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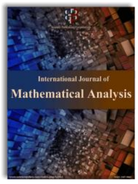


Non standard young tableaux of Γ_1 – non deranged permutation group $G_p^{\Gamma_1}$

Isah Abor Garba, Ojonugwa Ejima *, Kazeem Olalekan Aremu, Hamisu Usman

Usmanu Danfodiyo University, Sokoto, Nigeria
**Corresponding author E-mail: unusoj1@yahoo.com*

Abstract

There is a link between the representation of the permutation group $\mathcal{G}_p^{\Gamma_1}$ ($p \geq 5$ *and* p *a prime*) and the combinatorial object called Young tableaux. In this paper, we describe the representation of Γ_1 - non deranged permutation group via the Young tableaux, and we establish that every Young tableaux of this permutation group is not standard.

Keywords: *Representations; Combinatorial Objects; Non - Standard Tableaux; Non-Deranged and Permutation Group.*

1. Introduction

Gauss in the early 19th century introduced an idea that has opened up the study of groups, he worked on the characters of finite abelian group, this idea was later extended to non finite abelina group. Representations of permutation groups is not limited to the beautiful results of the group characters alone; Alfred Young a mathematician from Cambridge University introduced a combinatoric diagram called Young tableaux [12], this diagrams has provided a convenient way to describe the representations of the symmetric group, general linear groups and to study their properties. This theory was then applied to the study of the symmetric group by [13] and was further developed by many other mathematicians. In recent years, it has been used by several other researchers to showcase the beauty of representation theory.

[15] explored an intimate connection between two seemingly unrelated objects: on one hand is the representations of $\mathcal{S}_n$; on the other hand is the combinatorial objects called Young tableaux, he described the construction of Specht modules which are irreducible representations of $\mathcal{S}_n$, he also introduced tabloids which he used to construct a representation of symmetric group known as the permutation module, some interesting results such as the branching rule and Young's rule were also established. [11] Described symmetric groups to be of great value in tensor analysis as a means to describe the tensor space in terms of symmetries under permutations of indices. He further develop a diagrammatic approach using Young tableaux to determine the irreducible representations of $\mathcal{S}_n$. [9] studied and established some interesting results, from his studies of Young's Natural Representations of $\mathcal{S}_4$. He calculated all inequivalent irreducible representations of $\mathcal{S}_4$ by specifying the matrices for adjacent transpositions and indicated how to obtain general permutations in $\mathcal{S}_4$ from these transpositions. He also employed standard Young tableaux methods as found in [2] to pictorially represent $\mathcal{S}_4$. Another approach to representation through Young tableaux was used by [8], they presented a different description of finite-dimensional complex irreducible representations of the symmetric groups. In their work they established an alternative construction to the combinatorial objects, which uses tabloids, polytabloids, and Specht modules, they were able to show how the combinatorial objects of the theory (Young diagrams and tableaux) arose from the internal structure of the symmetric group. this field of mathematics was also extended to graphs, this is clear in the work of [4] who showed the connection between the combinatorial objects to webs, a type of planar graph that occur naturally in knot theory and representation theory.

Over time through further studies other form of permutation patterns and groups has been discovered and established, for instance, The Poincaré group, named after Henri Poincaré was first defined by [7] being the group of Minkowski spacetime isometries. It is a ten-generator non-abelian Lie group which has a fundamental applications in physics; [14] through an extensive group-theoretical treatment of linear wave equations on Minkowski spacetime of arbitrary dimension $D \geq 3$ were able to use the induced representations to reduces the problem of classifying the representations of the Poincar´e group to a classification and representations the stability subgroups only. Similarly, the application of algebra to all works of life cannot be over emphasized; example of such applications is the group theoretical interpretation of Bara'at Model by [6] that gave birth to a new phase in algebra. This interpretation has lead to the discovery of a new permutation pattern called the Aunu numbers. The Aunu permutation patterns first arose, out of attempts to provide some combinatorial interpretations of some succession schemes and today series of results ranging from its avoiding class to its properties which have also been studied. More recently the Catalan numbers were used by [3] to develop the scheme for prime numbers $P \geq 5$ and $\Omega \subseteq N$ which generate the cycles of permutation patterns using $\omega_i = ((1)(1+i)_{mp}(1+2i)_{mp}...(1+(p-1)i)_{mp})$ to determine the arrangements.

This permutation pattern was further studied by [10] to establish a permutation group, this they achieved by embedding an identity element {1} in the collection of $\omega_i = (1)(1+i)_{mp}(1+2i)_{mp}...(1+(p-1)i)_{mp}$

Besides the general representations of groups and symmetric groups, Researchers have over time looked also at permutation groups with some certain properties; one that comes to mind is the permutation patterns that have any of the element fixed or the ones that has no fixed element, here the idea of deranged and non-deranged permutations surfaces. It is in line with this understanding that [1] extended the Aunu permutation pattern of [3] to a two line notation and to establish a different permutation group (Γ_1 - non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$), this enabled them a further study on the theoretical and algebraic properties of this two lines structure. Besides the representations of this permutation group $\mathcal{G}_p^{\Gamma_1}$ have also been studied by [5]; in their work they established $\mathcal{G}_p^{\Gamma_1}$ as a $\mathcal{FG}_p$ - module and revealed some interesting results about the character of each element of $\mathcal{G}_p^{\Gamma_1}$. However, in this paper, we will study the Young tableaux of this Γ_1 - non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$.

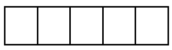
2. PRELIMINARIES

In this section before we outline the main results in this research, we attempt to define some basic concepts and notations that will help in further understanding of this work;

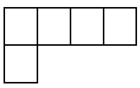
Definition 2.1: *A partition of a positive integer n is a sequence of positive integers $\lambda = (\lambda_1, \lambda_2, ..., \lambda_t)$ satisfying $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_t > 0$ and $n = \lambda_1 + \lambda_2 + ... + \lambda$. We write $\lambda \vdash n$ to denote that λ is a partition of n . For instance, the number 5 has seven partitions: (5), (4,1), (3,1,1), (3,2), (2,1,1,1), (2,2,1), (1,1,1,1,1). We can also represent partitions pictorially using Young diagrams as follows.*

Definition 2.2: *The Young diagram is a finite collection of boxes arranged in left-justified rows, with the row sizes weakly decreasing number of boxes in each row. The Young diagram associated to the partition $\lambda = (\lambda_1, \lambda_2, ..., \lambda_t)$ is the one that has l rows, and λ_i boxes on the ith row.*

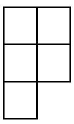
For instance, some of the Young diagrams corresponding to the partitions of 5 are



(5)



(4, 1)



(2, 2, 1)

Definition 2.3: *A Young tableau is obtained by filling in the boxes of the Young diagram with symbols taken from ordered set, it is a Young diagram in which the n boxes have been filled with the numbers 1, . . . , n, each number used exactly once; simply put as a filling of n distinct entries, arbitrarily assigned to boxes of the Young diagram.*

Suppose $\lambda \vdash n$. A Young tableaux t of shape λ is obtained by filling in the boxes of a Young diagram of λ with 1, 2... n with each number occurring once. For example, here are some of the tableaux corresponding to the partition (3, 2)

1	2	3
4	5	

1	1	2
2	3	

1	2	5
3	4	

Definition 2.4: *A standard Young tableaux is a Young tableaux whose the entries are increasing across each row and each column For instance, the only standard Young tableaux for (3,2) as given the example above are:*

1	2	3
4	5	

1	3	5
2	4	

Definition 2.5: *A semi standard Young tableaux is a filling which is weakly increasing across each row and strictly increasing down each column, but may have repeated entries; for instance, the semi standard Young tableaux for (3,2) is as follows.*

1	3	4
3	5	

1	3	3
2	4	

1	1	3
4	5	

Definition 2.6 [5]: *Γ_1 - non-deranged permutation group $\mathcal{G}_p^{\Gamma_1}$ is a permutation group with a fixed element on the first column from the left.*

Definition 2.7 $\omega_i \in \mathcal{G}_p^{\Gamma_1}$ **[5]:** *Let Ω be a non-empty ordered set such that $\Omega \subset N$. Let $\mathcal{G}_p^{\Gamma_1}$ = ω_i $1 \leq i \leq (p-1)$ be a subgroup of symmetry group Sp such that every ω_i is generated by arbitrary set Ω for any prime $p \geq 5$ using the following*

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & (1+i)_{mp} & (1+2i)_{mp} & . & . & . & (1+(p-1)i)_{mp} \end{pmatrix} \quad (1)$$

EXAMPLE 2.7.1

For p=5 equation (1) will generate permutation group $\mathcal{G}_5$

$$\omega_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} = \{e\} \text{ (the identity permutation)}$$

$$\omega_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & 2 & 4 \end{pmatrix},$$

$$\omega_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 2 & 5 & 3 \end{pmatrix},$$

$$\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 4 & 3 & 2 \end{pmatrix}$$

Theorem 2.8 [2]

Every permutation on a finite set can be written as a product of disjoint cycles.

Definition 2.9: *From theorem 2.8; permutations can be decompose into disjoint cycles. For instance the example 2.6.1 above*

$\omega_1 = (e)$, $\omega_2 = (2354)$ $\omega_3 = (3245)$ and $\omega_4 = (43)(25)$

i.e., ω_2 is decomposed into (2354) which denotes the permutation that sends $1 \to 1$, $2 \to 3$, $3 \to 5$, $5 \to 4$ and $4 \to 2$

Likewise ω_4 is decomposed into (25) (34) which denotes the permutation that sends $1 \to 1$, $2 \to 5$, $5 \to 2$, $3 \to 4$ and $4 \to 3$.
Note the number of element(s) in each cycles denote the length of such cycle, hence the cycle type of ω_4 = (43) (25) is the partition whose parts are the length of the decomposed cycles. (25)(34) = $\omega_4 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 4 & 3 & 2 \end{pmatrix} \in \mathcal{G}_5^{\Gamma_1}$ has partition (2, 2, 1).

3. Results

3.1. Theorem

Let $\mathcal{G}_p$ be a Γ_1 - non deranged permutation group $p \geq 5$ (p a prime). Every $\omega_{p-1} \in \mathcal{G}_p^{\Gamma_1}$ has $(\frac{p-1}{2})$ cycles of length 2.

Proof
From equation (1) above ω_i of the permutation group $\mathcal{G}_p^{\Gamma_1}$ is given as

$$\omega_i = \begin{pmatrix} 1 & 2 & 3 & . & . & . & p \\ 1 & (1+i)_{mp} & (1+2i)_{mp} & . & . & . & (1+(p-1)i)_{mp} \end{pmatrix}$$

It is clear that every ω_{p-1} is always of the

□

Form

$$\omega_{p-1} = \begin{pmatrix} 1 & 2 & 3 & . & . & . & (p-1) & p \\ 1 & p & (p-1) & . & . & . & 3 & 2 \end{pmatrix}$$

Then ω_{p-1} in cycle form will be (2 p) (3 (p-1)) ((4 (p-2)) ...

3.2. Proposition

The length of cycle(s) of any $\omega_i \in \mathcal{G}_p^{\Gamma_1}$ is a factor of the cardinality of the group $\mathcal{G}_p^{\Gamma_1}$

3.3. Theorem

The Young tableaux of Γ_1 - non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$ is always non-standard for all $\omega_i \neq \omega_1$.

Proof
From definition 2.6 above, the permutation group $\mathcal{G}_p^{\Gamma_1}$ is a non-deranged type of permutation group, and for every $\omega_i \neq \omega_1$, then it's obvious that the cycle notation of each $\omega_i \neq \omega_1$ will have cycle length of 1 because of the fixed $1 \to 1$; it's representations which using the Young tableaux the 1 will always be below the other cycle length greater than 1 and this contradict the definition of standard tableaux, hence the proof.

4. Conclusion

Young tableaux from the review of previous research is a combinatorics object which can be used to diagrammatically show weather a permutation group has a standard, semi standard or non-standard tableaux. In this work it has been shown that the representations of the Γ_1 - non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$ is always non-standard for all $\omega_i \neq \omega_1$ and this is simply because of the derangement at the first entry of the permutations.

References

[1] Aremu, K.O. Ahmad, R., Ejima, O. & Usman, H. (2016): Abstract group theoretical properties of Γ_1 - non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$: Nigerian Journal of Basic and Applied Sciences.

[2] Bruce, S (2001). The Symmetric Group: Rep resentations, Combinatorial Algorithms, and Symmetric Functions: *Springer*

[3] Garba, A.I., & Ibrahim A.A (2010). A new method of constructing a Variety of Finite Group Based on Some Succession Scheme: *International Journal of Physical Science* 2(3) 23 – 26.

[4] Heather, M.R., Matthew H. & Julianna, T. (2015). The Robinson Schensted Correspondence and A2-Web Bases. http://scholarship.richmond.edu/mathcs-faculty publications.

[5] Ibrahim, A.A, Ejima, O, Aremu, K.O, (2016). On the Representations of Γ_1 – non deranged permutation group $\mathcal{G}_p^{\Gamma_1}$ Advances in Pure Mathematics, 6, 608-614.

[6] Ibrahim, A.A (2004). Group Theoretical In terpretation of Bara'at Model: Mathematical Association of Nigeria, Proceedings of Annual National Conference, 35 – 46.

[7] Minkowski, H (1909). Space and time http://minkowskiinstitute.org/mip/books/

[8] Okounkov, A., & Vershik, A. (1996). A new approach to representation theory of symmetric Groups: Selecta Math.,New Ser., 2, 581–605. https://doi.org/10.1007/bf02433451.

[9] Quinton, W. (2008) Young's Natural Representations of S4: arXiv: 1112.0687v1 [math.RT].

[10] Usman, A. & Ibrahim, A.A (2011) a New Generating Function for Aunu Patterns: Application in Integer Group Modulo n. Nigerian Journal of Basic and Applied Sciences, 19(1): 1 – 4. https://doi.org/10.4314/njbas.v19i1.69337.

[11] William, F. (1997). Young Tableaux with applications to representations theory and geometry: The Press Syndicate of the University of Cambridge.

[12] Wikipedia, the free encyclopedia. https://en.wikipedia.org/wiki/Alfred-Young-disambiguation.

[13] Wikipedia, the free encyclopedia. https://en.wikipedia.org/wiki/Frobenius's-thoerem-(group-theory).

[14] Xavier, B. & Nicolas, B. (2006) the unitary representations of the Poincar´e group in any spacetime dimension. arXiv:hep-th/0611263v1

[15] Yufei, Z. (2008): Young Tableaux and the representations of the Symmetric Group. https://pdfs.semanticscholar.org/4703/420974ab8fcd6057f35644be9b251d35edd1.pdf.