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Research paper

Holographic Dark Energy Model With Time Varying Deceleration Parameter

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Abstract

Two minimally interacting fluids; dark matter and holographic dark energy components has been studied in a spatially homogeneous and anisotropic Bianchi type-I space-time. The solutions of the Einstein's field equations are obtained under the assumption of time varying deceleration parameter (Abdussattar and S. Prajapati, Astrophys. Space Sci. 331, 65, 2011) which represents transition of the universe from the early decelerating phase to the recent accelerating phase. It is shown that for large expansion the model reduces to ΛCDM model while for suitable choice of interaction between dark matter and holographic dark energy the anisotropy parameter of the universe approaches to zero for large cosmic time and the coincidence parameter increases with increase in time. Allowing for time dependent deceleration parameter the solutions of the field equations and some physical and geometric properties of the model along with physical acceptability of the solutions have also been discussed in details.

Keywords: *Bianchi Type-I Space-Time; Time Varying Deceleration Parameter; Holographic Dark Energy.*

1. Introduction

The recent cosmological observations of Type Ia supernovae [1-2] indicate that the universe is currently accelerating. These results, when combined with the observations of Cosmic Microwave Background [3, 4] and Large Scale Structure [5-6], strongly suggest that the universe is spatially flat and dominated by an exotic component with large negative pressure called as dark energy [7-10]. Since it has been proven that the expansion of the universe is accelerated, the physicists and astronomers started considering the dark energy cosmological observations indicated that at about $2/3$ of the total energy of the universe is attributed by dark energy and $1/3$ is due to dark matter [11].There are many candidates for dark energy. Among the many different approaches to describe the dark cosmological sector, so called holographic dark energy models have received considerable attention [12-15].

The holographic principle emerged in the context of black-holes, where it was noted that a local quantum field theory cannot fully describe the black holes [16]. Some long standing debates regarding the time evolution of a system, where a black hole forms and then evaporates, played the key role in the development of the holographic principle [17-19]. Cosmological versions of holographic principle have been discussed in various literatures [20-22]. Easther et al. [22] proposed that the holographic principle be replaced by the generalized second law of thermodynamics when applied to time-dependent backgrounds and found that the proposition agreed with the cosmological holographic principle proposed by Fischler and Susskind [20] for an isotropic open and flat universe with a fixed equation of state. Numerous cosmological observations have established the accelerated expansion of the universe [22], [23]. In recent times, considerable interest has been stimulated in explaining the observed dark energy by the holographic dark energy model.

In order to obtain exact solutions of the Einstein's field equations, many authors assume various physical or mathematical conditions. Many authors use condition on deceleration parameter. Among this, constant deceleration parameter which was proposed by Berman (1983), linearly varying deceleration parameter proposed by Akarsu and Dereli (2012) and a special form of deceleration parameter proposed by Singha and Debnath (2009) are mostly used by many authors.

Holographic dark energy models have been tested and constrained by various astronomical observations.A special class is models in which holographic dark energy is allowed to interact with dark matter [26-48]. Recently, Sarkar [49] have studied non-interacting holographic dark energy with linearly varying deceleration parameter in Bianchi type-I, Adhav et al. [50] presented interacting holographic dark energy with constant and special form of deceleration parameter. Raut et al. [51] also extended the study of interacting holographic dark energy with Hybrid Expansion Law.

Motivated from the study outlined above, in this paper two minimally interacting fluids i.e. dark matter and holographic dark energy components has been studied in a spatially homogeneous and anisotropic Bianchi type-I space-time. The solutions of the Einstein's field equations are obtained under the assumption of time varying deceleration parameter proposed by [53] which representing transition of the universe from the early decelerating phase to the recent accelerating phase. Allowing for time dependent deceleration parameter the solutions of the field equations, physical acceptability of some physical and geometric properties of the model have also been discussed in details.

2. Metric and basic equations

The anisotropy plays a significant role in the early stage of evolution of the universe and hence the study of anisotropic and homogeneous cosmological models becomes important. The Bianchi type universe models are spatially homogeneous cosmological models that are in general anisotropic. The Bianchi type-I space-time is the straight forward generalization of Robertson-Walkar (RW) metric which is one of the simplest models of the anisotropic universe. Therefore I confine myself to Bianchi type-I model of the form

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2, \quad (1)$$

where a_1, a_2 and a_3 are the directional scale factors and are the function of cosmic time t only.

In case of a radial symmetry between the metric potentials, universe (1) is equal to RW universe.

Some parameters for the LRS Bianchi type-I universe which are important in cosmological observations are given as fallows.

The average scale factor a and spatial volume scale factor V respectively are defined as

$$a = (a_1 a_2 a_3)^{\frac{1}{3}}, \text{and} V = a^3 = a_1 a_2 a_3 \quad (2)$$

The anisotropy parameter of the expansion is expressed as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \quad (3)$$

where

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a}_1}{a} + \frac{\dot{a}_2}{a} + \frac{\dot{a}_3}{a}\right), \quad (4)$$

be the mean Hubble parameter and $H_i\ (i = 1, 2, 3)$ represent the directional Hubble parameters in the directions of x, y and z axes respectively.

The kinematical parameters which are observational interest in cosmology to define the behavior of the universe are the expansion scalar θ, the shear scalar σ^2 and the deceleration parameter q which are respectively given as

$$\theta = \left(\frac{\dot{a}_1}{a} + \frac{\dot{a}_2}{a} + \frac{\dot{a}_3}{a}\right), \quad (5)$$

$$\sigma^2 = \frac{1}{2}\left[H_1^2 + H_2^2 + H_3^2\right] - \frac{\theta^2}{6}, \quad (6)$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \quad (7)$$

3. Field equations and their quadrature forms

Einstein's field equation is given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -\left(T_{ij} + \bar{T}_{ij}\right), \quad (8)$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar and $8\pi G = 1$ and $c = 1$ for relativistic units. The energy momentum tensor for matter and the holographic dark energy are respectively defined as

$$T_{ij} = \rho_m u_i u_j \text{ and } \bar{T}_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j + g_{ij} p_\Lambda, \quad (9)$$

where ρ_m, ρ_Λ are the energy densities of matter and the holographic dark energy and p_Λ is the pressure of the holographic dark energy.

In a co-moving coordinate system, Einstein's field equation (8) with (9) for the considered model (1) leads to following system of field equations.

$$\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} = \rho_m + \rho_\Lambda, \quad (10)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1 a_2} = -p_\Lambda, \quad (11)$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} = -p_\Lambda, \quad (12)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} = -p_\Lambda. \quad (13)$$

Here, overhead dot (.) denotes derivative with respect to time t. Subtracting equation (13) from the equation (12), one may get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0. \quad (14)$$

Now, equation (2) and (14) gives

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V} = 0, \quad (15)$$

which on integrating yield

$$\frac{a_1}{a_2} = \alpha_1 \exp\left(\beta_1 \int \frac{dt}{V}\right). \quad (16)$$

Similarly Subtracting equation (12) from equation (11) and (11) from (13) yields

$$\frac{a_1}{a_3} = \alpha_2 \exp\left(\beta_2 \int \frac{dt}{V}\right), \quad (17)$$

$$\frac{a_2}{a_3} = \alpha_3 \exp\left(\beta_3 \int \frac{dt}{V}\right), \quad (18)$$

where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2$ and β_3 are the integration constants.

In view of $V = a_1 a_2 a_3$, following relation between the constants $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2$ and β_3 are obtained as $\alpha_2 = \alpha_1\alpha_3$, $\beta_2 = \beta_1 + \beta_3$.

Using equations (16) to (18), the values of scale factors a_1, a_2 and a_3 can be written explicitly as

$$a_1 = d_1 a \exp\left(X_1 \int a^{-3} dt\right), \quad (19)$$

$$a_2 = d_2 a \exp\left(X_2 \int a^{-3} dt\right), \quad (20)$$

$$a_3 = d_3 a \exp\left(X_3 \int a^{-3} dt\right), \quad (21)$$

where the relations $d_1 d_2 d_3 = 1$ and $X_1 + X_2 + X_3 = 0$ are satisfied by the constants $d_i\ (i = 1,2,3)$ and $X_i\ (i = 1,2,3)$ respectively.

Using equations (10) to (13) and the barotropic equation of state $p_\Lambda = \omega\rho_\Lambda$, the continuity equation can be obtained as

$$\dot{\rho}_m + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\rho_m + \dot{\rho}_\Lambda + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)(1+\omega)\rho_\Lambda = 0. \quad (22)$$

For minimally interaction between the matter and dark energy, the continuity equation of matter and holographic dark energy can be obtained as

$$\dot{\rho}_m + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\rho_m = 0, \quad (23)$$

$$\dot{\rho}_\Lambda + \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)(1+\omega)\rho_\Lambda. \quad (24)$$

4. Cosmological solutions

For $M_p^{-2} = 8\pi G = 1$ [61], the holographic dark energy density is given by

$$\rho_\Lambda = 3(\alpha H^2 + \beta \dot{H}) \ , \tag{25}$$

where α and β are constants and H is the mean Hubble parameter.
Using equation (24) in (23) yield

$$\omega = -1 - \frac{2\alpha H\dot{H} + \beta \ddot{H}}{3H(\alpha H^2 + \beta \dot{H})} \ . \tag{26}$$

According to the work of Akarsu and Dereli [52], the deceleration parameter which is linear in time with a negative slope is considered as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \ . \tag{27}$$

As time dependence of the scale factor reflects main events in history of the Universe. Moreover it is the deceleration parameter who dictates the expansion rate of the Hubble sphere and determines the dynamics of the observable galaxy number variation: depending on the sign of the deceleration parameter this number either grows (in the case of decelerated expansion), or we are going to stay absolutely alone in the cosmos (if the expansion is accelerated).
One can classify models of Universe on the basis of time dependence of the two parameters. All models can be characterized by whether they expand or contract, and accelerate or decelerate:

a) $H > 0,\ q > 0$: expanding and decelerating
b) $H > 0,\ q < 0$: expanding and accelerating
c) $H < 0,\ q > 0$: contracting and decelerating
d) $H < 0,\ q < 0$: contracting and accelerating
e) $H > 0,\ q = 0$: expanding, zero deceleration
f) $H < 0,\ q = 0$: contracting, zero deceleration
g) $H = 0,\ q = 0$: static

Integrating equation (27) gives the average scale factor as

$$a = \exp\int \frac{dt}{[\int (1+q)dt + \gamma]} \ . \tag{28}$$

Where, γ be the arbitrary constant. It is an easy choice that provides a as an explicit function of time for constant value of deceleration parameter. But, when q is taken to vary with time, an explicit determination of a leads to a possible choice of q as Abdussattar and Prajapati [53]

$$q = -\frac{m}{t^2} + (n-1) \ . \tag{29}$$

Here $m > 0$ is a parameter having the dimension of square of time and $n > 1$ is dimensionless constant. Obviously, the different values of m and n will give rise to different models. For $n > 1$ the model shows decelerating behavior but for $n \le 1$ it shows accelerating behavior. Equation (28) can be integrated to give the time variation of the scale factor as

$$a = \exp\left[\frac{1}{n}\int \frac{dt}{\left(t^2 + \frac{\gamma}{n} + \frac{m}{n}\right)}\right] . \tag{30}$$

Setting $\gamma = 0$ and integrating, the average scale factor a can be obtain as

$$a = \left(t^2 + \frac{m}{n}\right)^{1/2n} . \tag{31}$$

Using equation (31) into equations (19) to (21), the directional scale factors are obtained as

$$a_1 = d_1\left(t^2 + \frac{m}{n}\right)^{1/2n} \exp X_1\left\{t\left(\frac{m}{n} + t^2\right)^{-3/2n}\left(\frac{nt^2}{m} + 1\right)^{3/2n} {}_2F_1\left(\frac{1}{2}, \frac{3}{2n}; \frac{3}{2}; \frac{-nt^2}{m}\right)\right\} \tag{32}$$

$$a_2 = d_2\left(t^2 + \frac{m}{n}\right)^{1/2n} \exp X_2\left\{t\left(\frac{m}{n} + t^2\right)^{-3/2n}\left(\frac{nt^2}{m} + 1\right)^{3/2n} {}_2F_1\left(\frac{1}{2}, \frac{3}{2n}; \frac{3}{2}; \frac{-nt^2}{m}\right)\right\} \tag{33}$$

$$a_3 = d_3\left(t^2 + \frac{m}{n}\right)^{1/2n} \exp X_3\left\{t\left(\frac{m}{n} + t^2\right)^{-3/2n}\left(\frac{nt^2}{m} + 1\right)^{3/2n} {}_2F_1\left(\frac{1}{2}, \frac{3}{2n}; \frac{3}{2}; \frac{-nt^2}{m}\right)\right\} \tag{34}$$

Where ${}_2F_1$ is hyper geometric function.
Here it is observed that all the directional scale factors are the product of exponential and power term. At the initial time $t = 0$ that is when the universe starts to expand, the entire directional scale factors remains constant and approaches to isotropy. Hence, initially the model has no singularity. For $t^2 = -\frac{m}{n}$, all the directional scale factors reduces to zero thus the derived model has a point type singularity at $t^2 = -\frac{m}{n}$.

5. Behavior of geometrical and physical parameters of the model

The volume of the universe is obtained as

$$V = \left(t^2 + \frac{m}{n}\right)^{3/2n} , \tag{35}$$

It is observed that the spatial volume V and the directional scale factors a_1 , a_2 and a_3 are remains constant as $t \to 0$. However, the volume scale factor expands exponentially as t increases and becomes infinitely large at $t = \infty$ which shows the late time acceleration of the universe.
The expression for the mean Hubble parameter and Anisotropy parameter are respectively obtained as

$$H = \frac{t}{n}\left(t^2 + \frac{m}{n}\right)^{-1} , \tag{36}$$

$$A_m = \frac{\left(X_1^2 + X_2^2 + X_3^2\right)}{3t^2}\left(t^2 + \frac{m}{n}\right)^{(2-3/n)} . \tag{37}$$

From the observation the universe starts its expansion from constant volume, the mean Hubble parameter and expansion scalar both are initially large and decreases with the expansion. For large cosmic time the Hubble parameter, expansion scalar and the mean anisotropy parameter all are approaches to zero. Therefore for large cosmic time the anisotropy of the universe damp out and the universe approaches to an isotropic universe. Although the early universe was anisotropy, it approaches to an isotropic universe as dark energy starts to dominate the energy density of the universe at ΛCDM model. Figure (ii) shows the behavior of anisotropy parameter.
Using equation (35) in equation (26), the dark energy Equation of State (EoS) parameter can be obtained as

$$\omega = -1 - \left\{\frac{\left(\frac{2\alpha}{n^2} - \frac{6\beta}{n}\right)\left(t^2 + \frac{m}{n}\right)^{-2} + \left(\frac{8\beta}{n} - \frac{4\alpha}{n^2}\right)t^2\left(t^2 + \frac{m}{n}\right)^{-3}}{\frac{3\beta}{n^2}\left(t^2 + \frac{m}{n}\right)^{-2} + \left(\frac{3\alpha}{n^3} - \frac{6\beta}{n^2}\right)t^2\left(t^2 + \frac{m}{n}\right)^{-3}}\right\} . \tag{38}$$

A large class of scalar field dark energy models has been studied including quintessence $(\omega > -1)$, phantom $(\omega < -1)$ and Quinton (which can cross from the phantom region to the quintessence region). The Quinton scenario of dark energy is designed to understand the nature o dark energy with ω cross -1. From figure (iii) it is observed that in this derived model the dark energy EoS parameter is time dependent. At the initial stage when the universe started to expand, the EoS parameter of the universe has the value $\omega > 0$ i.e. the model behaves like matter dominated once, while late time it become $\omega < 0$. At late time the EoS parameter varies from phantom $\omega < -1$ region and approaches to -1, hence at late time model becomes a ΛCDM model. It is interesting to note that EoS parameter takes a negative value, which is in good agreement within Supernova observations.

From equation (23) and (25), the matter density ρ_m and the holographic dark energy density ρ_Λ are found as

$$\rho_m = k\left(t^2 + \frac{m}{n}\right)^{-3/2n}. \tag{39}$$

$$\rho_\Lambda = \frac{3}{n^2}\left\{(\alpha - 2\beta n)t^2\left(t^2 + \frac{m}{n}\right)^{-2} + \beta n\left(t^2 + \frac{m}{n}\right)^{-1}\right\}. \tag{40}$$

The variation of matter energy density ρ_m and dark energy density ρ_Λ versus time is given in figure (vii).It is observed that holographic dark energy density ρ_Λ is inverse function of cosmic time and initially at $t \to 0$ it starts from some constant value while for infinite expansion of the universe it converges to zero which shows that the model is asymptotically empty and the model is filled with dust matter.

The matter density parameter Ω_m and the holographic dark energy parameter Ω_Λ are respectively given by

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{n^2 k}{3t^2}\left(t^2 + \frac{m}{n}\right)^{(2-3/2n)}$$

And

$$\Omega_\Lambda = \frac{\rho_\Lambda}{3H^2} = (\alpha - 2\beta n) + \beta n\left(1 + \frac{m}{n t^2}\right). \tag{41}$$

With the help of equation (4.11), (4.14) and (4.15), the overall density parameter is found as

$$\Omega = \Omega_m + \Omega_\Lambda = \frac{n^2 k}{3t^2}\left(t^2 + \frac{m}{n}\right)^{(2-\frac{3}{2n})} + (\alpha - 2\beta n) + \beta n\left(1 + \frac{m}{nt^2}\right). \tag{42}$$

It is observed that the energy density is always positive and decreasing function of time t. At the initial stage $t \to 0$ the universe has infinitely large total energy density i.e. the universe start with Big Bang and with the expansion of the universe, the total energy density tends to a finite value(≈ 1). Hence, after some finite time, the models approach to a steady state which is clearly shown in figure (iv).

Coincidence parameter:

The recent observations demands that the ratio of two energy densities i.e. the coincidence parameter $\bar{r} = \frac{\rho_\Lambda}{\rho_m}$ stays constant or varies very slowly, around the present time, with respect to the universe expansion. But, the leading candidate for dark energy, the popular ΛCDM model is not consistent with this observation. This coincidence problem has led numerous authors to consider alternatives to ΛCDM which preserve its stunning successes (Type IaSNe, CMB anisotropies, large-scale structure) but avoid the above difficulty. To avoid the coincidence problem, matter and dark energy must scale each other over a considerably long period of time during the later stage of evolution of the universe. In other words, the ratio of two energy densities $\bar{r} = \frac{\rho_\Lambda}{\rho_m}$ remains constant in spite of their different rates of time evolution.

For this model the coincidence parameter $\bar{r}$ is obtained as

$$\bar{r} = \frac{3}{kn^2}\left\{(\alpha - 2\beta n)t^2\left(t^2 + \frac{m}{n}\right)^{(\frac{3}{2n}-2)} + \beta n\left(t^2 + \frac{m}{n}\right)^{(\frac{3}{2n}-1)}\right\}. \tag{43}$$

The variation of coincidence parameter $\bar{r}$ with respect to cosmic time t is as shown in Figure (v). Observational data shows that present value of the coincidence parameter $\bar{r}$ and the cosmic time t_0 are 2.33 and 13.798 gaga years [55]. It is observed that coincidence parameter $\bar{r}$ at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution of the universe. It is seen from this figure that our result is consistent with the current observational data and also resembles with the work of [55].

The jerk parameter:

Observations confirm that in the early years of the universe, the dark energy would have been too small to counteract the gravity of the matter in the universe, and the expansion would have initially slowed. After the universe grew big enough, though, the dark energy would dominate, and the universe would start to accelerate from some five to six billion years ago [54]. Cosmologists believe that the universe transitioned from deceleration to acceleration in a cosmic jerk. The knowledge of how and when the jerk occurred was an important step in figuring out just what the dark energy is. The deceleration to acceleration transition of the universe occurs for different models with a positive value of the jerk parameter and negative value of the deceleration parameter [56-60]. For example flat ΛCDM models have a constant jerk $j = 1$. Cosmic jerk parameter is a dimensionless third derivative of the scale factor with respect to the cosmic time, defined as

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a}. \tag{44}$$

Above equation can be express in terms of Hubble and deceleration parameter as

$$j(t) = q + 2q^2 - \frac{\dot{q}}{H}. \tag{45}$$

For this model the jerk parameter is obtain as

$$j(t) = 2n^2 + 3n + 1 + \frac{3m}{t^2}(1 - 2n). \tag{46}$$

Figure (vi) shows that the jerk parameter is positive throughout the entire history of the universe and for large cosmic time, the jerk parameter is approaches to some constant positive value.

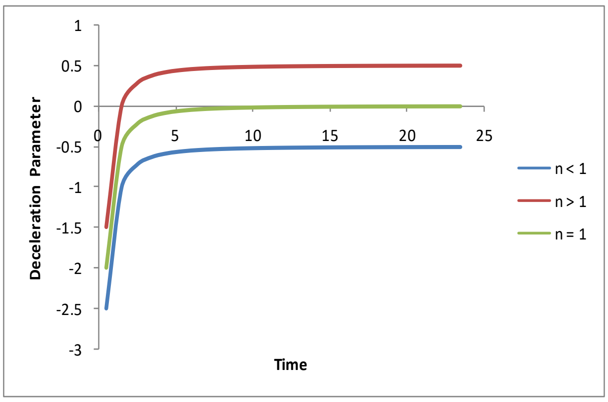

Fig. 1: Variation of Deceleration Parameter versus Time for Appropriate Values of Constants.

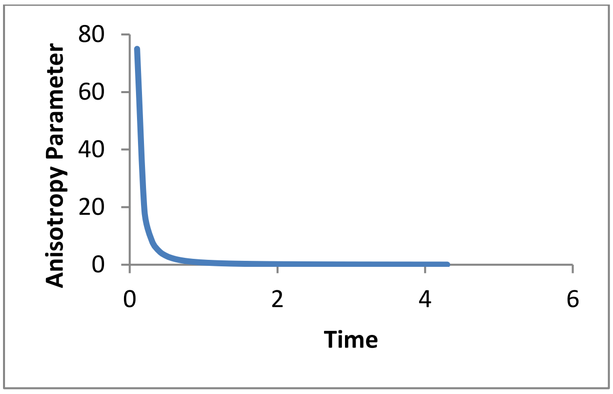

Fig. 2: Anisotropy Parameter versus Time for Appropriate Values of Constants.

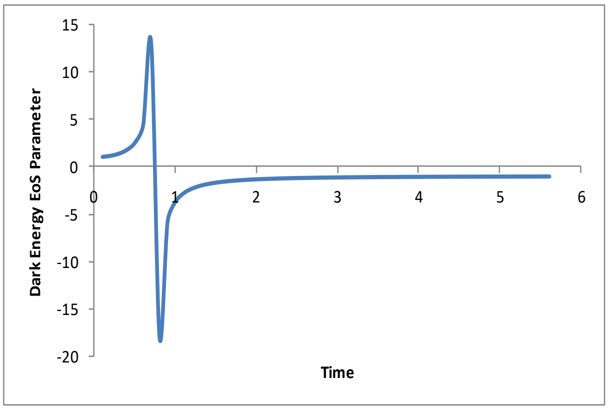

Fig. 3: Dark Energy Eos Parameter versus Time for Appropriate Values of Constants.

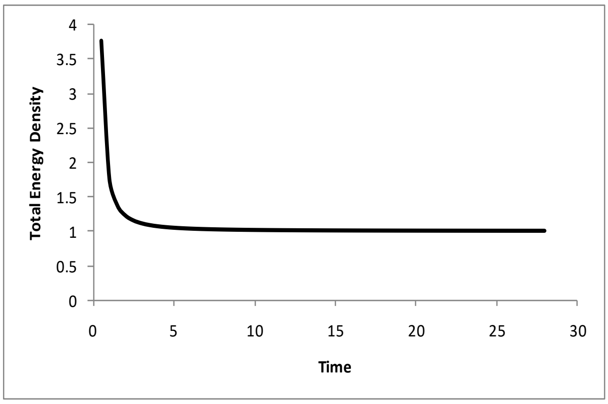

Fig. 4: Total Energy Density versus Time for Appropriate Values of Constants.

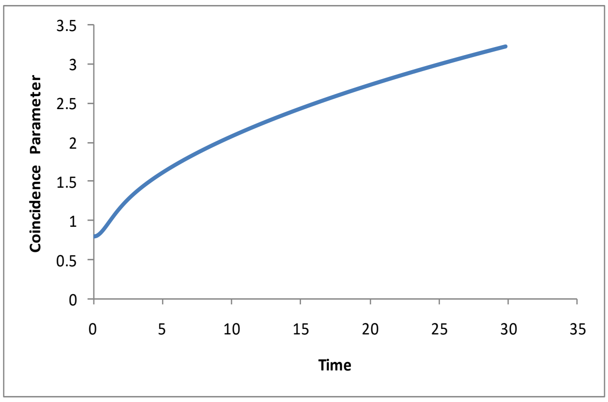

Fig. 5: Coincidence Parameter versus Time for Appropriate Values of Constants.

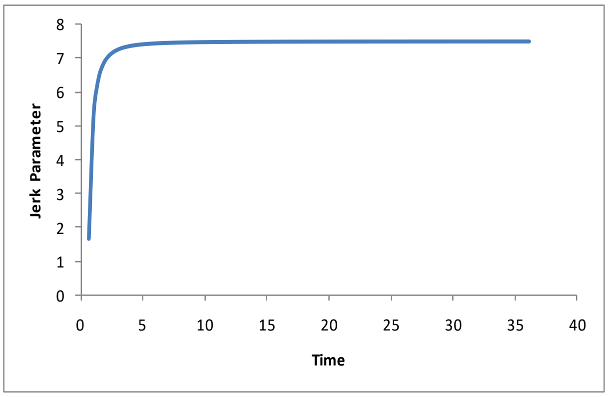

Fig. 6: Jerk Parameter versus Time for Appropriate Values of Constants.

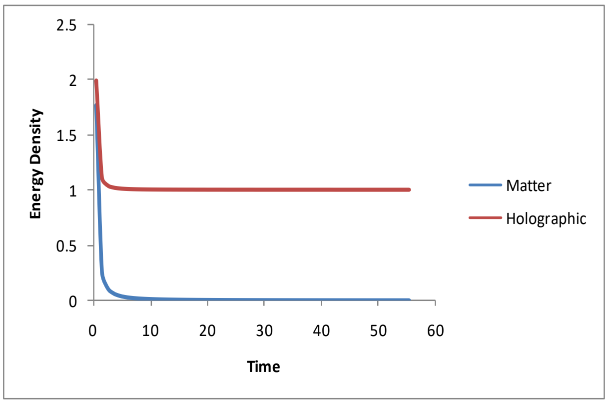

Fig. 7: Matter Energy Density ρ_m & Dark Energy Density ρ_Λ versus Time for Appropriate Values of Constants.

6. Conclusions

The present work deals with two minimally interacting fluids dark matter and holographic dark energy components in a spatially homogeneous and anisotropic Bianchi type-I space-time under the assumption of time varying deceleration parameter. It is observed that the model represent transition from the early decelerating phase to the recent accelerating phase. Also from the expression of holographic dark energy density parameter it is observed that the model is asymptotically empty thus the model is filled with dust matter. The anisotropy parameter has been found to be dynamical and decreases as the universe expands. It is found that the universe approaches to isotropy for large cosmic time as suggested by different observational data. Thus we can say that the Bianchi type-I space-time reduces to flat RW soon after inflation. Also the derived model is accelerating, expanding and has no initial singularity and has a point type singularity at $t^2 = -\frac{m}{n}$.

In this derived model, the inflation of universe is depend on the value of n . For $n < 1$, the mode of universe is accelerating which is consistent with the recent CMB and WMAP observations whereas for $n > 1$ it is decelerating which is also consistent with the high red-shift supernovae-Ia data [1-5] while for $n = 1$ the mode of universe is constant exponent (de-Sitter expansion) which is depicted in figure (i).

The coincidence parameter of this derived model increases with increase in time whereas the jerk parameter at very early stage of evolution varies but after some finite time it converges to a constant value and remains constant throughout the entire evolution.

The Lorentz Invariant Vacuum Energy (LIVE), which can be represented by a cosmological constant (Λ), with a constant Eos parameter $\omega = -1$, the so-called ΛCDM model, which in a flat universe model contains both LIVE and cold dark matter CDM , i.e. dust. It is observed that the EoS parameter is function of time. From the obtained value of EoS parameter for large expansion it is observed that the model reduces to ΛCDM model which is also shown from the value of total energy density parameter. Some other limits of ω obtained from observational results that came from SNeIa data [62] and combination of SNeIa data with CMB anisotropy and Galaxy clustering statistics [5-6] are $-1.67 < \omega < -0.62$ and $-1.33 < \omega < -0.79$ respectively. The latest result obtained after a combination of cosmological data sets coming from CMB anisotropy, luminosity high red-shift SNeIa and galaxy clustering constrain the dark energy EoS parameter to $-1.44 < \omega < -0.92$ [63]. If the present model is compared with above experimental results one can conclude that the limit of ω provided by equation (38) may be accumulated with the acceptable range of EoS parameter. This model confirms the high red-shift supernova experiment.

7. Conflict of interests

The author declares that there is no conflict of interest regarding the publication of this paper.

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