{(33)-(22)}$ are obtained from equations (35) or (41) on inserting the values of the source coefficients L_0, M_0, P_0, and Q_0 from Table I corresponding to the sources (23) + (32) and (33)−(22) respectively. These yields,

$$U = \frac{\alpha\mu b' ds}{\pi}\left[\cos 2\delta\left\{\pm\frac{y|z-h|}{R^2}-\frac{(A_1h+z)y}{S^2}+\frac{4A_2hyz(z+h)}{S^4}\right\}\right.$$
$$\left.+\sin 2\delta\left\{\frac{(z-h)^2}{R^2}+\frac{(A_1h+z)(z+h)+2A_2hz}{S^2}-\frac{4A_2hz(z+h)^2}{S^4}\right\}\right] \tag{45}$$

$$U^* = \frac{2\alpha\mu b' ds}{\pi}\left[\cos 2\delta\left\{hy\left(\frac{B_2}{T^2} - \frac{B_1}{H^2}\right)\right\} - \sin 2\delta\left\{\frac{B_2 h(h-az)}{T^2} - \frac{B_1 h(h-bz)}{H^2}\right\}\right] \tag{46}$$

Similar expressions can be obtained for stresses and displacements.

3.2. Tensile dislocation

The Airy stress function for a long line tensile source of arbitrary dip can be expressed as a linear combination of

i) $b'dsU_{22}$, the Airy stress function for a vertical tensile fault ($\delta = 90°$) with dislocation in the x_2-direction;
ii) $b'dsU_{33}$, the Airy stress function for a horizontal tensile fault ($\delta = 0°$) with dislocation in the x_3-direction;
iii) $b'dsU_{23}$, the Airy stress function for a vertical dip-slip fault, as given below:

$$U = \mu b' ds[U_{22} sin^2\delta - U_{23} sin2\delta + U_{33} cos^2\delta\ ] \tag{47}$$

Using the values of the source coefficients L_0, M_0, P_0, Q_0, L^-, M^-, P^-, and Q^- given in Table I. Equations (33) and (38) yield the Airy stress function due to a long tensile line source of arbitrary dip parallel to x_1-axis and acting at the point (0,0,h) located in the isotropic half-space welded with the orthotropic half-space in the form ,
For the isotropic half-space,

$$U = \frac{\alpha\mu b' ds}{\pi}\left[-\ln R - A_1 \ln S - \frac{2A_2 z(z+h)}{S^2} + \cos 2\delta\left\{\frac{(z-h)^2}{R^2} + \frac{(A_1 h + z)(z+h) + 2A_2 hz}{S^2} - \frac{4A_2 hz(z+h)^2}{S^4}\right\} - \sin 2\delta\left\{\frac{y(z-h)}{R^2} - \frac{(A_1 h + z)y}{S^2} + \frac{4A_2 hyz(z+h)}{S^4}\right\}\right] \tag{48}$$

And for the orthotropic half space,

$$U^* = \frac{2\alpha\mu b' ds}{\pi}\left[B_2 \ln T - B_1 \ln H - \cos 2\delta\left\{\frac{B_2 h(h-az)}{T^2} - \frac{B_1 h(h-bz)}{H^2}\right\} - \sin 2\delta\left\{hy\left(\frac{B_2}{T^2} - \frac{B_1}{H^2}\right)\right\}\right] \tag{49}$$

Similar results can be obtained for stresses and displacements.

4. Numerical results and discussions

Numerically we will consider the case of vertical dip slip line source. We compare the displacement field due to a long vertical dip-slip line source located at (0, 0, h) in the isotropic half-space welded with orthotropic half-space with the corresponding displacement field when both the half-spaces are isotropic. We assume the isotropic half-space to be poissonian so that $\sigma = 0.25$. For the orthotropic half-space, we use the values of elastic constants given by Love (1944).
For Topaz,
$c_{11} = 2870,\ c_{22} = 3560,\ c_{33} = 3000,$
$c_{12} = 1280,\ c_{23} = 900,\ c_{13} = 860,$
$c_{44} = 1100,\ c_{55} = 1350,\ c_{66} = 1330,$
in terms of a unit of 10^6grammes wt/cm^2, this yields a = 1.2992 and b = 0.8385.

For Barytes,
$c_{11} = 907,\ c_{22} = 800,\ c_{33} = 1074,$
$c_{12} = 468,\ c_{23} = 273,\ c_{13} = 275,$
$c_{44} = 122,\ c_{55} = 293,\ c_{66} = 283,$
in terms of a unit of 10^6grammes wt/cm^2, this yields a = 2.3118 and b= 0.3735.

When the lower half-space is also isotropic,

$$c_{11} = c_{22} = c_{33} = \frac{2\mu'(1-\sigma')}{1-2\sigma'}$$

$$c_{12} = c_{13} = c_{23} == \frac{2\mu'\sigma'}{1-2\sigma'}$$

$$c_{44} = c_{55} = c_{66} = \mu'$$

We take $\sigma' = 0.25$ and c_{44}/μ =0.5 for numerical computations.

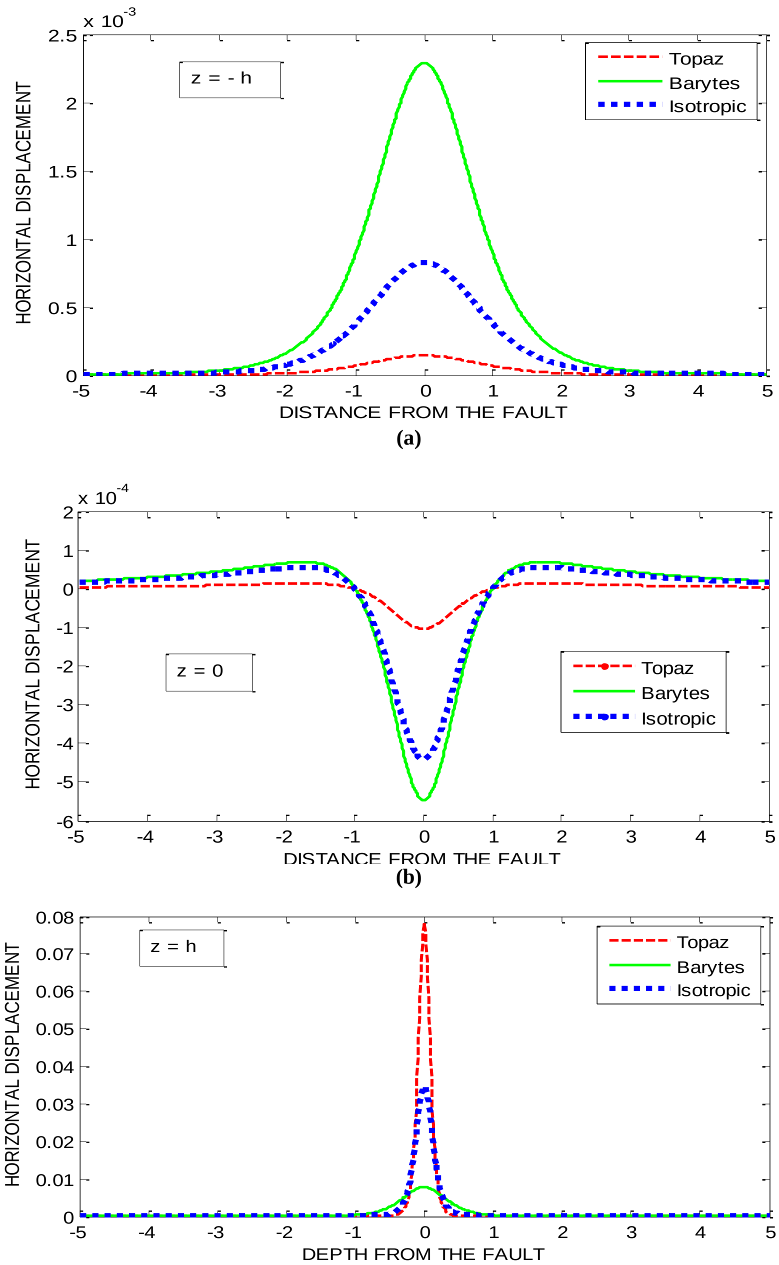
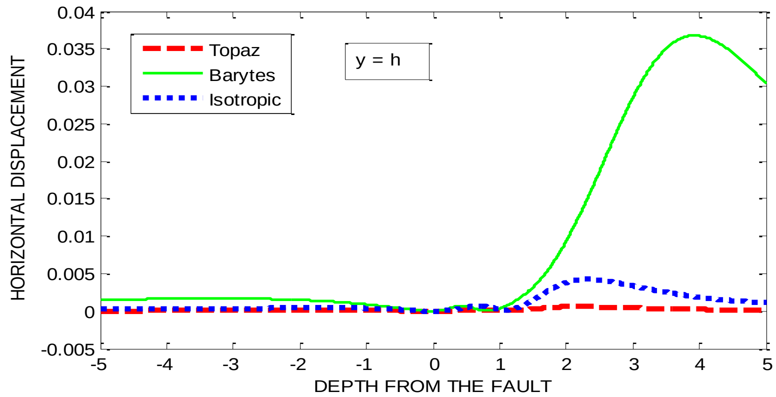
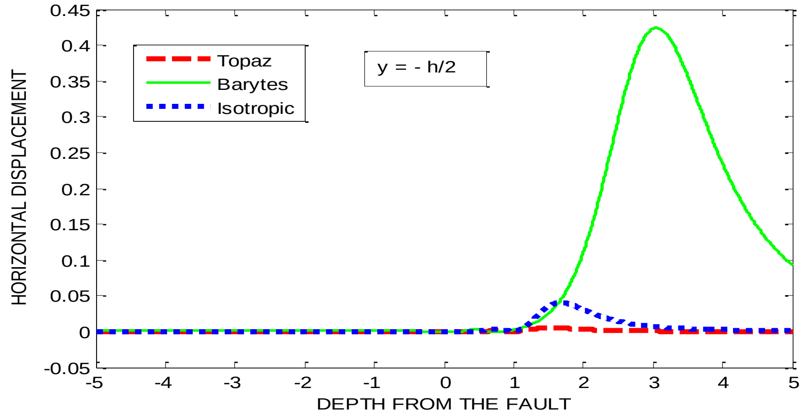


Fig. 1: Variation of Horizontal Displacement ($u_2'/b'ds$) with Distance from the Fault (*y/h*) Due to Vertical Dip-Slip Fault at (a) *z* = - *h*, (b) *z* = 0, (C) *z* = *h*.

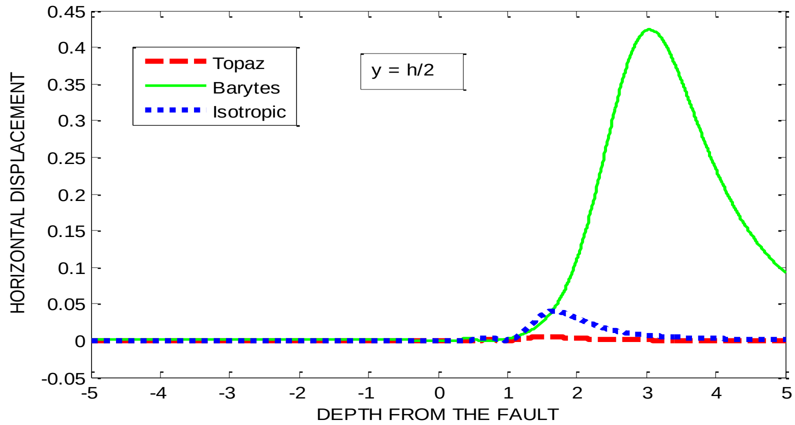
Figures 1a-c are showing the variation of horizontal displacement ($u_2'/b'ds$) with distance from the fault *y/h* due to vertical dip-slip line source. The displacement is symmetric about the line *y* = 0. In fig. 1(a) the observer is in the orthotropic half-space at *z* = - *h*. The horizontal displacement for Barytes varies most significantly in magnitude rather than Topaz from the corresponding one for the isotropic case. In fig 1(b), the observer is at the interface, displacement varies as in fig.1 (a) but with negative magnitude approximately. In fig.1(c), observer is in isotropic half-space at *z* = *h*, horizontal displacement attains maximum value at origin and tends to zero as *y* approaches to infinity for all cases. It is observed that effect of anisotropy on the displacement field is more pronounced when the observer is in the orthotropic half-space.



(a)



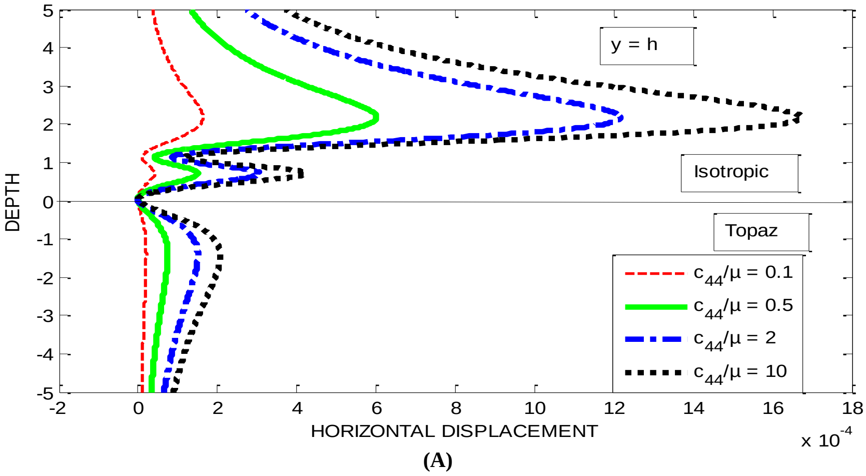
(b)



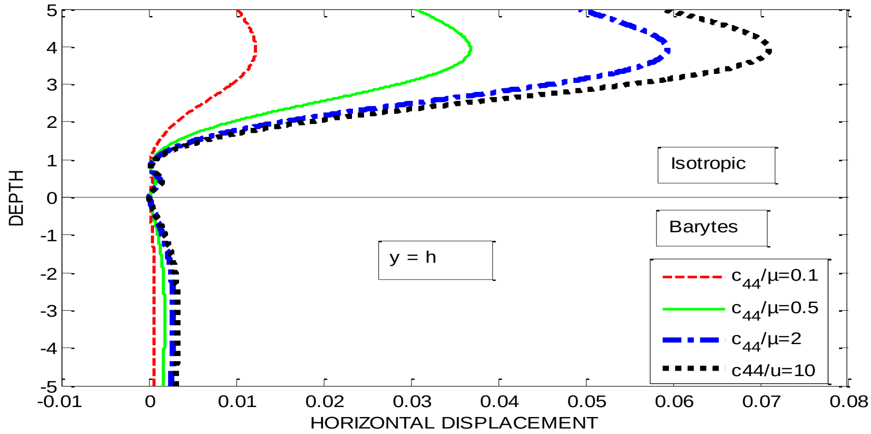
(c)

Fig. 2: Variation of Horizontal Displacement ($u_2'/b'ds$) with Depth from the Fault *z/h* Due to Vertical Dip-Slip Fault at (a) $y = h$, (b) $y = -h/2$, (c) $y = h/2$.

Figures 2a-c display horizontal displacement ($u_2'/b'ds$) with depth from the fault *z/h*. The displacement is anti-symmetric about the line $z = 0$. In fig. 2(a), (c) the observer is in the isotropic half-space at $y = h$, $h/2$ respectively. In fig 2(b), the observer is in the orthotropic half-space at $y = -h$. In each, pattern is same and tends to zero as z approaches to infinity.



(A)



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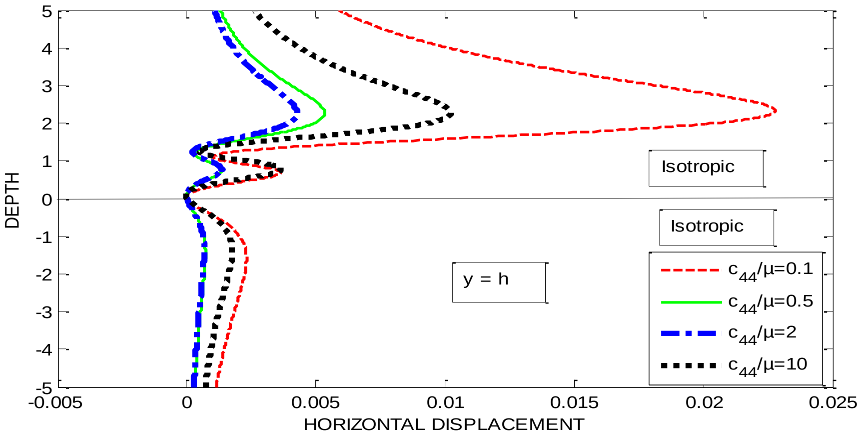
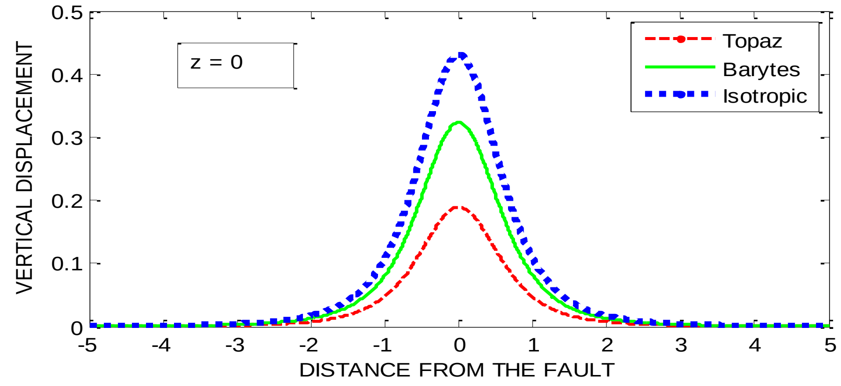


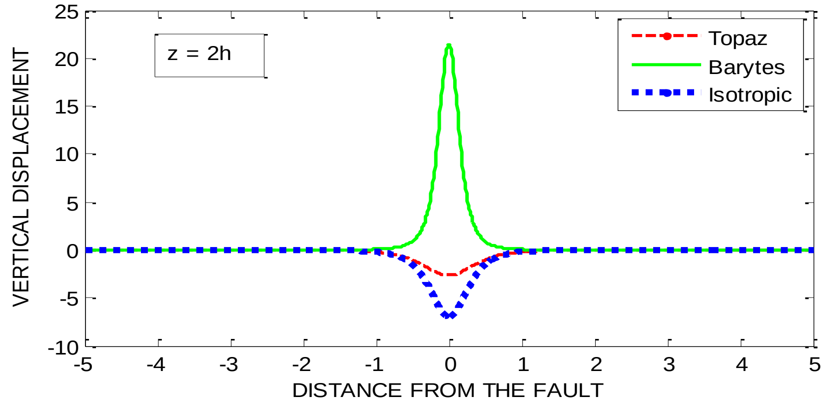
(C)

Fig. 3: Variation of Horizontal Displacement $(u_2'/b'ds)$ with Depth from the Fault *Z/H* Due to Vertical Dip Slip Fault at (a) $y = h$ for Different Values of Ratio of Rigidities $\frac{c_{44}}{\mu}$ = 0.1, 0.5, 2 and 10. Taking the Orthotropic Material as (a) Topaz, (b) Barytes, (c) Isotropic.

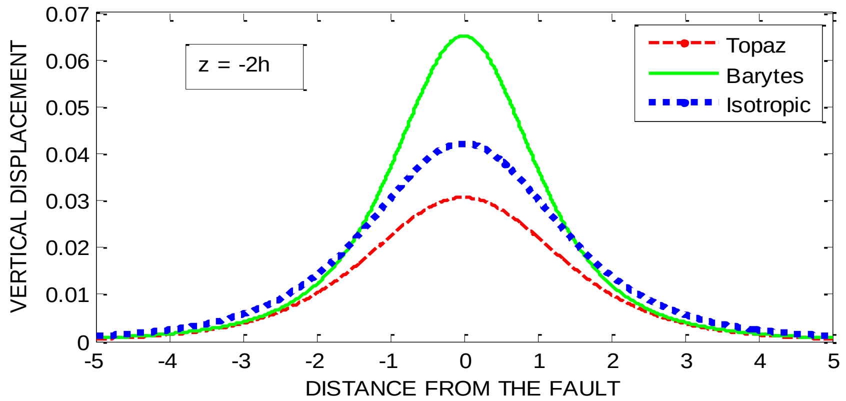
Figures 3a-c exhibit the variation of the horizontal displacement with depth *z/h* for four values of rigidities $\frac{c_{44}}{\mu} = 0.1, 0.5, 2$ and 10 for Topaz and Barytes and isotropic respectively due to dip-slip fault. It is observed that with increase in the value of ratio of rigidities, there is increase in horizontal displacement for Topaz and Barytes. Also, for the same value of ratio of rigidities horizontal displacement for Barytes is more than Topaz and isotropic in magnitude.



(a)



(b)



(c)

Fig. 4: Variation of vertical displacement ($u_3'/b'ds$) with distance from the fault y/h due to vertical dip-slip fault at (a) *z = 0* (b) *z = 2h*, (c) *z = -2h*.

Figures 4a-c shows vertical displacement ($u_3'/b'ds$) with distance from the fault y/h due to vertical dip-slip line source. The displacement is symmetric about the line y = 0. In fig. 4(c) the observer is in the orthotropic half-space at z = -2h. The vertical displacement for Barytes varies most significantly in magnitude rather than Topaz from the corresponding one for the isotropic case. In fig 4(a), the observer is at the interface and In fig.4(b), observer is in isotropic half-space at z =2h, vertical displacement attains maximum value at origin and tends to zero as y approaches to infinity for all cases. It is observed that effect of anisotropy on the displacement field is more pronounced when the observer is in the orthotropic half-space.

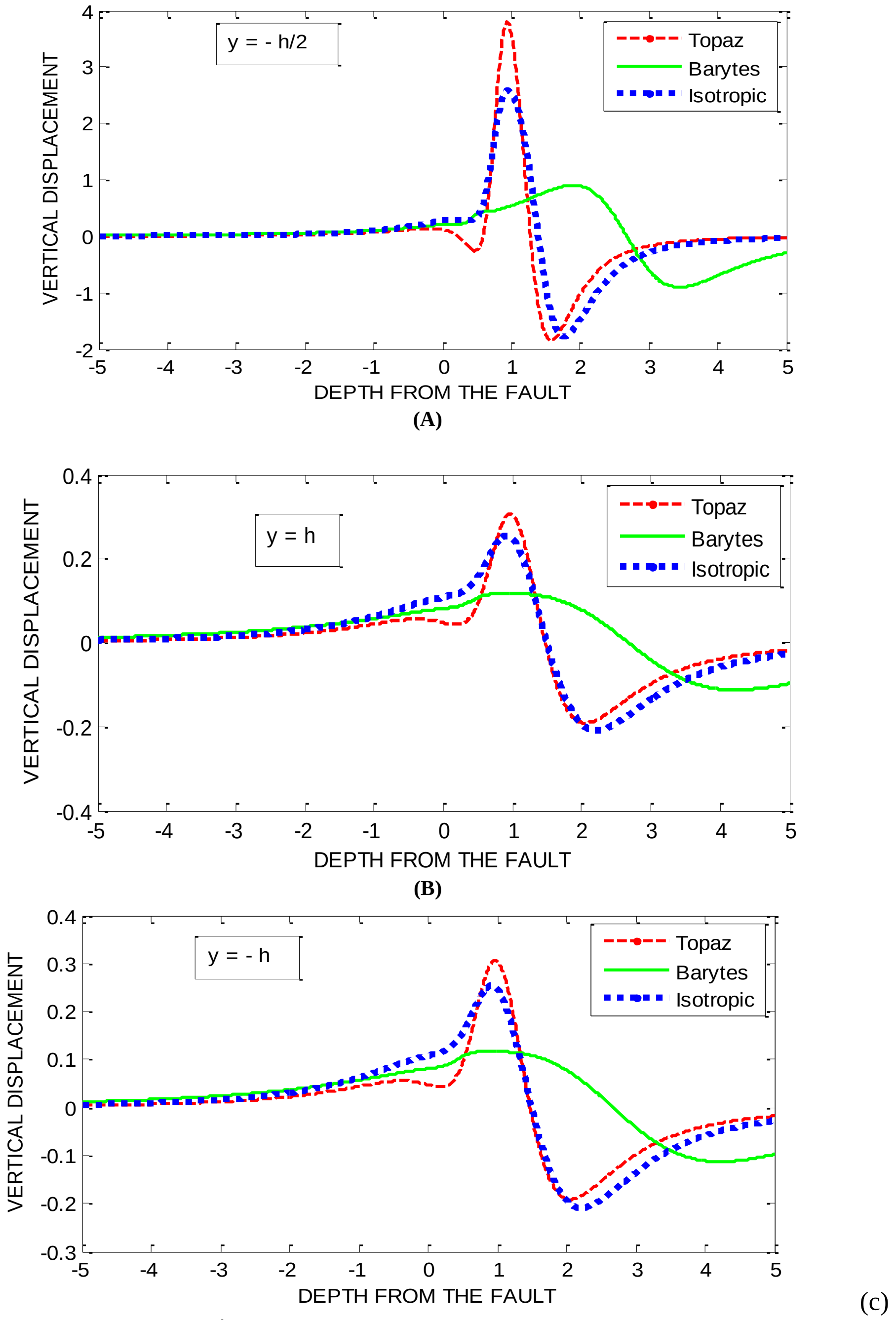
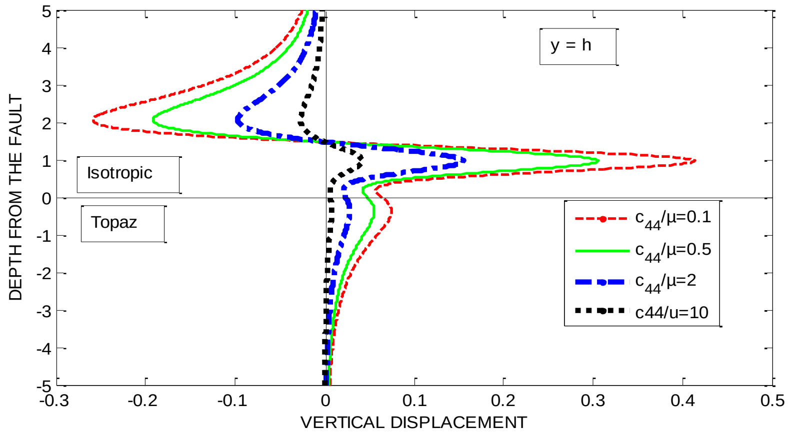
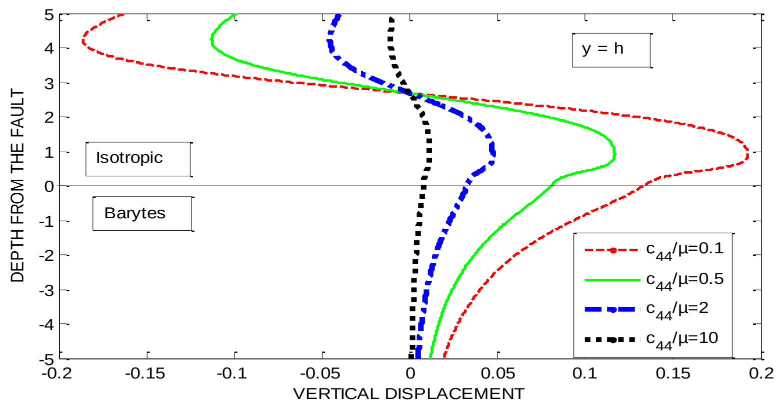


Fig. 5: Variation of Vertical Displacement ($u_3'/b'ds$) with Depth from the Fault Z/H Due to Vertical Dip-Slip Fault at (a) y = - h/2, (b) $y = h$, (c) y = - h.

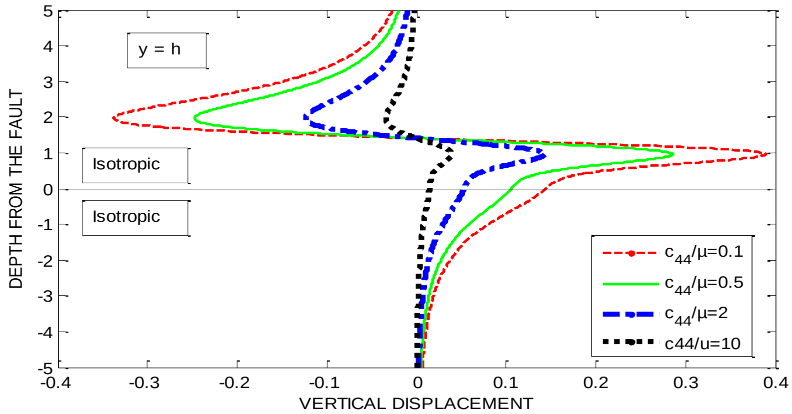
Figures 5a-c display the vertical displacement ($u_3'/b'ds$) with depth from the fault *z/h*. The displacement is anti-symmetric about the line $y = 0$. In fig. 5(a), (c) the observer is in the orthotropic half-space at y = - *h/2*, -*h* respectively. In fig 5(b), the observer is in the isotropic half-space at $y = h$. In each, pattern is same and displacement tends to zero as y approaches to infinity.



(A)



(B)



(C)

Fig. 6: Variation of Vertical Displacement $(u_3'/b'ds)$ with Depth from the Fault *z/h* Due to Vertical Dip-Slip Fault at $y = h$ for Different Values of Ratio of Rigidities $\frac{c_{44}}{\mu}$ = 0.1, 0.5, 2 and 10. Taking the Orthotropic Material as (a) Topaz, (b) Barytes, (c) Isotropic.

Figures 6a-c exhibit the variation of the vertical displacement with depth *z/h* for four values of rigidities $\frac{c_{44}}{\mu} = 0.1, 0.5, 2$ and 10 for Topaz and Barytes and isotropic respectively due to dip-slip fault. It is observed that with increase in the value of ratio of rigidities, there is increase in vertical displacement for Topaz and Barytes. Also, for the same value of ratio of rigidities vertical displacement for Barytes is more than Topaz and isotropic in magnitude.

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Appendix I

[$z > 0$, $y^2+z^2=R^2$]

1.) $\int_0^\infty e^{-kz}\frac{\sin ky}{k}dk = \tan^{-1}\left(\frac{y}{z}\right)$

2.) $\int_0^\infty e^{-kz}\frac{\cos ky}{k}dk = -\ln R$

3.) $\int_0^\infty e^{-kz}\, sinky\; dk = \frac{y}{R^2}$

4.) $\int_0^\infty e^{-kz}\cos ky\; dk = \frac{z}{R^2}$

5.) $\int_0^\infty e^{-kz}\, sinkykdk = \frac{2yz}{R^4}$

6.) $\int_0^\infty e^{-kz}\cos ky\; k\; dk = \frac{1}{R^2}\left(\frac{2z^2}{R^2}-1\right)$

7.) $\int_0^\infty e^{-kz}\sin ky\; k^2\; dk = \frac{2y}{R^4}\left(\frac{4z^2}{R^2}-1\right)$

8.) $\int_0^\infty e^{-kz}\cos ky\;\; k^2\; dk = \frac{2z}{R^4}\left(\frac{4z^2}{R^2}-3\right)$

9.) $\int_0^\infty e^{-kz}\sin kyk^3 dk = \frac{24yz}{R^6}\left(\frac{2z^2}{R^2}-1\right)$

10.) $\int_0^\infty e^{-kz}\cos kyk^3 dk = \frac{6}{R^4}\left(\frac{8z^4}{R^4}-\frac{8z^2}{R^2}+1\right)$

Appendix II

Source coefficients for various sources. The upper sign is for z > h and the lower sign for z < h.

Source	L_0	M_0	P_0	Q_0
Single couple (23)	$\mp\frac{F_{23}}{2\pi}$	$\pm\alpha\frac{F_{23}}{2\pi}$	0	0
Single couple (32)	$\pm\frac{F_{32}}{2\pi}$	$\pm\alpha\frac{F_{32}}{2\pi}$	0	0
Double couple (23)+(32) $F_{23}=F_{32}=D_{23}$	0	$\pm\frac{\alpha}{\pi}D_{23}$	0	0
Centre of rotation (32)-(23) $F_{23}=F_{32}=R_{23}$	$\pm\frac{\alpha}{\pi}R_{23}$	0	0	0
Dipole (22)	0	0	$(1-\alpha)\frac{F_{22}}{2\pi}$	$-\frac{\alpha}{2\pi}F_{22}$
Dipole (33)	0	0	$(1-\alpha)\frac{F_{33}}{2\pi}$	$\frac{\alpha}{2\pi}F_{33}$
Centre of dilatation (22)+(33) $F_{23}=F_{32}=C_0$	0	0	$(1-\alpha)\frac{C_0}{\pi}$	0
Dipole (22) $F_{23}=F_{32}=D'_{23}$	0	0	0	$\frac{\alpha}{\pi}D'_{23}$