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Research Paper

The generalized triple difference of χ^3 sequence spaces

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Abstract

In this paper we define some new sequence spaces and give some topological properties of the sequence spaces $\chi^3(\Delta_v^m, s, p)$ and $\Lambda^3(\Delta_v^m, s, p)$ and investigate some inclusion relations.

Keywords: *analytic sequence; difference sequence; gai sequence; triple sequence.*
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1. Introduction

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex triple sequences (x_{mnk}), where $m, n, k \in \mathbb{N}$, the set of positive integers.Then, w^3 is a linear space under the coordinate wise addition and scalar multiplication.

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq}(m, n, k = 1, 2, 3, ...).$$

A sequence $x = (x_{mnk})$ is said to be triple analytic if

$$sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \to 0 \text{ as } m, n, k \to \infty.$$

The vector space of all triple entire sequences are usually denoted by Γ^3. The space Λ^3 and Γ^3 is a metric space with the metric

$$d(x, y) = sup_{m,n,k} \left\{ |x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m, n, k : 1, 2, 3, ... \right\}, \tag{1}$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in Γ^3. Let $\phi = \{\text{finite sequences}\}$.

Consider a triple sequence $x = (x_{mnk})$. The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\delta_{ijq}$ for all $m, n, k \in \mathbb{N}$,

$$\delta_{mnk} = \begin{pmatrix} 0 & 0 & ...0 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \\ . & . & & . & \\ . & & & & \\ . & & & & \\ 0 & 0 & ...1 & 0 & ... \\ 0 & 0 & ...0 & 0 & ... \\ & & & & \end{pmatrix}$$

with 1 in the $(m, n, k)^{th}$ position and zero otherwise.

A sequence $x = (x_{mnk})$ is called triple gai sequence if $((m + n + k)!\, |x_{mnk}|)^{\frac{1}{m+n+k}} \to 0$ as $m, n, k \to \infty$. The triple gai sequences will be denoted by χ^3.

Consider a triple sequence $x = (x_{mnk})$. The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\Im_{ijq}$ for all $m, n, k \in \mathbb{N}$; where $\Im_{ijq}$ denotes the triple sequence whose only non zero term is a $\frac{1}{(i+j+k)!}$ in the $(i, j, k)^{th}$ place for each $i, j, k \in \mathbb{N}$.

An FK-space(or a metric space) X is said to have AK property if $(\Im_{mnk})$ is a Schauder basis for X, or equivalently $x^{[m,n,k]} \to x$.

An FDK-space is a triple sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings are continuous.

If X is a sequence space, we give the following definitions:

(i) X' is continuous dual of X;

(ii) $X^\alpha = \left\{a = (a_{mnk}) : \sum_{m,n,k=1}^{\infty} |a_{mnk}x_{mnk}| < \infty,\ for\ \ each\ \ x \in X\right\}$;

(iii) $X^\beta = \left\{a = (a_{mnk}) : \sum_{m,n,k=1}^{\infty} a_{mnk}x_{mnk}\ is\ convergent,\ for\ each\ x \in X\right\}$;

(iv) $X^\gamma = \left\{a = (a_{mn}) : sup_{m,n\geq 1} \left|\sum_{m,n,k=1}^{M,N,K} a_{mnk}x_{mnk}\right| < \infty,\ for\ \ each\ \ x \in X\right\}$;

(v) Let X be an FK-space $\supset \phi$; then $X^f = \left\{f(\Im_{mnk}) : f \in X'\right\}$;

(vi) $X^\delta = \left\{a = (a_{mnk}) : sup_{m,n,k} |a_{mnk}x_{mnk}|^{1/m+n+k} < \infty,\ for\ \ each\ \ x \in X\right\}$;

$X^\alpha.X^\beta, X^\gamma$ are called $\alpha-$ (or Köthe-Toeplitz) dual of X, $\beta-$ (or generalized-Köthe-Toeplitz) dual of X, $\gamma-$ dual of X, $\delta-$ dual of X respectively. X^α is defined by Gupta and Kamptan [10]. It is clear that $X^\alpha \subset X^\beta$ and $X^\alpha \subset X^\gamma$, but $X^\alpha \subset X^\gamma$ does not hold.

2. Definitions and preliminaries

A sequence $x = (x_{mnk})$ is said to be triple analytic if $sup_{mnk} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty$. The vector space of all triple analytic sequences is usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}} \to 0$ as $m, n, k \to \infty$. The vector space of triple entire sequences is usually denoted by Γ^3. A sequence $x = (x_{mnk})$ is called triple gai sequence if $((m + n + k)!\, |x_{mnk}|)^{\frac{1}{m+n+k}} \to 0$ as $m, n, k \to \infty$. The vector space of triple gai sequences is usually denoted by χ^3. The space χ^3 is a metric space with the metric

$$d(x, y) = sup_{m,n,k} \left\{((m + n + k)!\, |x_{mnk} - y_{mnk}|)^{\frac{1}{m+n+k}} : m, n, k : 1, 2, 3, ...\right\} \tag{2}$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in χ^3.

Throughout the article $w^3, \chi^3(\Delta), \Lambda^3(\Delta)$ denote the spaces of all, triple gai difference sequence spaces and triple analytic difference sequence spaces respectively.

For a triple sequence $x \in w^3$, we define the sets

$\chi^3(\Delta) = \left\{x \in w^3 : ((m + n + k)!\, |\Delta x_{mnk}|)^{1/m+n+k} \to 0\, as\, m, n, k \to \infty\right\}$

$\Lambda^3(\Delta) = \left\{x \in w^3 : sup_{m,n,k} |\Delta x_{mnk}|^{1/m+n+k} < \infty\right\}$.

The space $\Lambda^3(\Delta)$ is a metric space with the metric

$$d(x,y) = sup_{m,n,k}\left\{|\Delta x_{mnk} - \Delta y_{mnk}|^{1/m+n} : m,n,k = 1,2,\cdots\right\}$$

for all $x = (x_{mnk})$ and $y = (y_{mnk})$ in $\Lambda^3(\Delta)$.

The space $\chi^3(\Delta)$ is a metric space with the metric

$$d(x,y) = sup_{mnk}\left\{((m+n+k)!\,|\Delta x_{mnk} - \Delta y_{mnk}|)^{1/m+n+k} : m,n,k = 1,2,\cdots\right\}$$

for all $x = (x_{mnk})$ and $y = (y_{mnk})$ in $\chi^3(\Delta)$.

Now we define the following sequence spaces: Let $s \geq 0$ be real number and $v = (v_{mnk})$ be non-zero real number sequence, then

$$\chi^3(\Delta_v^m, s, p) = \left\{x = (x_{mnk}) \in w^3 : (mnk)^{-s}\left(((m+n+k)!\,|\Delta_v^m x_{mnk}|)^{1/m+n+k}\right)^{p_{mnk}} \to 0 \quad (m,n,k\to\infty), s \geq 0\right\}$$

$$\Lambda^3(\Delta_v^m, s, p) = \left\{x = (x_{mnk}) \in w^3 : sup_{m,n,k}(mnk)^{-s}\left(|\Delta_v^m x_{mnk}|^{1/m+n+k}\right)^{p_{mnk}} < \infty, s \geq 0\right\}$$

where $\Delta_v^0 x_{mnk} = (v_{mnk}x_{mnk})$, $\Delta_v x_{mnk} = v_{mn}x_{mn} - v_{mn+1}x_{mn+1} - v_{mn+2}x_{mn+2} - v_{m+1n}x_{m+1n} - v_{m+1n+1}x_{m+1n+1} - v_{m+1n+2}x_{m+1n+2} - v_{m+2n}x_{m+2n} - v_{m+2n+1}x_{m+2n+1} - v_{m+2n+2}x_{m+2n+2}$,

$\Delta_v^m x_{mn} = \Delta\Delta_v^{m-1}x_{mn} = \Delta_v^{m-1}x_{mn} - \Delta_v^{m-1}x_{mn+1} - \Delta_v^{m-1}x_{mn+2} - \Delta_v^{m-1}x_{m+1n} - \Delta_v^{m-1}x_{m+1n+1} - \Delta_v^{m-1}x_{m+1n+2} - \Delta_v^{m-1}x_{m+2n} - \Delta_v^{m-1}x_{m+2n+1} - \Delta_v^{m-1}x_{m+2n+2}$

We get the following sequence spaces from the above sequence spaces by choosing some special p, m, s and v.

If $s = 0, m = 1$ and

$$v = \begin{pmatrix} 1 & 1 & ...1 & 1 & 0... \\ 1 & 1 & ...1 & 1 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 1 & ...1 & 1 & 0... \\ 0 & 0 & ...0 & 0 & 0... \end{pmatrix}$$

with 1 upto $(m,n,k)^{th}$ position and zero otherwise and $p_{mnk} = 1$ for all m,n,k. We have

$$\chi^3(\Delta) = \left\{x = (x_{mnk}) : \Delta x \in \chi^3\right\},$$
$$\Lambda^3(\Delta) = \left\{x = (x_{mnk}) : \Delta x \in \Lambda^3\right\}.$$

If $s = 0$ and $p_{mnk} = 1$ for all m,n,k we have the following sequence spaces

$$\chi^3(\Delta_v^m) = \left\{x = (x_{mnk}) \in w^3 : \Delta_v^m x \in \chi^3\right\},$$
$$\Lambda^3(\Delta_v^m) = \left\{x = (x_{mnk}) \in w^3 : \Delta_v^m x \in \Lambda^3\right\}.$$

If $s = 0, m = 0$ and

$$v = \begin{pmatrix} 1 & 1 & ...1 & 1 & 0... \\ 1 & 1 & ...1 & 1 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 1 & ...1 & 1 & 0... \\ 0 & 0 & ...0 & 0 & 0... \end{pmatrix}$$

with 1 upto $(m,n,k)^{th}$ position and zero otherwise. We have the following sequence spaces

$$\chi^3(p) = \left\{x = (x_{mnk}) \in w^3 : ((m+n+k)!\,|x_{mnk}|)^{p_{mnk}/m+n+k} \to 0, (m,n,k\to\infty)\right\}$$

$$\Lambda^3(p) = \left\{x = (x_{mnk}) \in w^3 : sup_{m,n,k}|x_{mnk}|^{p_{mnk}/m+n+k} < \infty\right\}$$

If $m = 0$ and

$$v = \begin{pmatrix} 1 & 1 & ...1 & 1 & 0... \\ 1 & 1 & ...1 & 1 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 1 & ...1 & 1 & 0... \\ 0 & 0 & ...0 & 0 & 0... \\ & & & & \end{pmatrix}$$

with 1 upto $(m,n,k)^{th}$ position and zero otherwise. We have the following sequence spaces

$$\chi^3(p,s) = \left\{ x = (x_{mnk}) \in w^3 : (mnk)^{-s}\left((m+n+k)!\,|x_{mnk}|\right)^{p_{mnk}/m+n+k} \to 0, (m,n,k \to \infty), s \geq 0 \right\},$$
$$\Lambda^3(p,s) = \left\{ x = (x_{mnk}) \in w^3 : sup_{m,n,k}(mnk)^{-s}|x_{mnk}|^{p_{mnk}/m+n+k} < \infty, s \geq 0 \right\},$$

If $s = 0, m = 0$ and $p_{mnk} = 1$

$$v = \begin{pmatrix} 1 & 1 & ...1 & 1 & 0... \\ 1 & 1 & ...1 & 1 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 1 & ...1 & 1 & 0... \\ 0 & 0 & ...0 & 0 & 0... \\ & & & & \end{pmatrix} \text{ for all } m,n,k$$

with 1 upto $(m,n,k)^{th}$ position and zero otherwise. We have χ^3 and Λ^3.

If $s = 0$ we have $\chi^3(\Delta_v^m, p)$ and $\Lambda^3(\Delta_v^m, p)$

For a subspace ψ of a linear space is said to be sequence algebra if $x, y \in \psi$ implies that $x \cdot y = (x_{mnk}y_{mnk}) \in \psi$, see Kamptan and Gupta [10].

A sequence E is said to be solid (or normal) if $(\lambda_{mnk}x_{mnk}) \in E$, whenever $(x_{mnk}) \in E$ for all sequences of scalars $(\lambda_{mnk} = k)$ with $|\lambda_{mnk}| \leq 1$.

If X is a linear space over the field $\mathbb{C}$, then a paranorm on X is a function $g : g(\theta) = 0$ where $\theta = (0,0,0,\cdots), g(-x) = g(x), g(x+y) \leq g(x) + g(y)$ and $|\lambda - \lambda_0| \to 0, g(x - x_0)$ imply $g(\lambda x - \lambda_0 x_0) \to 0$, where $\lambda, \lambda_0 \in C$ and $x, x_0 \in X$. A paranormed space is a linear space X with a paranorm g and is written (X, g).

In this paper, we define some new sequence spaces and give some topological properties of the sequence spaces $\chi^3(\Delta_v^m, s, p)$ and $\Lambda^3(\Delta_v^m, s, p)$ and investigate some inclusion relations.

3. Main results

Theorem 3.1. *The following statements are hold*

(i)$\chi^3(\Delta_v^m, s) \subset \Lambda^3(\Delta_v^m, s)$ *and the inclusion is strict.*

(ii)$X(\Delta_v^m, s, p) \subset X\left(\Delta_v^{m+1}, s, p\right)$ *does not hold in general for any* $X = \chi^3$ *and* Λ^3.

Proof. (i) If we choose $s = 0$,

$$x = \begin{pmatrix} 1 & 0 & ...1 & 0 & 0... \\ 1 & 0 & ...1 & 0 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 0 & ...1 & 0 & 0... \\ 0 & 0 & ...0 & 0 & 0... \\ & & & & \end{pmatrix} \text{ and } v = \begin{pmatrix} 1 & 1 & ...1 & 1 & 0... \\ 1 & 1 & ...1 & 1 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 1 & ...1 & 1 & 0... \\ 0 & 0 & ...0 & 0 & 0... \\ & & & & \end{pmatrix}$$

Hence $x \in \Lambda^3(\Delta_v^m, s)$, but $x \notin \chi^3(\Delta_v^m, s)$

(ii) Let $v = \begin{pmatrix} 1 & 1 & ...1 & 1 & 0... \\ 1 & 1 & ...1 & 1 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 1 & ...1 & 1 & 0... \\ 0 & 0 & ...0 & 0 & 0... \\ & & & & \end{pmatrix}$, $p = (p_{mnk})$ and $x = (x_{mnk})$ given by

$p_{mnk} = 1, ((m+n+k)! \, |x_{mnk}|)^{1/m+n+k} = m^2 n^2 k^2$ if m, n, k is odd

$p_{mnk} = 2, ((m+n+k)! \, |x_{mnk}|)^{1/m+n+k} = mnk$ if m, n, k if even

0 otherwise

Since for $m, n, k \geq 1$, $\left((m+n+k)! \left|\Delta_v^0 x_{mnk}\right|\right)^{p_{mnk}/m+n+k} = ((m+n+k)! \, |x_{mnk}|)^{p_{mnk}/m+n+k} = m^2 n^2 k^2$

$m^{-3} n^{-3} k^{-3} \left((m+n+k)! \left|\Delta_v^0 x_{mnk}\right|\right)^{p_{mnk}/m+n+k} = m^{-3} n^{-3} k^{-3} m^2 n^2 k^2 = m^{-1} n^{-1} k^{-1} \to 0 \, (m, n, k \to \infty)$ and for $j \geq 1$

$((6j)! \, |\Delta_v x_{2j,2j,2j}|)^{p_{2j,2j,2j}/6j} = \left(6j^3 + 6j^2 + 1\right)^2, (6j)^{-3} \left((6j)! \, |\Delta_v x_{2j,2j,2j}|\right)^{p_{2j,2j,2j}/6j} \geq 6j \to \infty \, (j \to \infty)$.

Now, we can see that $x \in \chi^3 \left(\Delta_v^0, 3, p\right)$ and $x \notin \Lambda^3 \left(\Delta_v^0, 3, p\right)$, which imply that $X \left(\Delta_v^m, s, p\right)$ is not a subset of $X \left(\Delta_v^{m+1}, s, p\right)$. This completes the proof. □

Theorem 3.2. *$\chi^3 \left(\Delta_v^m, s, p\right)$ and $\Lambda^3 \left(\Delta_v^m, s, p\right)$ are linear spaces over the complex field $\mathbb{C}$.*

Proof. Suppose that $M = max \left(1, sup_{m,n,k \geq \mathbb{N}} p_{mnk}\right)$ Since $p_{mnk}/M \leq 1$, we have for all m, n, k

$$\left|\Delta_v^m \left(x_{mnk} + y_{mnk}\right)\right|^{p_{mnk}/M} \leq \left(\left|\Delta_v^m x_{mnk}\right|^{p_{mnk}/M} + \left|\Delta_v^m y_{mnk}\right|^{p_{mnk}/M}\right) \tag{3}$$

and for all $\lambda \in \mathbb{C}$

$$|\lambda|^{p_{mnk}/M} \leq Max \left(1, |\lambda|\right) \tag{4}$$

Now the linearity follows from (3) and (4). This completes the proof. □

Theorem 3.3. *Let $N_1 = min \left\{n_0 : sup_{m,n,k \geq n_0} (mnk)^{-s} \left(\left((m+n+k)! \, |\Delta_v^m x_{mnk}|\right)^{1/m+n+k}\right)^{p_{mnk}} < \infty\right\}$, $N_2 = min \left\{n_0 : sup_{m,n,k \geq n_0} p_{mnk} < \infty\right\}$, $N_3 = min \left\{n_0 : sup_{m,n,k \geq n_0} < \infty\right\}$ and $N = max \left\{N_1, N_2, N_3\right\}$ $\chi^3 \left(\Delta_v^m, s, p\right)$ is a paranormed space with*

$$g(x) = \sum_{m=1}^{i} \sum_{n=1}^{j} \sum_{k=1}^{r} (m+n+k)! \, |x_{mnk}| + lim_{N \to \infty} sup_{m,n,k \geq N} (mnk)^{-S/M} \left((m+n+k)! \, |\Delta_v^m x_{mnk}|\right)^{p_{mnk}/M} \tag{5}$$

if and only if $\mu > 0$, where $\mu = lim_{N \to \infty} inf_{m,n,k \geq N} p_{mnk}$ and $M = max \left(1, sup_{m,n,k \geq N} p_{mnk}\right)$

Proof. (i)**Necessity:** Let $\chi^3 \left(\Delta_v^m, s, p\right)$be a paranormed space with (5) and suppose that $\mu = 0$. Then $\alpha = inf_{m,n,k \geq N} p_{mnk} = 0$ for all $N \in \mathbb{N}$ and hence we obtain $g(\lambda x) = \sum_{m=1}^{i} \sum_{n=1}^{j} \sum_{k=1}^{r} (m+n+k)! \, |x_{mnk}| + lim_{N \to \infty} sup_{m,n,k \geq N} (mnk)^{-s} |\lambda|^{p_{mnk}/M} = 1$ for all $\lambda \in (0, 1]$, where $x = \alpha \in \chi^3 \left(\Delta_v^m, s, p\right)$. whence $\lambda \to 0$ does not imply $\lambda x \to \theta$, when x is fixed. But this contradicts to (5) to be a paranorm.

Sufficiency: Let $\mu > 0$. It is trivial that $g(\theta) = 0, g(-x) = g(x)$ and $g(x+y) \leq g(x) + g(y)$. Since $\mu > 0$ there exists a positive number β such that $p_{mnk} > \alpha, \beta$ for sufficiently large positive integer m, n, k. Hence for any $\lambda \in \mathbb{C}$, we may write $|\lambda|^{p_{mnk}} \leq max \left(|\lambda|^M, |\lambda|^\alpha, |\lambda|^\beta\right)$ for sufficiently large positive integers $m, n, k \geq \mathbb{N}$. Therefore, we obtain that $g(\lambda x) \leq max \left(|\lambda|, |\lambda|^{\alpha/M}, |\lambda|^{\beta/M}\right) g(x)$ using this, one can prove that $\lambda x \to \theta$, whenever x is fixed and $\lambda \to 0$, (or) $\lambda \to 0$ and $x \to \theta$, or λ is fixed and $x \to \theta$. This completes the proof. □

Theorem 3.4. *Let $0 < p_{mnk} \leq q_{mnk} \leq 1$ for all $m, n, k \in \mathbb{N}$, then (i)$\Lambda^3 \left(\Delta_v^m, s, p\right) \subseteq \Lambda^3 \left(\Delta_v^m, s, q\right)$ (ii)$\chi^3 \left(\Delta_v^m, s, p\right) \subseteq \chi^3 \left(\Delta_v^m, s, q\right)$*

Proof. **(i):** Let $x \in \Lambda^3 (\Delta_v^m, s, p)$. Then there exists a constant $M > 1$ such that

$(mnk)^{-s} |\Delta_v^m x_{mn}|^{q_{mnk}/m+n+k} \leq M$ for all m, n, k

and so

$(mnk)^{-s} |\Delta_v^m x_{mnk}|^{q_{mnk}/m+n+k} \leq M$ for all m, n, k

suppose that $x^i \in \Lambda^3 (\Delta_v^m, s, q)$ and $x^i \to x \in \Lambda^3 (\Delta_v^m, s, p)$. Then for every $o < \epsilon < 1$, there exist N such that for all m, n, k

$$(mnk)^{-s} \left|\Delta_v^m \left(x_{mnk}^{(i)} - x_{mnk}\right)\right|^{p_{mnk}/m+n+k} < \epsilon \text{ for all } i > N$$

Now,

$$(mnk)^{-s} \left|\Delta_v^m \left(x_{mnk}^{(i)} - x_{mnk}\right)\right|^{q_{mnk}/m+n+k} < (mnk)^{-s} \left|\Delta_v^m \left(x_{mnk}^{(i)} - x_{mnk}\right)\right|^{p_{mnk}/m+n+k} < \epsilon \text{ (for all } i > N)$$

Therefore $x \in \Lambda^3 (\Delta_v^m, s, q)$. This completes the proof.

(ii): It is easy. Therefore omit the proof. □

Proposition 3.5. *For $X = \chi^3$ and Λ^3, then we obtain (i)$X (\Delta_v^m, s, p)$ is not sequence algebra, in general (ii) $X (\Delta_v^m, s, p)$ is not solid, in general.*

Proof. **(i)** This result is clear from the following example : □

Example 3.6. *(1) Let $p_{mnk} = 1, (m + n + k)!v_{mnk} = \frac{1}{(mnk)^{2(m+n+k)}}, (m + n + k)!x_{mnk} = (mnk)^{2(m+n+k)}$ and $(m + n + k)!y_{mnk} = (mnk)^{2(m+n+k)}$ for all m, n, k. Then we have $x, y \in \chi^3 (\Delta, 0, p)$ but $x, y \notin \chi^3 (\Delta, 0, p)$ with $m = 1$ and $s = 0$.*

Proof. **(ii)** This result is clear from the following example □

Example 3.7. *(2) Consider* $x_{mnk} = \begin{pmatrix} 1 & 1 & ...1 & 1 & 0... \\ 1 & 1 & ...1 & 1 & 0... \\ . & & & & \\ . & & & & \\ . & & & & \\ 1 & 1 & ...1 & 1 & 0... \\ 0 & 0 & ...0 & 0 & 0... \\ & & & & \end{pmatrix} \in \chi^3 (\Delta_v^m, s, p)$ *Let* $p_{mnk} = 1, \alpha_{mnk} = (-1)^{m+n+k}$, *then* $\alpha_{mnk}x_{mnk} \notin \chi^3 (\Delta_v^m, s, p)$ *with* $m = 1$ *and* $s = 0$.

The following proposition's proof is a routine verification.

Proposition 3.8. *For $X = \chi^3$ and Λ^3, then we obtain*

(i) $s_1 < s_2$ implies $X (\Delta_v^m, s_1, p) \subset X (\Delta_v^m, s_2, p)$,

(ii) Let $0 < inf p_{mnk} < p_{mnk} \leq 1$ then $X (\Delta_v^m, s, p) \subset X (\Delta_v^m, s)$,

(iii) Let $1 \leq p_{mnk} \leq sup_{mnk} p_{mnk} < \infty$, then $X (\Delta_v^m, s) \subset X (\Delta_v^m, s, p)$,

(iv) Let $0 < p_{mnk} \leq q_{mnk}$ and $\left(\frac{q_{mnk}}{p_{mnk}}\right)$ be bounded, then $X (\Delta_v^m, s, q) \subset X (\Delta_v^m, s, p)$.

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