

Modelling Infectiology of Bursal Disease in Chicks with Control Measures

Alfred K. Hugo^{1*}, Raymond E. Kitengeso² and Eva Lusekelo³

^{1,2,3}Department of Mathematics and Statistics, University of Dodoma

*Corresponding author E-mail: alfredkhugo@gmail.com

Abstract

This paper focuses on mathematical control of infectious bursal disease in chicks. The model boundedness and the control measures to reduce the spreads of the disease have well analytically examined. The theory of Pontryagin's maximum principle used in analysing necessary conditions to combat the disease. Numerically, forward backward sweep method and fourth-order Runge-Kutta scheme using the forward solution of the state equations was applied. The outcome indicates that the combination of vaccination of chicks and environmental sanitation as the most cost-effectiveness strategy to combat the spread of IBD with limited resources. Therefore, IBD can be controlled if the poultry farmers will effectively apply vaccination of chicks and environmental sanitation.

Keywords: Bursal Disease; Optimal Control; Cost Effectiveness.

1. Introduction

Infection bursal disease is a highly transmissible bird disease that mostly affects chickens; particularly young chickens usually of age between three to six weeks [20]. The Infection bursal disease (IBD) also known as 'Gumboro disease' due to the geographical location of the first recorded outbreak in 1961, which occurred in and around Gumboro, Delaware, USA [20]. The disease is caused by infection bursal disease virus (IBDV), the virus belongs to the genus avibirnavirus of the family birnaviridae, indeed the virus was also identified in the Middle East, Southern and western Africa, India, the Far East, and Australia from 1966 to 1974 [22], [23]. Moreover the infectious bursal disease is currently a global problem as about 95% of the 65 countries that affected by this disease [21]. The IBD viral also occurs in other lymphoid structures including the spleen, thymus, harderian gland, and ceca tonsil [24].

The incubation period for IBDV ranges from 2-3 days, and the infections before 3 weeks of age are usually subclinical (no detectable symptoms) [25]. The severity of the disease may vary with age and breed of chickens, the virulence of the strain, and degree of passive immunity. Moreover the susceptible chicken can be infected through direct contact, feces, contaminated environment and is possibly also carried in the dust. Bird to bird is the most recognized transmission way of the disease through contact with contaminated drinking water. The virus can also transferred from the house on fomites and rodents [26], apart from that lesser mealworm (*Alphitobus diapering*) has been shown to carry the virus and transmit mechanically among the farms by people, equipment, and vehicles [20]. Furthermore, it shows that there is no vertical transmission from parents direct off spring.

This disease is characterized by the following symptoms; nasal discharge, sneezing, diarrhoea with urates in mucus, in appetite and prostrated (extremely weakness), feathers are ruffled, which leads to a drop in egg production, decreased feed and water consumption [25]. Although IBDV represents one of the most severe poultry diseases and is responsible for marked economic losses, few studies of IBDV have been done on chickens in Tanzania, which hinders the implementation of effective disease [28].

According to [27] IBDV in Tanzania was found in the Eastern Zone (Dar-es Salaam and Kibaha) in 1988 that was first affected broiler flocks disease. Further more in 2007, it was found that the pathotypes exist in Tanzania are African VV-IBDV (VV1 & VV2) and European/Asian VV-IBDVs [19]. Since IBD is a viral disease there is no treatment, the only way to reduce the impact of the disease is to keep the flock of chicken free of this disease through effective vaccination. The governments through different means have tried to emphasize on the vaccination but still the disease has continued causing economic depression for both government and individual who are invested in chicken especially in the village areas for those people depend on poultry. Due to important of IBD constraints for commercial and local chicken production in Tanzania [29], this study aims to develop a mathematical model as an attempt of controlling the spread of the disease that will help the poultry farm to plan for vaccination programs to fight against the outbreak of IBDV disease.

2. Model Formulation

In this section we formulate a deterministic compartmental mathematical model to describe the transmission dynamics of IBD. We assume that the chicks population is homogeneously mixing and reflects increasing dynamics. We have chick population divided into four epidemiological classes and vector population in the environment divided into two epidemiological classes. For the chicks population we have the susceptible chicks who may get the disease S_C , the chicks who are exposed to the disease E_C , the chicks who are infectious and may transfer the disease to others I_C and the chicks who are vaccinated V_C . In the environment there are various vectors who may carry the disease and transmit to chicks so we have susceptible vectors in the environment T_S and the infectious vectors in the environment T_I . All new chicks Π and vectors Λ who enters the population are susceptible to the disease. The chicks and vector populations may decrease naturally by death rates μ and ψ respectively.

- (i) The proportion of susceptible chicks θS_C who receive first dose of active vaccine may progress to vaccinated chicks V_C . These vaccinated chicks will stay immune to the disease until they receive the second dose of inactive vaccine. The proportion of vaccinated chicken $(1 - \theta)S_C$ who may not receive second dose of vaccine may again become susceptible to the disease after the waning of first dose at rate α .
- (i) When the proportion of susceptible chicks λS_C come into contact with infectious chicks I_C at rate β_1 or the infectious vectors in the environment T_I at rate β_2 they may progress to be exposed chicks E_C . The force of infection $\lambda = \beta_1 I_C + \beta_2 T_I$.
- (i) After some time the proportion of exposed chicks σE_C may progress to become infectious chicks I_C who may infect other chicks or a proportion of susceptible vectors in the environment λT_S .

Table 1: Ranges of parameters used in the model

Symbol	Description	Estimated range
Π	The rate of increase of chicks	0 – 1
λ	The force of infection	0.2 – 0.8
β_1	The rate of infectious chicks to transmit the disease	0.4 – 0.9
β_2	The rate of infectious vectors to transmit the disease	0.1 – 0.8
θ	Proportion of vaccinated chicks	0 – 1
α	The rate of waning of first dose of vaccine	0.5 – 1
μ	Natural death rate of chicks	0 – 1
σ	The rate of progression from latent to infectious	0.5 – 1
δ	Death rate of chicks due to disease	0 – 1
Λ	The rate of increase of susceptible vectors	0 – 1
ψ	Natural death rate of vectors	0 – 1

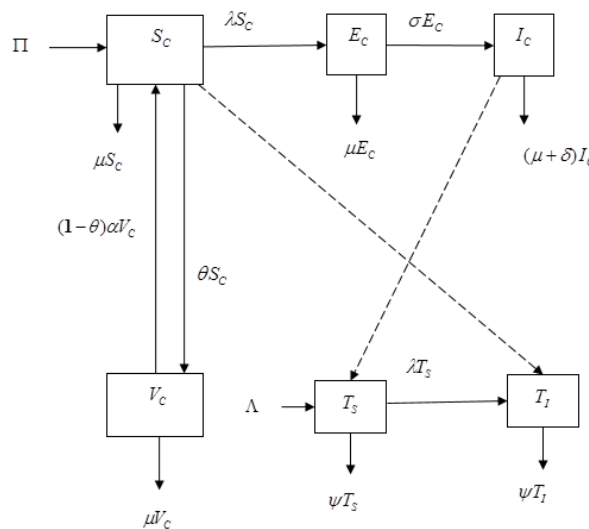


Figure 1: Compartmental Diagram IBD model.

Basing on these assumptions and compartmental model Figure 1 the following system of the model equations generated:

$$\begin{aligned}
 \frac{dS_C}{dt} &= \Pi - \lambda S_C - \theta S_C + (1 - \theta)\alpha V_C - \mu S_C \\
 \frac{dE_C}{dt} &= \lambda S_C - \sigma E_C - \mu E_C \\
 \frac{dI_C}{dt} &= \sigma E_C - (\mu + \delta)I_C \\
 \frac{dV_C}{dt} &= \theta S_C - (1 - \theta)\alpha V_C - \mu V_C \\
 \frac{dT_S}{dt} &= \Lambda - \lambda T_S - \psi T_S \\
 \frac{dT_I}{dt} &= \lambda T_S - \psi T_I
 \end{aligned} \tag{1}$$

We examine the boundedness of the model (1) using the following lemma.

Lemma:

All solutions of the system (1) which starts in \mathbf{R}_+^6 are uniformly bounded.

Proof:

Let

$$N(t) = P(S_C, E_C, I_C, V_C) + Q(T_S, T_I) \tag{2}$$

Differentiating and solving (2) we get,

$$N(t) = \frac{\Pi}{\mu}(1 - e^{-\mu t}) + P(0)e^{-\mu t} + \frac{\Lambda}{\psi}(1 - e^{-\psi t}) + Q(0)e^{-\psi t} \tag{3}$$

Then in the equation (3) as $t \rightarrow \infty$ we consequently have,

$$0 \leq N(t) \leq \frac{\Pi\psi + \Lambda\mu}{\mu\psi} \tag{4}$$

Implying that all solutions of the system (1) are uniformly bounded in the interior of \mathbf{R}_+^6 , then

$$r = \left\{ (S_C, E_C, I_C, V_C, T_S, T_I) \in \mathbf{R}_+^6 : N \leq \frac{\Pi\psi + \Lambda\mu}{\mu\psi} + \varepsilon \right\} \text{ for any } \varepsilon > 0 \text{ is bounded.}$$

3. Application of Optimal Control to the Infectiology Bursal Disease (IBD) Model

Controlling chicks from IBD may contribute to gross economy for nation or individual bases of the farmers. The time-dependent control to the model (1) for the aim of minimizing the spread of the disease in local or indigeneous chicks basing on the Tanzanian context is analysed. In formulating the control strategies the following assumptions considered as the guideline.

It is assumed that infected chickens may be controlled through treatment and denoted as $u_3(t)$ whereas susceptible populations are protected through vaccination $u_1(t)$. Also, the disease can be controlled through sanitation which is denoted by $u_2(t)$. Furthermore, it is assumed that a fraction of susceptible population being infectious is $1 - u_1(t)$ while the remaining population turns to a class of susceptible. The incorporated control time is bounded by $t \in [0, T]$ where T is the final time of the intervention program. The vaccination control will be evaluated at its optimal level when $u_1 = 1$ and at the minimum level when $u_1 = 0$. The control associated with chickens environmental sanitation attain its maximum level whenever $u_2 = 1$ and the optimal level of treatment achieved when $u_2 = 1$. Otherwise, it is assumed that intervention is at a low or intermediate level.

Hence, incorporating these assumptions in the model (1), we generate the following model equations

$$\begin{aligned}
 \frac{dS_C}{dt} &= \Pi - (1 - u_1)\lambda S_C - \theta S_C + (1 - \theta)\alpha V_C - \mu S_C \\
 \frac{dE_C}{dt} &= (1 - u_1)\lambda S_C - \sigma E_C - \mu E_C \\
 \frac{dI_C}{dt} &= \sigma E_C - (\mu + \delta + u_2)I_C \\
 \frac{dV_C}{dt} &= \theta S_C - (1 - \theta)\alpha V_C - \mu V_C \\
 \frac{dT_S}{dt} &= \Lambda - (1 - u_3)\lambda T_S - \psi T_S \\
 \frac{dT_I}{dt} &= (1 - u_3)\lambda T_S - \psi T_I
 \end{aligned} \tag{5}$$

It is assumed that the control strategies that are chicken vaccination, treatment of infected chicken and chicken environmental sanitation has maximum limitations in a given period of time. The limitations are evaluated under a Lebesgue measurable control variable presented as

$$u = \{u = (u_1, u_2, u_3), 0 \leq u_{i\max}, i = 1, 2, 3\}.$$

This leads to the minimization of the number of the infected chicken population while minimizing the associated cost of interventions u_1, u_2 and u_3 and in a specified period of time. Thus, the optimal control problem is set to minimize the objective functional presented as

$$J(u) = \int_0^T \left(A_1 I_C + A_2 T_I + \frac{1}{2} B_1 u_1^2 + \frac{1}{2} B_2 u_2^2 + \frac{1}{2} B_3 u_3^2 \right) dt \quad (6)$$

where $A_i > 0$ represents the weight of control of infected chicken and its environment, $B_i > 0$ represents the relative weight of control cost and benefit of the control while $\frac{1}{2} B_1 u_1^2$ is the minimization cost of vaccination control, $\frac{1}{2} B_2 u_2^2$ is the minimization cost of environment sanitation and $\frac{1}{2} B_3 u_3^2$ is the minimization cost of treatment control. The aim is to find the pair of optimal control $u^*(t) = (u_1^*(t), u_2^*(t), u_3^*(t))$ such that

$$J(u^*) = \min_U J(u_1, u_2, u_3) \quad (7)$$

The basic setup of the optimal control problem is to check the existence and uniqueness of the optimal controls and to characterize them [30]. The Pontryagin maximum principle converts the control set U into a problem of minimizing the Hamiltonian H , point wise with respect to u_1, u_2, u_3 . The particular study applies the optimal controls that rely on Pontryagin's maximum principle as presented by [12] and applied by many other authors. To apply this theory, we convert the optimal control problem (5) and objective functional (6) into a problem of minimizing point-wise a Hamiltonian (H), with respect to $u(t)$. Hamiltonian equation formed

$$\begin{aligned} H = & A_1 I_C + A_2 T_I + \frac{1}{2} B_1 u_1^2 + \frac{1}{2} B_2 u_2^2 + \frac{1}{2} B_3 u_3^2 \\ & + \tau_1 (\Pi - (1 - u_1) \lambda S_C - \theta S_C + (1 - \theta) \alpha V_C - \mu S_C) \\ & + \tau_2 ((1 - u_1) \lambda S_C - \sigma E_C - \mu E_C) \\ & + \tau_3 (\sigma E_C - (\mu + \delta + u_2) I_C) \\ & + \tau_4 (\theta S_C - (1 - \theta) \alpha V_C - \mu V_C) \\ & + \tau_5 (\Lambda - (1 - u_3) \lambda T_S - \psi T_S) \\ & + \tau_6 ((1 - u_3) \lambda T_S - \psi T_I) \end{aligned} \quad (8)$$

where $\tau_i, i = 1, 2, 3, 4, 5, 6$ are the co-state variables associated by S_C, E_C, I_C, V_C, T_S and T_I .

3.1. Existence of an optimal control

The existence of an optimal control for the state system is checked by using the results obtained by Fleming and Rishel [13] through the following theorem.

Theorem 1. *Let the optimal control problem that minimizes the objective functional J be defined over a time horizon $[0, T]$. If the objective function is defined on a set of bounded and Lebesgue measurable control u and subjected to the dynamic constraint of some state equations, then there exists an optimal solution u^* such that $\mathbb{J}(u^*) = \min_U u$ provided that the following conditions hold:*

- (i) *The control set is convex and closed.*
- (ii) *The right-hand side of the state system is bounded by a linear function in the state and control variable.*
- (iii) *The state variables used in the system (5), together with their control variables are not empty.*
- (iv) *There exist some constants $x_1, x_2 > 0$ and $y > 1$, for which the integrand of the objective function is convex and satisfies the boundary condition:*

$$J(u) = x_1 \left(\sum_{i=1}^n |u_i|^2 \right)^{\frac{y}{2}} - x_2. \quad (9)$$

The reader is therefore advised to go through the proof of the theorem 1 from the book of [13] entitled Deterministic and Stochastic optimal control, pages 62, 69 and [12]. Conversely, for the analysis of particular paper, the conditions that guarantees the existence of an optimal solution for the objective functional are verified.

Consider an optimal control problem described by Equation (9), which is subject to the state constraint given by system (5).

1. By definition, the control variables u_1, u_2, u_3 are convex and closed.
2. Clearly, the solutions of the state system are bounded since the state functions are linear with respect to the control variables. Hence, the second condition is satisfied.
3. It is obvious that the state and our corresponding set of control variables U in the system (5) are presumed bounded and not empty.
4. Since the state equations are bounded, we can find some positive constants $a_1, a_2 > 0$ and $b > 1$, for which the integrand of the objective functional is convex and satisfies

$$A_1 I_C + A_2 T_I + \frac{1}{2} B_1 u_1^2 + \frac{1}{2} B_2 u_2^2 + \frac{1}{2} B_3 u_3^2 \geq a_1 \left(\sum_{i=1}^3 |u_i|^2 \right)^{\frac{b}{2}} - a_2. \quad (10)$$

Therefore, it worth to conclude that there exists an optimal solution which lies between 0 and 1 that minimizes the objective functional articulated in (6).

3.2. Necessary Optimality Conditions

The optimality condition of the solution of the model, is established by the following theorem.

Theorem 2. Let u_i be the set of optimal control and X_i be the corresponding solution of the set of equations that minimizes the objective function J over the set of controls, then there exist λ_i adjoint variables such that optimality system is

$$\begin{cases} \frac{d\lambda_X(t)}{dt} = -\frac{\partial H}{\partial X} \\ \lambda_X(T) = 0 \\ \frac{\partial H}{\partial u_i} = 0. \end{cases}$$

By applying Pontryagin’s maximum principle, the following adjoint system obtained with corresponding optimal solutions of the state equations:

$$\begin{aligned} \frac{\partial H}{\partial S_C} &= -\tau_1 (-(1-u_1)(I_C \beta_1 + T_I \beta_2) - \theta - \mu) - \tau_2 (1-u_1)(I_C \beta_1 + T_I \beta_2) - \tau_4 \theta \\ \frac{\partial H}{\partial E_C} &= -\tau_2 (-\sigma - \mu) - \tau_3 \sigma \\ \frac{\partial H}{\partial I_C} &= -A_I + \tau_1 (1-u_1) \beta_1 S_C - \tau_2 (1-u_1) \beta_1 S_C - \tau_3 (-\mu - \delta - u_2) \\ &\quad + \tau_5 (1-u_3) \beta_1 T_S - \tau_6 (1-u_3) \beta_1 T_S \\ \frac{\partial H}{\partial V_C} &= -\tau_1 (1-\theta) \alpha - \tau_4 (-(1-\theta) \alpha - \mu) \\ \frac{\partial H}{\partial T_S} &= -\tau_5 (-(1-u_3)(I_C \beta_1 + T_I \beta_2) - \psi) - \tau_6 (1-u_3)(I_C \beta_1 + T_I \beta_2) \\ \frac{\partial H}{\partial T_I} &= -A_2 + \tau_1 (1-u_1) \beta_2 S_C - \tau_2 (1-u_1) \beta_2 S_C + \tau_5 (1-u_3) \beta_2 T_S \\ &\quad - \tau_6 ((1-u_3) \beta_2 T_S - \psi) \end{aligned} \tag{11}$$

Through Pontryagin’s maximum principle, the optimality system for the optimal control problem obtained. Furthermore, the optimality system involves the state equations of the system (5) including initial conditions $S_c \geq 0, E_c \geq 0, I_c \geq 0, V_c \geq 0, T_s \geq 0, T_I \geq 0$, together with its adjoint (co-state) equations (8). Consequently, the adjoint system bounded by final values or transversality conditions as

$$\tau_1 S_c(T) = \tau_2 E_c(T) = \tau_3 I_c(T) = \tau_4 V_c(T) = \tau_5 T_s(T) = \tau_6 T_I(T) = 0.$$

3.3. Characterization of the Optimal Control

The optimal solution for the Hamiltonian (H) is evaluated through the partial derivative of the Hamiltonian (H) with respect to the control (u_1, u_2, u_3) . The optimal solution is obtained by solving $\frac{\partial H}{\partial u_i} = 0$ for $i = 1, 2, 3$. Therefore the solution is characterized as

$$\begin{aligned} u_1^* &= \max \left\{ 0, \min \left\{ 1, \frac{S_C(I_C \beta_1 \tau_1 - I_C \beta_1 \tau_2 + T_I \beta_2 \tau_1 - T_I \beta_2 \tau_2)}{B_1} \right\} \right\} \\ u_2^* &= \max \left\{ 0, \min \left\{ 1, \frac{\tau_3 I_C}{B_2} \right\} \right\} \\ u_3^* &= \max \left\{ 0, \min \left\{ 1, \frac{-T_S(I_C \beta_1) \tau_5 - I_C \beta_1 \tau_6 + T_I \beta_2 \tau_5 - T_I \beta_2 \tau_6}{B_3} \right\} \right\} \end{aligned} \tag{12}$$

3.4. Uniqueness of the Optimal Control Solution

In this subsection, the uniqueness of the optimal control solution is evaluated following the method applied by [14]. Thus combining system (11) together with the optimality system results to;

$$\begin{cases} s_h = p_1(t, s_h, e_h, i_h, s_v, e_v, i_h) \\ e_h = p_2(t, s_h, e_h, i_h, s_v, e_v, i_h) \\ i_h = p_3(t, s_h, e_h, i_h, s_v, e_v, i_h) \\ r_h = p_4(t, s_h, e_h, i_h, s_v, e_v, i_h) \\ s_v = p_5(t, s_h, e_h, i_h, s_v, e_v, i_h) \\ e_v = p_6(t, s_h, e_h, i_h, s_v, e_v, i_h) \\ i_v = p_7(t, s_h, e_h, i_h, s_v, e_v, i_h) \\ s_h(0), e_h(0), i_h(0), r_h(0), s_v(0), e_v(0), i_v(0), \\ s_h(T), e_h(T), i_h(T), r_h(T), s_v(T), e_v(T), i_v(T) \text{ and } t \text{ is fixed.} \end{cases} \tag{13}$$

where $s_h \in \mathbb{R}^{n_i}, e_h \in \mathbb{R}^{n_i}, i_h \in \mathbb{R}^{n_i}, r_h \in \mathbb{R}^{n_i}, s_v \in \mathbb{R}^{n_i}, e_v \in \mathbb{R}^{n_i}, i_v \in \mathbb{R}^{n_i}$ for $i = 1, 2, 3, 4, 5, 6, 7$ as dimension of vector space \mathbb{R}^{n_i} and

$$\begin{cases} q_1 = R \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \times \mathbb{R}^{n_4} \times \mathbb{R}^{n_5} \times \mathbb{R}^{n_6} \times \mathbb{R}^{n_7} \longrightarrow \mathbb{R}^{n_1} \\ q_2 = R \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \times \mathbb{R}^{n_4} \times \mathbb{R}^{n_5} \times \mathbb{R}^{n_6} \times \mathbb{R}^{n_7} \longrightarrow \mathbb{R}^{n_2} \\ q_3 = R \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \times \mathbb{R}^{n_4} \times \mathbb{R}^{n_5} \times \mathbb{R}^{n_6} \times \mathbb{R}^{n_7} \longrightarrow \mathbb{R}^{n_3} \\ q_4 = R \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \times \mathbb{R}^{n_4} \times \mathbb{R}^{n_5} \times \mathbb{R}^{n_6} \times \mathbb{R}^{n_7} \longrightarrow \mathbb{R}^{n_4} \\ q_5 = R \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \times \mathbb{R}^{n_4} \times \mathbb{R}^{n_5} \times \mathbb{R}^{n_6} \times \mathbb{R}^{n_7} \longrightarrow \mathbb{R}^{n_5} \\ q_6 = R \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \times \mathbb{R}^{n_4} \times \mathbb{R}^{n_5} \times \mathbb{R}^{n_6} \times \mathbb{R}^{n_7} \longrightarrow \mathbb{R}^{n_6} \\ q_7 = R \times \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^{n_3} \times \mathbb{R}^{n_4} \times \mathbb{R}^{n_5} \times \mathbb{R}^{n_6} \times \mathbb{R}^{n_7} \longrightarrow \mathbb{R}^{n_7} \end{cases} \quad (14)$$

are continuous.

Theorem 3. Given that $q_1, q_2, q_3, q_4, q_5, q_6, q_7$ are bounded, satisfying Lipschitz condition in relation to $s_h, e_h, i_h, s_v, e_v, i_v$ with constant $L > 0$ then the solution of (13) are unique if the final time, T is sufficiently small.

Proof. Assume that the system (13) has two solutions; $s_{h1}(t), e_{h1}(t), i_{h1}(t), s_{v1}(t), e_{v1}(t), i_{v1}(t)$ and $s_{h2}(t), e_{h2}(t), i_{h2}(t), s_{v2}(t), e_{v2}(t), i_{v2}(t)$. Using the approach presented in [15] and by applying Lipschitz condition for q_1 we obtain

$$\begin{aligned} \|s_{h1}(t) - s_{h2}(t)\| &\leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

Applying Lipschitz condition for q_2 we get

$$\begin{aligned} \|e_{h1}(t) - e_{h2}(t)\| &\leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

Similarly applying Lipschitz condition for $q_3 - q_7$ we get

$$\begin{aligned} \|i_{h1}(t) - i_{h2}(t)\| &\leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

$$\begin{aligned} \|r_{h1}(t) - r_{h2}(t)\| &\leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

$$\begin{aligned} \|s_{v1}(t) - s_{v2}(t)\| &\leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

$$\begin{aligned} \|e_{v1}(t) - e_{v2}(t)\| &\leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

$$\begin{aligned} \|i_{v1}(t) - i_{v2}(t)\| &\leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

By adding the above equations, we have

$$\begin{aligned} &\|s_{h1}(t) - s_{h2}(t)\| + \|e_{h1}(t) - e_{h2}(t)\| + \|i_{h1}(t) - i_{h2}(t)\| + \|r_{h1}(t) - r_{h2}(t)\| + \|s_{v1}(t) - s_{v2}(t)\| \\ &+ \|e_{v1}(t) - e_{v2}(t)\| + \|i_{v1}(t) - i_{v2}(t)\| \leq \int_0^T L(\|s_{h1}(m) - s_{h2}(m)\| + \|e_{h1}(m) - e_{h2}(m)\| + \|i_{h1}(m) - i_{h2}(m)\| \\ &\quad + \|r_{h1}(m) - r_{h2}(m)\| + \|s_{v1}(m) - s_{v2}(m)\| + \|e_{v1}(m) - e_{v2}(m)\| + \|i_{v1}(m) - i_{v2}(m)\|) dm \end{aligned}$$

According to the mean value theorem, $\exists c$ for $0 \leq c \leq T$ such that

$$\begin{aligned} & \|s_{h1}(t) - s_{h2}(t)\| + \|e_{h1}(t) - e_{h2}(t)\| + \|i_{h1}(t) - i_{h2}(t)\| + \|r_{h1}(t) - r_{h2}(t)\| + \|s_{v1}(t) - s_{v2}(t)\| \\ & + \|e_{v1}(t) - e_{v2}(t)\| + \|i_{v1}(t) - i_{v2}(t)\| \leq LT(\|s_{h1}(c) - s_{h2}(c)\| + \|e_{h1}(c) - e_{h2}(c)\| + \|i_{h1}(c) - i_{h2}(c)\| \\ & + \|r_{h1}(c) - r_{h2}(c)\| + \|s_{v1}(c) - s_{v2}(c)\| + \|e_{v1}(c) - e_{v2}(c)\| + \|i_{v1}(c) - i_{v2}(c)\|) \end{aligned}$$

For all $t \in [0, T]$. The proof will be complete if T is small enough such that $t < 1$ where T denote the final time. □

4. Numerical Results

The numerical effects of optimal control strategies are analysed and discussed. The solution of the optimal control problem was obtained by solving the optimality system of state and adjoint systems through forward-backwards sweep method. The adjoint systems were solved by fourth-order Runge-Kutta scheme using the forward solution of the state equations. The optimality condition is satisfied through the convex update of the previous control values

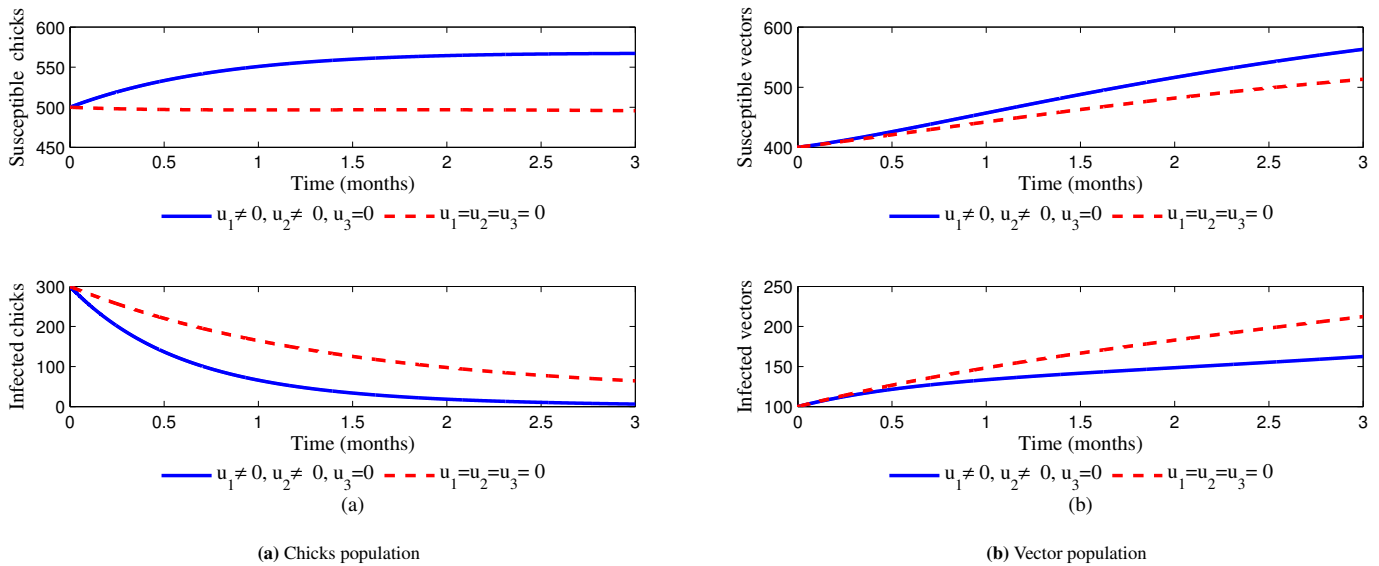


Figure 2: The effects of vaccination of chicks and removal of infected chicks

Strategy A: Combination of vaccination of chicks and removal of infected chicks. Figure 2(a) and 2(b) shows the positive effect of vaccination of chicks and removal of infected chicks when u_1 and u_2 are applied to the system and u_3 is set to zero. Figure 2(a) shows that, when the control is applied, the susceptible chicks increases while infected chicks decreases. The significant difference is also observed in susceptible and infected vector, when the control is applied, infected vector decreases as well as susceptible vector decreases as shown in figure 2(b). This result shows that the optimal control measure is effective in chicks and vectors vaccination of chicks and environmental sanitation.

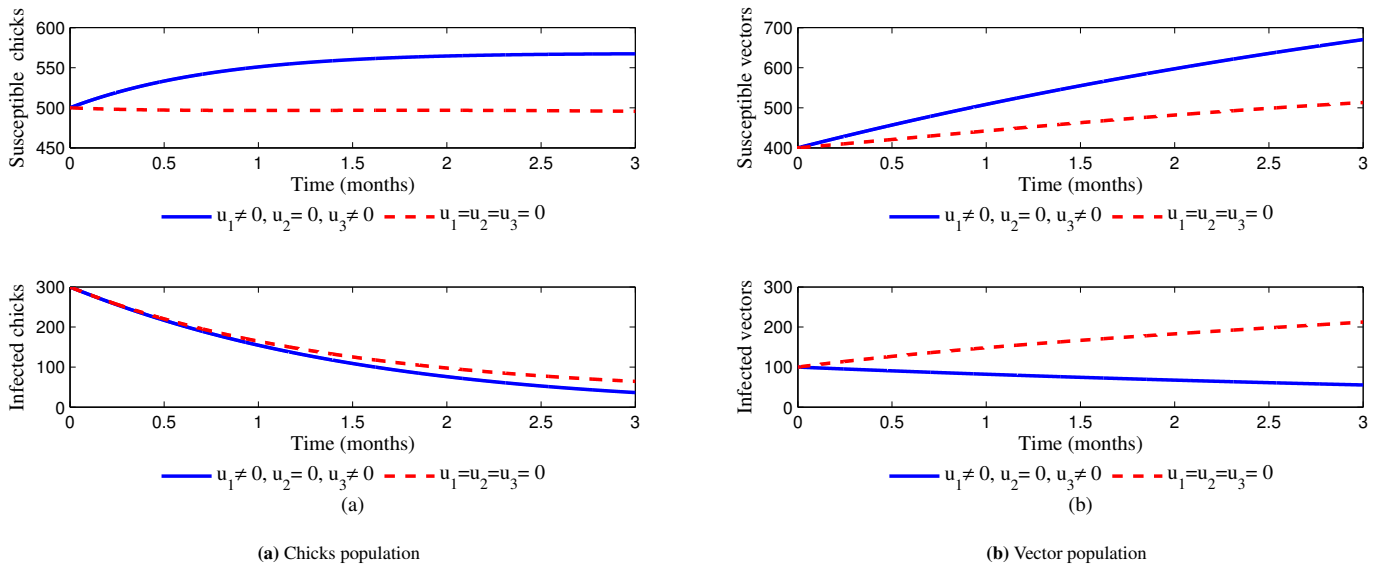


Figure 3: The effects of vaccination of chicks and environmental sanitation

Strategy *B*: Combination of vaccination of chicks and environmental sanitation. We can observe from Figure 3(a) and 3(b) when u_1 and u_3 are applied to the system and u_2 is set to zero. Figure 3(a) shows the number of susceptible chicks increase while infected chicks decrease as a result, reducing the transmission of the virus to other chicks. Figure 3(b) shows that, when the control is applied, the infected vector in the environment decreases.

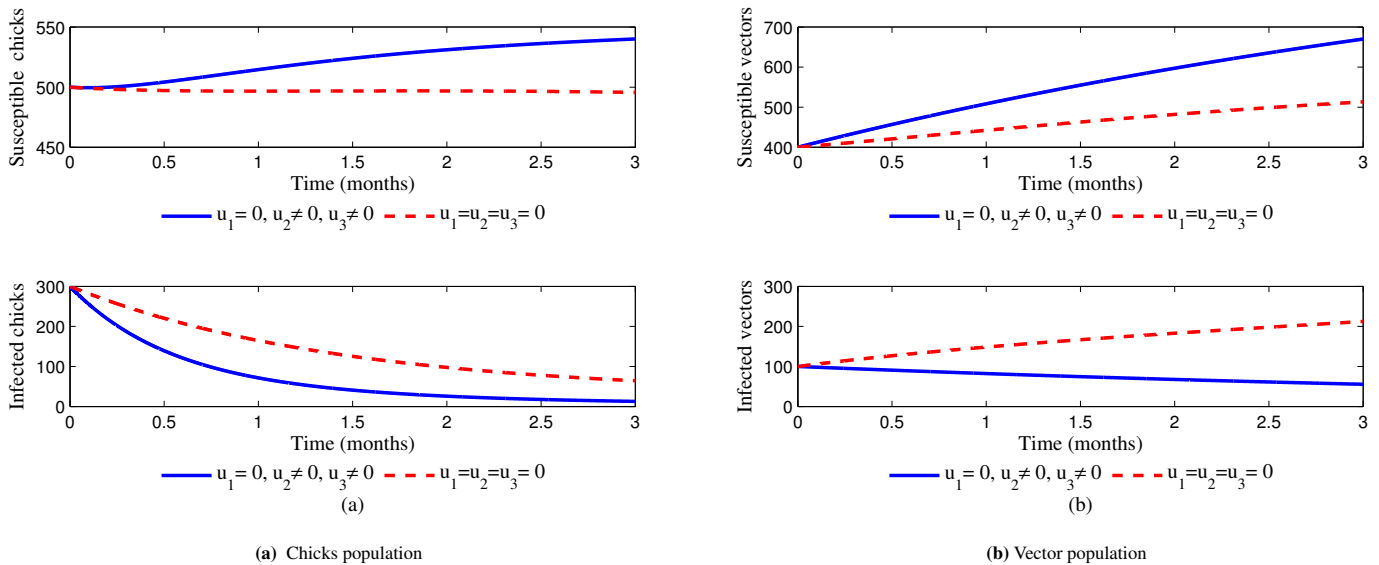


Figure 4: The effects of removing infected chicks and environmental sanitation

Strategy *C*: Combination of removing infected chicks and environmental sanitation. The results show that removing infected chicks and environmental sanitation in the system will reduce the spread of the disease. We can observe this from Figure 4(a) where the infected chicks decreases because of removing the infected chicks; and infected vector decreases by employing environmental sanitation to the system. Moreover, the combination of strategies when u_2 and u_3 are applied to the system and u_1 is set to zero give good results to optimize the objective function J .

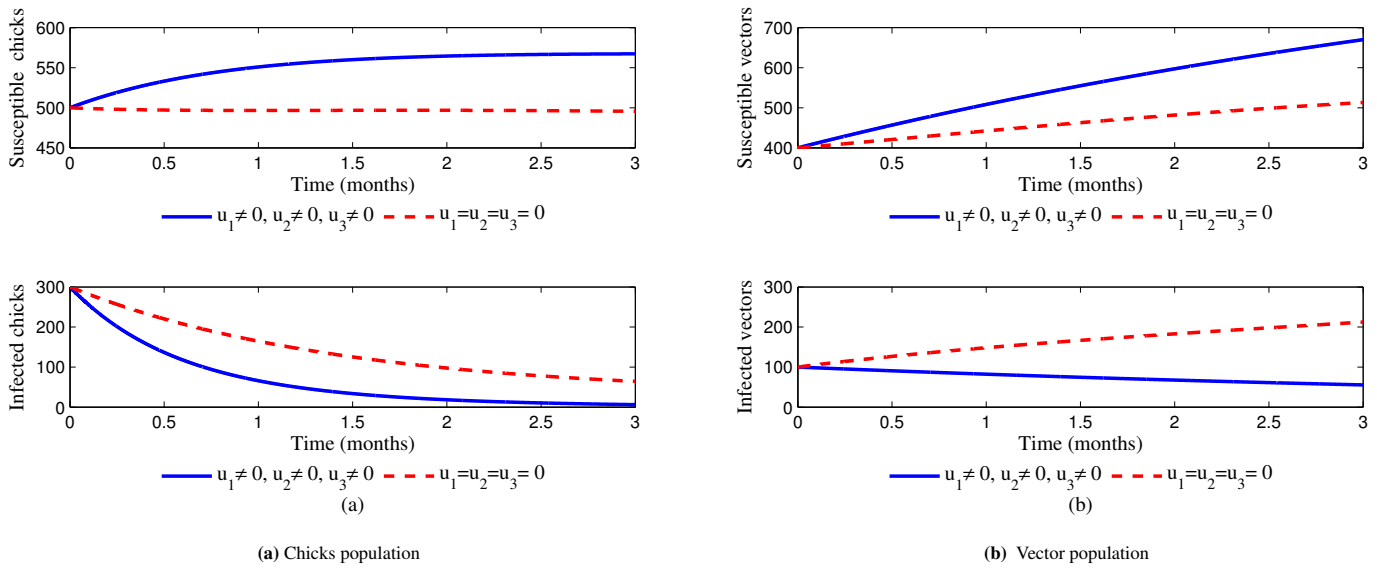


Figure 5: The effects of vaccination of chicks, removing infected chicks and environmental sanitation

Strategy *D*: Combination of vaccination of chicks, removing infected chicks and environmental sanitation. The results show that the use of vaccination of chicks, removing infected chicks and environmental sanitation in the system will reduce the spread of the disease. We can observe this tendency from Figure 5(a), which displays that the infected chicks decreases by intensifying the removal of the infected chicks and infected vector decreases by strengthening environmental sanitation to the system. Moreover, Figure 5(b) shows that when the control is applied, the infected vector decreases. The combination of strategies u_1 , u_2 and u_3 give the best results to optimize the objective function J .

5. Cost-effective Analysis

The cost effectiveness analysis helps to show the economic benefit of each control measure. It is used to make comparisons between the relative costs and outcomes of different strategies. In making decision on which intervention to implement in limited resources, the economic evaluation of IBID is carried out to find the most cost effective strategy. In this study, the cost effectiveness is thoroughly analysed using incremental cost effectiveness ratio (ICER) which compares the differences between the costs and health outcomes of the two competing intervention strategies. Each intervention is compared with the next less effective alternative [12]. The averted plant is computed by finding the difference between the total number of plants without control and the total number of plants with control. The total control cost is evaluated as:

$$C(u) = \min_{u_1, u_2, u_3} = \int_0^3 \left(\frac{1}{2}B_1u_1^2 + \frac{1}{2}B_2u_2^2 + \frac{1}{2}B_3u_3^2 \right) dt \tag{15}$$

The total control costs $B_1u_1^2$, $B_2u_2^2$ and $B_3u_3^2$ are relative cost weight for each control measure. The numerical output for the control strategies are ranked in increasing order of effectiveness in form of infection averted as shown in table 2

Table 2: Control strategies in order of increasing averted

Strategy	Infections averted	Control cost	Total cost
<i>C</i>	44.4801	29.996	18639
<i>A</i>	71.5448	29.9926	16575
<i>D</i>	71.731	44.9919	18489
<i>B</i>	71.7801	29.9975	20338

$$ICER = \frac{\text{Difference in cost in strategy } i \text{ and } j}{\text{Difference infected in strategy } i \text{ and } j} \tag{16}$$

$$ICER(C) = \frac{29.996}{44.4801} = 0.674368987, ICER(A) = \frac{29.9926 - 29.996}{71.5448 - 44.4801} = -0.000125625$$

The negative ICER for strategy *A* indicates that strategy *C* is strongly dominated and less effective than strategy *A*. Therefore, strategy *C* is excluded from the set of alternatives. We exclude *C* and compare strategy *A* and *D*, and ICER recalculated as follows

Table 3: Total infection averted, total cost and ICER

Strategy	Infections averted	Control cost	ICER
A	71.5448	29.9926	0.419214255
D	71.731	44.9919	80.55477981

The comparison between strategies A and D indicate that strategy D is strongly dominated and is more costly than strategy A as $ICER(A) < ICER(D)$ then strategy D is excluded in set of alternative hence A and B are compared.

Table 4: Total infection averted, total cost and ICER

Strategy	Infections averted	Control cost	ICER
A	71.5448	29.9926	0.419214255
B	71.7801	29.9975	0.020824479

Comparison between strategies A and B shows that strategy A is more costly and less effective than strategy B as $ICER(B) < ICER(A)$. Therefore strategy A is excluded from the set of alternatives and strategy B is cost effective. Now, basing on these results we therefore conclude that strategy B (vaccination of chicks and environmental sanitation) is most cost effective of all strategies for IBD.

6. Conclusion

In this paper, deterministic model for the transmission of IBD was formulated and three control strategies have been investigated. The cost effectiveness analysis was also a focal point of concentration to combat the disease in chicks. The Pontryagin's maximum principle was used in deriving and analysing the conditions for optimal control of IBD with control strategies such as vaccination of chicks, removal of infected chicks and environmental sanitation. The numerical analysis shows that each strategy has potential to control the transmission of the disease. Whenever control is applied, numerical results show that susceptible chicks increases while infected chicks decreases. Cost effectiveness analysis indicates that the use of vaccination of chicks and environmental sanitation is the cost effective optimal control strategy and is sufficient to combat the spread of IBD with limited resources

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