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Research Paper

Characteristics exponents of the triangular solution in the elliptical restricted three body problem under the radiation and oblateness of primaries

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Abstract

This paper studies effects of the oblateness and radiation of both the primaries on the stability of the infinitesimal motion about triangular equilibrium points (L4,5) in the elliptical restricted three body problem (ER3BP) around the binary system We have exploited analytical method for determining of characteristics exponent to the variational equations with periodic coefficients, developed by Bennet (! 965b), which is based on the Floquet's theory. The stability of the infinitesimal motion about the triangular points under the effects of radiation and oblateness of both the primaries around the binary systems Achird, Luyten726-8, Kruger 60, Alpha Centauri AB and Xi Bootis, has been studied. The stability of infinitesimal around the triangular points has been studied based on the analytical and numerical exploration is simulated by drawing transition curves bounding the region of stability in the (μ-e) plane. The region of stability changed with variations in eccentricity, oblateness and radiation pressures. It is observed that the equilibrium points stable in the shaded portion of the transition curve, whereas unstable outside the region of the transition curves.

Keywords: *Elliptical Restricted Three Body Problem; Stability; Radiation; Oblateness; Binary System.*

1. Introduction

The present paper is devoted to the analysis of the effects of the radiation and oblateness of both the primaries on the stability infinitesimal about the triangular equilibrium points of the planar ER3BP around the binary system. The ER3BP is a generalization of the classical problem. The eccentricity of the orbits plays a significant role which might not be seen in the circular case. The orbits of most celestial bodies are elliptical rather than the circular as such, the ER3BP describes the dynamical system more accurately.. We investigated the stability of triangular equilibrium points under the effects of the radiation and oblateness of both the primaries by exploiting analytical method for determining the characteristic exponents, which is based on the Floquet's theory.

The bodies of the elliptical restricted three body problem are generally considered to be spherical in shape, but in actual situations, it is observed that several heavenly bodies are either oblate spheroid or triaxial rigid bodies. The planets (Earth, Jupiter and Saturn) and stars (Archerner, Achird, Antares, Altair, Luyten etc.) are sufficiently oblate and are very significant in the study of celestial and stellar systems. The lacks of spherity of the heavenly bodies causes large perturbation. In addition to oblateness of heavenly body's triaxiality, the radiation forces of the bodies, the atmospheric drag and the solar wind are also caused of perturbation.

This motivates our study, the stability of triangular equilibrium points under the influence of oblateness and radiation of the pri maries in the elliptical restricted three body problem around the binary system.

The linear stability of the triangular equilibrium points of the ER3BP has been studied thoroughly by Danby (1964), Bennet (1965), Tschouner (1971), Roberts (1973), Meire (1981), Gyorgrey (1985), Kumar and Choudhary (1990), Khasan (1990, 1996), Markellos et al. (1992) with highlighting the transition curve separating region of stability in the (μ-e) plane. Conxita (1995), Jefferys (1965-1966), Selaru (1995) and Zsoft and Erdi (2003) have studied the different aspects of the same problem. Balint Erdi (2009) studied the parametric resonance stability around L4 in the ER3BP.

The influence of eccentricity of the orbits of the primaries with or without radiation pressure(s) on the existence of the equilibrium points and their stability was studied by Zimvoschikov and Thakai (2004), Markeev (2005), Ammar (2008). The influence of the eccentricity with oblateness and radiation parameters on the location and stability of collinear and triangular equilibrium points has been investigated by Narayan and Ramesh (2011-2012), Narayan and shrivastava (2012) and Jagadish and Umer (2012). Recently the linear stability of the triangular equilibrium points of the ER3BP has been studied by Narayan and Singh (2014a,2014b) and Narayan and Usha (2014).

The present study is devoted to the analysis of the stability of triangular points under radiating and oblate primaries by exploiting the analytical techniques developed by Bennet (1965). This method is mainly based on the Floquet's theory for a system with periodic coefficients.

This paper is organized in five sections, section-1 describes introduction, section -2 provides the equations of motion, while section-3 describes the calculation of characteristic exponents and section-4 provides the graphical representation of transition curves, which are divided into stable and unstable regions. The discussion and conclusion are drawn in section-5.

2. Equations of motion

The differential equations of motion for the elliptical restricted three body problem under the oblate and radiating primaries in barycentric, pulsating and non-dimensional coordinates are represented (Jagadish and Umer, (2012))as :

$$x'' - 2y' = \Omega_x^*, \quad y'' + 2x' = \Omega_y^*, \quad z'' = \Omega_z^* \tag{2.1}$$

where

$$\Omega = (1-e^2)^{-1/2}\left[\frac{x^2+y^2}{2} + \frac{1}{n^2}\left(\frac{(1-\mu)q_1}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)q_1 A_1}{2r_1^3} + \frac{\mu A_2 q_2}{2r_2^3}\right)\right] \tag{2.2}$$

The mean of motion, n is given by

$$n^2 = \frac{(1-e^2)^{1/2}\left(1+\frac{3}{2}A_1+\frac{3}{2}A_2\right)}{a(1-e^2)} \tag{2.3}$$

$$r_1^2 = (x-\mu)^2 + y^2 + z^2, \quad r_2^2 = (x-1+\mu)^2 + y^2 + z^2 \tag{2.4}$$

where $\mu = \dfrac{m_2}{m_1+m_2}$

where m_1 and m_2 are the masses of the bigger and smaller primaries situated at the point $(x_i, \mathbf{0}, \mathbf{0}), i = 1, 2,$ q_1 and q_2 are the mass radiation factor; A_1, A_2 are the oblateness parameter of the primaries; $r_i, (i = 1, 2)$ are the distances of the infinitesimal mass from the bigger and smaller primaries respectively; while a and e are respectively the semi- major axis and eccentricity of the orbits.

The co-ordinate of triangular equilibrium point is represented (Jagadish and Umer (2012)) as:

$$x_0 = \frac{1}{2} - \mu + \frac{1}{2}\left[\begin{array}{l}\left((aq_1)^{2/3}\right)\left(1-e^2-A_1-A_2+A_1(aq_1)^{-2/3}\right)\\ -(aq_2)^{2/3}\left(1-e^2-A_1-A_2+A_2(aq_2)^{-2/3}\right)\end{array}\right]$$

$$y_0 = \pm\Big[\left((aq_1)^{2/3}\right)\left(1-e^2-A_1-A_2+A_1(aq_1)^{-2/3}\right) - \frac{1}{4}\left(1+2(aq_1)^{2/3}\right)\left(1-e^2-A_1-A_2+A_2(aq_1)^{-2/3}\right) - 2(aq_2)^{2/3}\left(1-e^2-A_1-A_2+A_2(aq_2)^{-2/3}\right)\Big]^{1/2} \tag{2.5}$$

In order to investigate the stability of the infinitesimal equilibrium point (2.5) in the first approximation, we derived the equations for the variation in the coordinates.

Let ξ, η denote the small displacement in (x_0, y_0);

Then

$$x = x_0 + \xi \quad y = y_0 + \eta \tag{2.6}$$

Differentiating these with respect to v, we get;

$x' = \xi'; x'' = \xi''$ And $y'' = \eta''$;

where

$$\Omega_x = \Omega_x(x, y) = \Omega_x(x_0+\xi, y_0+\eta) \tag{2.7}$$

Expanding equation (2.7) by Taylor's theorem and retaining only upto the first order terms in the infinitesimal ξ and η, we get:

$$\Omega_x = \Omega_x^0 + \xi\Omega_{xx}^0 + \eta\Omega_{xy}^0$$

$$\text{and } \Omega_y = \Omega_y^0 + \xi\Omega_{yx}^0 + \eta\Omega_{yy}^0 \tag{2.8}$$

where Ω_x^0 and Ω_y^0 are the values of Ω_x and Ω_y at the equilibrium point $(x_0 y_0)$ which is given by (2.5).

At the equilibrium point $(x_0 y_0)$ we have:

$$\Omega_x^0 = 0 = \Omega_y^0 \tag{2.9}$$

Hence, the set of equation (2.1) with the help (2.8) and (2.9) is reduced to the following form:

$$\xi'' - 2\eta' = \varphi\left(\Omega_{xx}^0\xi + \Omega_{xy}^0\eta\right);$$

$$\eta'' + 2\xi = \varphi\left(\Omega_{xy}^0\xi + \Omega_{yy}^0\eta\right) \tag{2.10}$$

Differentiating partially Ω with respect to x and y, evaluating Ω_{xx}, Ω_{xy} and Ω_{yy} at the equilibrium point $(x_0 y_0)$ given by (2.5), which is obtained (Jagadish and Umer (2012)) as given below:

$$\Omega^0_{xx} = \left[\frac{3(1-\mu)}{4(aq_1)^{2/3}} + \frac{3(1-\mu)}{2} - \frac{3(1-\mu)q_2^{2/3}}{2q_1^{2/3}} + \frac{3\mu}{4(aq_2)^{2/3}} - \frac{3\mu q_1^{2/3}}{2q_2^{2/3}} + \frac{3\mu}{2}\right] + A_1\left[\frac{9(1-\mu)}{4(aq_1)^{2/3}} - \frac{3\mu}{4(aq_2)^{2/3}}\right] + A_2\left[\frac{9\mu}{4(aq_2)^{2/3}} - \frac{3\mu}{4(aq_1)^{2/3}}\right]$$

$$\Omega^0_{yy} = \left[\frac{3(1-\mu)}{2} + \frac{3(1-\mu)q_2^{2/3}}{2q_1^{2/3}} - \frac{3(1-\mu)}{4(aq_1)^{2/3}} + \frac{3\mu}{2} + \frac{3\mu q_1^{2/3}}{2q_2^{2/3}} - \frac{3\mu}{4(aq_2)^{2/3}}\right] + A_1\left[\frac{3(1-\mu)}{4(aq_1)^{2/3}} + \frac{3\mu}{4(aq_2)^{2/3}}\right] + A_2\left[\frac{3(1-\mu)}{4(aq_1)^{2/3}} + \frac{3\mu}{4(aq_2)^{2/3}}\right]$$

and

$$\Omega^0_{xy} = \left[\frac{3(1-\mu)}{2(aq_1)^{2/3}} + \frac{3(1-\mu)}{2} - \frac{3(1-\mu)q_2^{2/3}}{2q_1^{2/3}} - \frac{3\mu}{2(aq_2)^{2/3}} + \frac{3\mu q_1^{2/3}}{2q_2^{2/3}} - \frac{3\mu}{2}\right] + A_1\left[\frac{3(1-\mu)}{4(aq_1)^{2/3}}\right] + A_2\left[-\frac{3\mu}{4(aq_1)^{2/3}}\right] \tag{2.11}$$

We have investigated the stability of triangular equilibrium points under the radiation and oblateness of primaries in the elliptical restricted three body problem This investigation is based on Floquet theory, which determines characteristic components in the system with periodic coefficients. The transformed variational equation of motion of elliptical restricted three body problem under the oblate and radiating primaries which is represented in matrix form, as given as:

$$X' = P(X) \qquad (2.12)$$

where

$$X = \begin{Bmatrix} \xi \\ \eta \\ \xi' \\ \eta' \end{Bmatrix} \text{ and } X' = \begin{Bmatrix} \xi' \\ \xi'' \\ \eta' \\ \eta'' \end{Bmatrix}$$

and

$$p(v,e) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \varphi\Omega_{xx} & \varphi\Omega_{xy} & 0 & 2 \\ \varphi\Omega_{yx} & \varphi\Omega_{yy} & -2 & 0 \end{bmatrix} \qquad (2.13)$$

3. Determination of characteristic exponents

In order to find solution of variational equation of motion of the system, we have exploited the Floquet's theory of determining characteristic exponents of the system with periodic coefficients. We seek the solution of the system of equation (2.12) in the form:

$$x_k = y_k e^{\lambda_k v} \qquad (3.1)$$

where y_k is periodic with period 2π and λ_k are the characteristic exponents. Dropping the suffix in (3.1), we get:

$$x = ye^{\lambda v}$$

Differentiating with respect to v, we get:

$$x' = (y' + \lambda y)e^{\lambda v} \qquad (3.2)$$

Using the equation (3.2), the variational equation of motion takes the form:

$$y' = (P - \lambda I)y \qquad (3.3)$$

where I is the unit matrix of the same order as that of y .

Now, using the expressions, which are mentioned below:

$$y = y^{(0)} + ey^{(1)} + e^2 y^{(2)} + ...$$

$$\lambda = \lambda_0 + e\lambda_1 + e^2\lambda_2 + ... \qquad (3.4)$$

and the corresponding matrix P, which is expanded as:

$$P(v,e) = p^{(0)} + ep^{(1)} + e^2 p^{(2)} + ... \qquad (3.5)$$

$$\text{where } p^{(0)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \Omega^0_{xx} & \Omega^0_{xy} & 0 & 2 \\ \Omega^0_{yx} & \Omega^0_{yy} & -2 & 0 \end{bmatrix} \qquad (3.6)$$

$$\text{and } p^{(m)} = (-\cos v)^m c : \quad m = 1,2,3...... \qquad (3.7)$$

$$\text{where } C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Omega^0_{xx} & \Omega^0_{xy} & 0 & 0 \\ \Omega^0_{yx} & \Omega^0_{yy} & 0 & 0 \end{bmatrix} \qquad (3.8)$$

Substituting the equation (3.4) and (3.5) in (3.3), we get:

$$y'^{(0)} + ey'^{(1)} + e^2 y'^{(2)} .... =$$

$$\left[\left\{\left(P^{(0)} + eP^{(1)} + e^2P^{(2)} + ...\right) - I\left(\lambda_0 + e\lambda_1 + e^2\lambda_2 + ...\right)\right\}\left(y^{(0)} + ey^{(1)} + e^2y^{(2)} + ...\right)\right] \qquad (3.9)$$

Equating the coefficients of terms with the same power of e from (3.9) ,both the sides and using (3.3), we get:

$$y'^{(0)} + \left[I\lambda_0 - P^{(0)}\right]y^{(0)} = 0$$

$$y'^{(1)} + \left[I\lambda_0 - P^{(0)}\right]y^{(1)} = \left[-c\cos \mathrm{v} - \lambda_1 I\right]y^{(0)}$$

$$y'^{(2)} + \left[I\lambda_0 - P^{(0)}\right]y^{(2)} = \left[-c\cos \mathrm{v} - I\lambda_1\right]y^{(1)} + \left(c\cos^2 \mathrm{v} - I\lambda_2\right)y^{(0)};$$

$$y'^{(n)} + \left[I\lambda_0 - P^{(0)}\right]y^{(n)} = \sum_{m=1}^{n}\left[c\left(-\cos \mathrm{v}\right)^m - I\lambda_m\right]y^{(n-m)}$$

(3.10)

If a constant vector is assumed for the zeroth order solution, then for the nth order equation of non-homogenous terms have frequencies up to the order of e^2 and including $\frac{n}{2\pi}$. we take the particular solution as:

$$y^{(n)} = \sum_{k=-n}^{k=+n} a^{(n,k)} e^{ikv} \quad n = 0,1,2,3....... \qquad (3.11)$$

where

$$a^{(n,k)} = \begin{bmatrix} a_1^{(n,k)} \\ a_2^{(n,k)} \\ a_3^{(n,k)} \\ a_4^{n,k} \end{bmatrix} \qquad (3.12)$$

From the equation (3.11), we have;

$$y^{(0)} = a^{(0,0)}$$

$$y'^{(0)} = 0$$

$$y^{(1)} = a^{(0,-1)}e^{-iv} + a^{(0,0)} + a^{(1,1)}e^{iv}$$

$$y'^{(1)} = -i\,a^{(0,-1)}e^{-iv} + ia^{(1,1)}e^{iv}$$

$$y^{(2)} = \left[a^{(2,-2)}e^{-2iv} + a^{(2,-1)}e^{-iv} + a^{(2,0)} + a^{(2,1)}e^{iv} + a^{(2,2)}e^{2iv}\right]$$

$$y'^{(2)} = \left[-2ia^{(2,-2)}e^{-2iv} - ia^{(2,-1)}e^{-iv} + ia^{(2,1)}e^{iv} + 2ia^{(2,2)}e^{2iv}\right]$$

Substituting these values in the set of equations (3.10), we obtain a system of equations necessary for the determination of λ up to the order of e^2 given as follows:

$$\left[I\lambda_0 - P^{(0)}\right]a^{(0,0)} = 0 \qquad (3.13)$$

$$\left[I\lambda_0 - P^{(0)}\right]a^{(1,0)} = -\lambda_1 a^{(0,0)}$$

$$\left[I(\lambda_0 + i) - P^{(0)}\right]a^{(1,1)} = -\frac{1}{2}c \cdot a^{(0,0)}$$

$$\left[I(\lambda_0 - i) - P^{(0)}\right]a^{(1,-1)} = -\frac{1}{2}c \cdot a^{(0,0)}$$

$$\left[I\lambda_0 - P^{(0)}\right]a^{(2,0)} = -\lambda_1 a^{(1,0)} + \left[\left(\frac{1}{2}c - I\lambda_2\right)\right]a^{(0,0)} - \frac{1}{2}c\left(a^{(1,1)} + a^{(1,-1)}\right) \qquad (3.14)$$

From the equation (3.13), it is evident that for the existence of $a^{(0,0)}$, it is necessary that

$$\det\left(I\lambda_0 - P^{(0)}\right) = 0 \qquad (3.15)$$

i.e. $$\begin{vmatrix} \lambda_0 & 0 & -1 & 0 \\ 0 & \lambda_0 & 0 & -1 \\ -\Omega^0_{xx} & -\Omega^0_{xy} & \lambda_0 & -2 \\ -\Omega^0_{yx} & -\Omega^0_{yy} & 2 & \lambda_0 \end{vmatrix} = 0$$

From the above relation, we get:

$$\lambda_0^4 + \left(4 - \Omega^0_{xx} - \Omega_{yy}\right)\lambda_0^2 + \left\{\Omega^0_{xx}\Omega^0_{yy} - \left(\Omega_{xy}\right)^2\right\} = 0; \quad (3.16)$$

$$\therefore\ \lambda_0^4 - Q\lambda_0^2 + R = 0 \quad (3.17)$$

where $Q = \Omega^0_{xx} + \Omega^0_{yy} - 4$

$$Q = \frac{1}{\sqrt{(1-e^2)}}\left[3(1-\mu) + \frac{3(1-\mu)q_2^{2/3}}{q_1^{2/3}} + 3\mu\right] + A_1\left[\frac{3(1-\mu)}{(aq_1)^{2/3}}\right] + A_2\left[\frac{3\mu}{(aq_2)^{2/3}} + \frac{3(1-2\mu)}{4(aq_1)^{2/3}}\right] - 4$$

and $R = \Omega^0_{xx} . \Omega^0_{yy} - (\Omega^0_{xy})^2$

$$R = \frac{9}{8(1-e^2)}\Bigg[(1-2\mu)\left\{\frac{q_2^{2/3}}{(aq_1)^{2/3}} + \frac{10q_2^{2/3}}{q_1^{2/3}} - \frac{q_2^{2/3}}{q_1^{4/3}} + \frac{4q_2^{2/3}}{a^{2/3}q_1^{4/3}} - \frac{1}{2a^{4/3}q_1^{2/3}} - \frac{5}{(aq_1)^{2/3}} - \frac{2}{a^{4/3}q_1^{4/3}} - \frac{2q_2^{4/3}}{q_1^{4/3}} + 2\right\}$$
$$+\mu\left\{\frac{-2q_1^{2/3}}{q_2^{2/3}} \frac{-2q_2^{2/3}}{q_1^{2/3}} + \frac{3}{a^{4/3}(q_1q_2)^{2/3}} + \frac{1}{(aq_1)^{2/3}} + 8\right\}\Bigg] +$$
$$\frac{9A_1}{8\sqrt{(1-e^2)}}\Bigg[(1-2\mu)\left\{\frac{8q_2^{2/3}}{(aq_1)^{2/3}} + \frac{3q_2^{2/3}}{a^{2/3}q_1^{4/3}} - \frac{3}{(aq_1)^{2/3}} - \frac{9}{a^{4/3}q_1^{2/3}}\right\}$$
$$+\mu\left\{\frac{8}{a^{4/3}(q_1q_2)^{2/3}} + \frac{13}{(aq_1)^{2/3}} - \frac{5}{(aq_2)^{2/3}} - \frac{1}{(aq_1)^{4/3}}\right\}\Bigg] +$$
$$\frac{9A_2}{8\sqrt{(1-e^2)}}\Bigg[(1-2\mu)\left\{\frac{q_2^{2/3}}{a^{2/3}q_1^{4/3}} + \frac{1}{2a^{4/3}q_1^{4/3}} + \frac{1}{(aq_1)^{2/3}}\right\}$$
$$+\mu\left\{\frac{9}{(aq_1)^{2/3}} + \frac{4}{a^{4/3}q_2^{2/3}} - \frac{1}{a^{4/3}(q_1q_2)^{2/3}} + \frac{3}{a^{4/3}q_1^{2/3}} + \frac{17}{2(aq_1)^{4/3}} - \frac{9q_2^{2/3}}{a^{2/3}q_1^{4/3}}\right\}\Bigg]$$
$$+\frac{9e^2}{8\sqrt{(1-e^2)}}\Bigg[(1-2\mu)\left\{\frac{1}{2(aq_1)^{4/3}} + \frac{5q_2^{2/3}}{(aq_1)^{2/3}} - \frac{7}{(aq_1)^{2/3}}\right\} + \mu\left\{\frac{-3}{(aq_2)^{2/3}} - \frac{1}{(aq_1)^{2/3}}\right\}\Bigg]$$

(3.18)

The relation for the exponent in the elliptical restricted three body problems can be obtained using the equation (3.17):

$$\lambda_0^2 = \frac{Q \pm \sqrt{Q^2 - 4R}}{2}$$

From the first equation of the system of equation (3.14), we observe that it is necessary that the determinant of the coefficient on the left with any column replaced by the non-homogenous terms on the right must be zero.

It is represented as follows:

$$\text{det.}\left[I\lambda_0 - p^{(0)}\right] - \lambda_1 a^{(0,0)} = 0 \quad (3.20)$$

Since λ enters as a factor in all elements of the replaced column, therefore

$$\lambda_1 \ \text{det.}\left[I\lambda_0 - p^{(0)}\right] a^{(0,0)} = 0 \quad (3.21)$$

Since the determinant of the equation (3.20) is not zero, in general, we therefore conclude that

$$\lambda_1 = 0 \quad (3.22)$$

Again from the second and the third equation of (3.14), we have the solutions from $a^{(1,1)}$ *and* $a^{(1,-1)}$ are

$$a^{(1,1)} = -\frac{1}{2}\left[I\left(\lambda_0 + i\right) - p^{(0)}\right]^{-1} c\, a^{(0,0)}$$
$$a^{(1,-1)} = -\frac{1}{2}\left[I\left(\lambda_0 - i\right) - p^{(0)}\right]^{-1} c\, a^{(0,0)} \quad (3.23)$$

Substituting this value of $a^{(1,1)}$ and $a^{(1,-1)}$ from (3.23) in the last equation of (3.14),we get;

$$\left(I\lambda_0 - p^{(0)}\right)a^{(2,0)} = \frac{1}{4}c\left[\left(I\left(\lambda_0 + i\right) - p^{(0)}\right)^{-1} + \left(I\left(\lambda_0 - i\right) - p^{(0)}\right)^{-1}\right] + c\ a^{(0,0)} + \left(\frac{c}{2} - I\lambda_2\right)a^{(0,0)} \quad (3.24)$$

The matrices within the square bracket are complex conjugate so that only the real parts are considered, then equation (3.14) can be written as:

$$\left(I\lambda_0 - p^{(0)}\right)a^{(2,0)} = \left[\left\{\frac{1}{2}c\ R_c\left(I\lambda_0 + i\right) - p^{(0)}\right\}^{-1} + \left(\frac{c}{2} - I\lambda_2\right)\right]a^{(0,0)} \quad (3.25)$$

After some mathematical manipulations, we from (3.25) obtained the value of λ_2, given by

$$\lambda_2 = -\left[\frac{\left(Q^2 - 4R - 16\right)\lambda_0^2 + (A_0F_0 + A_1F_1 + A_2F_2)}{4\left(Q^2 - 4Q - 4R\right)\lambda_0^2 + 32R}\right]\lambda_0$$
$$= A\lambda_0 \quad (3.26)$$

where

$$A_0 = \left[(Q+4)^2(Q-4) - 4QR\right]\lambda_0^2 - R\left(Q+4\right) - 4R^2;$$
$$A_1 = -8\lambda_0 R\left[2\lambda_0^2 - (Q+4)\right];$$
$$A_2 = -\lambda_0^2 R\left[Q^2 - 4R - 16\right];$$
$$F_0 = \frac{1}{N}\left[\lambda_0^2(Q+1) + (Q+1) + 2R\right];$$
$$F_1 = -\frac{\lambda_0}{N}\left[2\lambda_0^2 + (Q+3)\right];$$
$$F_2 = -\frac{1}{N}\left[\lambda_0^2 - (Q+1)\right];$$
$$N = \lambda_0^2\left[4Q^2 + 8Q + 4 - 16R\right] - 4R + (Q+1)^2 \quad (3.27)$$

Using this value of

$$Q = 3(1-\mu)\left[1 + \frac{q_2^{2/3}}{q_1^{2/3}}\right] + 3\mu + A_1\left[\frac{3(1-\mu)}{(aq_1)^{2/3}}\right] + A_2\left[\frac{3\mu}{(aq_2)^{2/3}} + \frac{3(1-2\mu)}{4(aq_1)^{2/3}}\right] - 4 \quad (3.19)$$

We find the value of the parameter A, which is calculated as mentioned below;

Hence,

$$-A = \frac{\left(Q^2 - 4R - 16\right)\lambda_0^2 + (A_0F_0 + A_1F_1 + A_2F_2)}{4\left(Q^2 - 4Q - 4R\right)\lambda_0^2 + 32R} \qquad (3.28)$$

Thus $\lambda_2 = A\lambda_0$ (3.29)

where A is given by the equation (3.28). Hence the solution of this system becomes:

$\lambda = \lambda_0 + e^2\lambda_2$ (3.30)

where λ_2 is given by this equation (3.29).

4. Transition curves separating stable and unstable regions

The transition curves separating stable and unstable regions, which describes the stability of the triangular equilibrium points in the elliptical restricted three body problem under the stable primaries, can be found by simply equating the expression for the characteristic roots or exponents to the value of periodic solutions. In the range $0 \le \mu < \frac{1}{2}$, the periodic solution provides

$\lambda^* = \pm\frac{i}{2}$ (4.1)

Replacing λ by λ^* in (3.30), we obtain:

$$\pm\frac{i}{2} = \lambda_0 + e^2A\lambda_0;\ or\ \ \pm\frac{i}{2} = \left(1+e^2A\right)\lambda_0$$

Squaring both sides we obtain:

$$\left(1+e^2A\right)^2\lambda_0^2 \,\underline{\text{v}}\, -\frac{1}{4};$$

$$\left(1+e^2A\right) = \pm\left(-\frac{1}{4}\lambda_0^2\right)^{\frac{1}{2}}.$$

$$e^2 = \left[\pm\left(-\frac{1}{4\lambda_0^2}\right)^{\frac{1}{2}} - 1\right]\left(\frac{1}{A}\right) \qquad (4.2)$$

Now evaluating the values of A from the equation (3.28), and those of λ_0^2 from (3.19), we evaluate e easily for different values of μ.

The dependence of μ upon e given by equation (4.2) is shown in graph for various values of oblateness parameters A_1 and A_2.

The triangular equilibrium points L_4 and L_5 are stable in the shaded region, whereas the region outside the shaded portion is unstable. The Fig 1, Fig2 and Fig 3 are depicting the stable and unstable region for the binary system Achird, similarly the Fig.4, Fig.5 Fig.6 are representing for Luyten, Fig.7, Fig.8 Fig.9 are representing for Alpha Cen-AB, Fig.10 Fig.11, Fig.12 are representing for Kruger-60 and , Fig.13 Fig.14, Fig.15 are representing for Xi-Booties.

The combined effects of the oblateness and radiation of the primaries around the binary system, introduced a visible left shift in the bifurcation points in each of the binary systems.

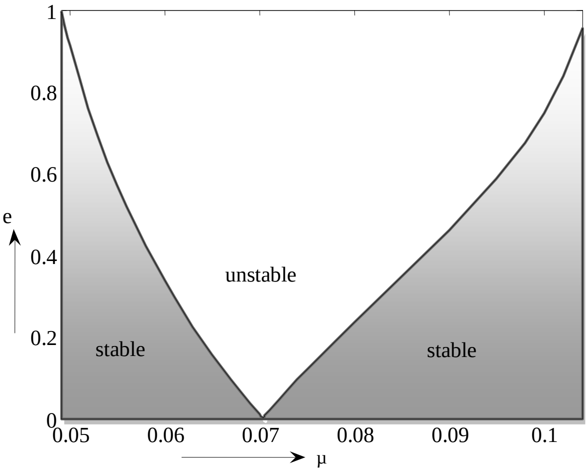

Fig. 1: Transition Curve of Achird-I

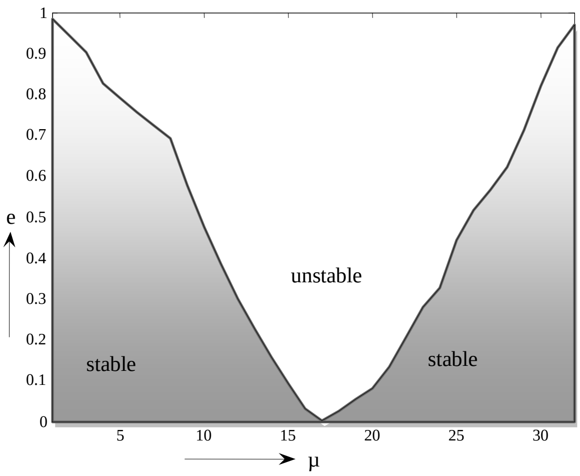

Fig. 2: Transition Curve of Achird-II

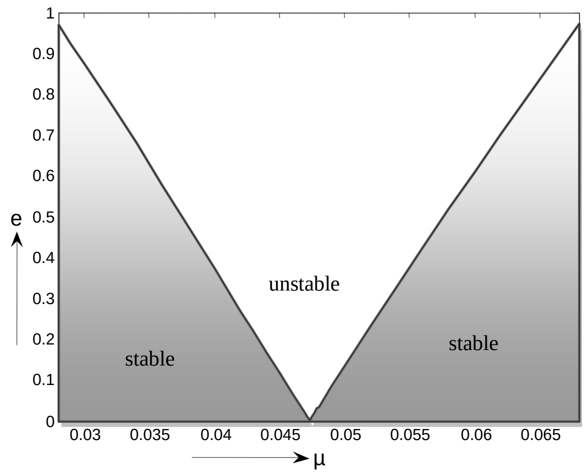

Fig. 3: Transition Curve of Achird-III

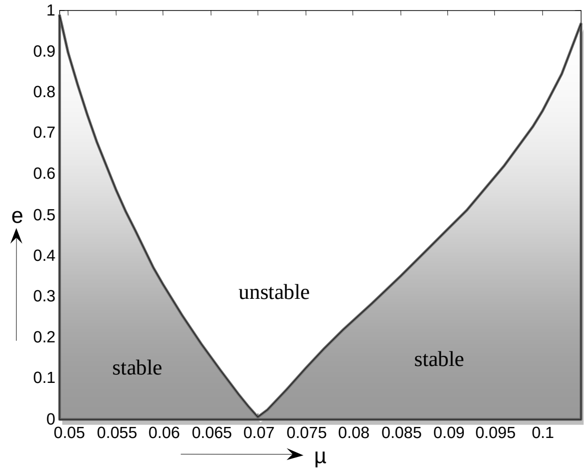

Fig. 4: Transition Curve of Luyten-I

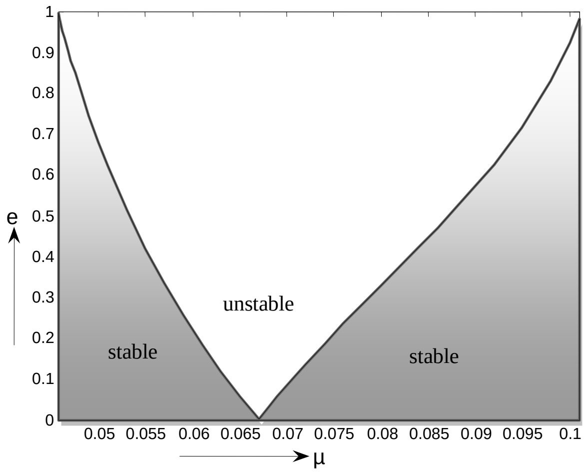

Fig. 5: Transition Curve of Luyten-II

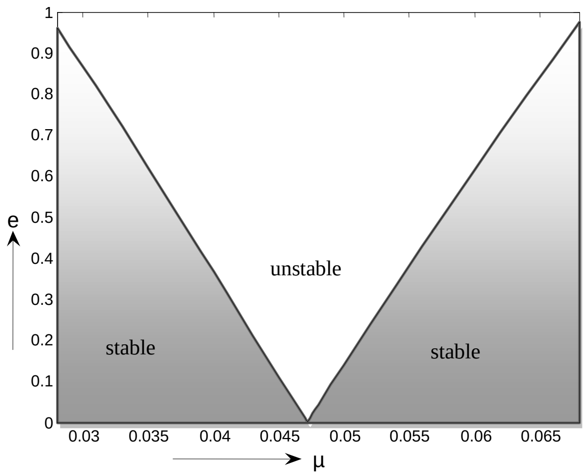

Fig. 6: Transition Curve of Luyten-III

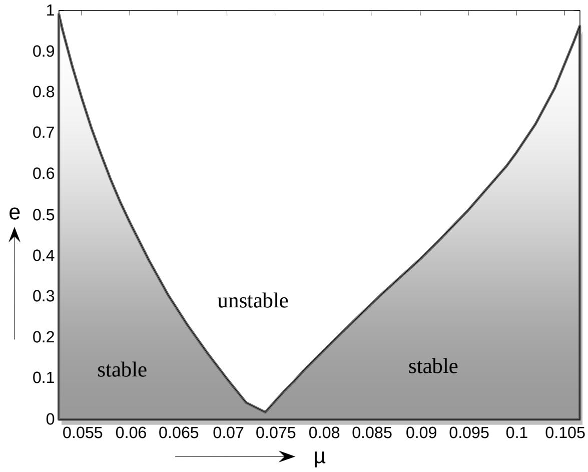

Fig. 7: Transition Curve of Alphan Cen-AB-I

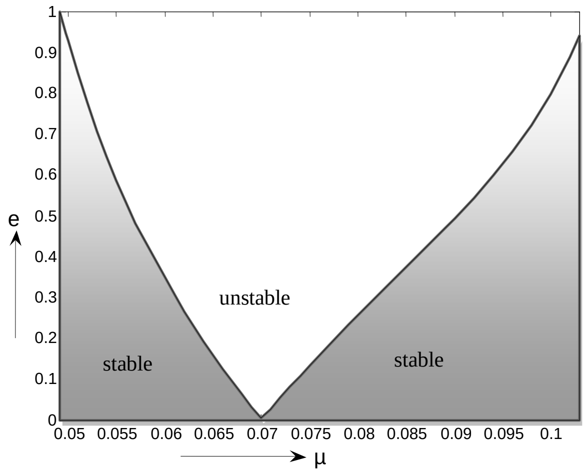

Fig. 8: Transition Curve of Alpha Cen-AB-II

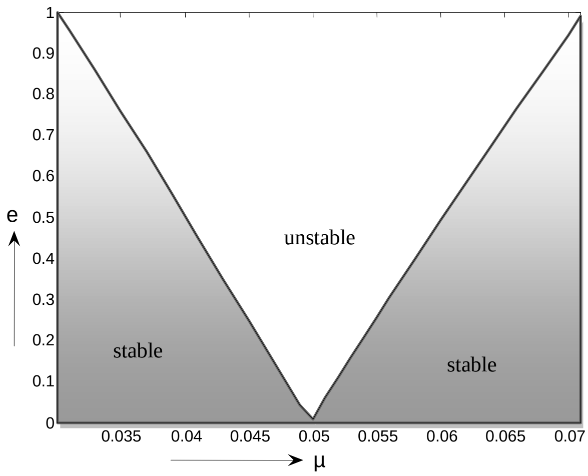

Fig. 9: Transition Curve of Alpha Cen-AB-III

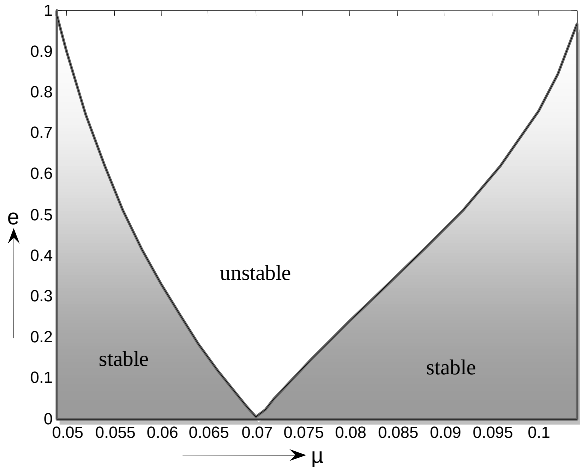

Fig. 10: Transition Curve of Kruger 60-I

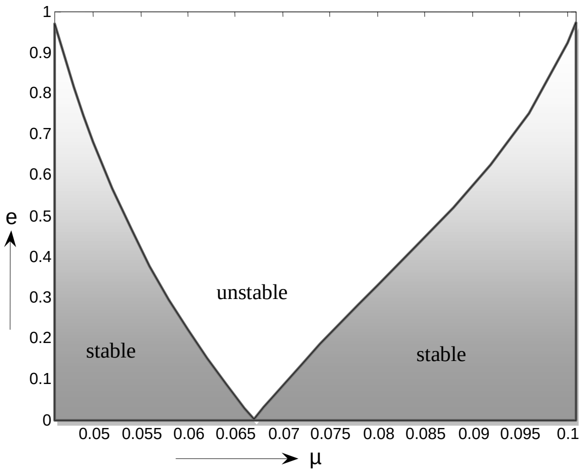

Fig. 11: Transition Curve of Kuger 60-II

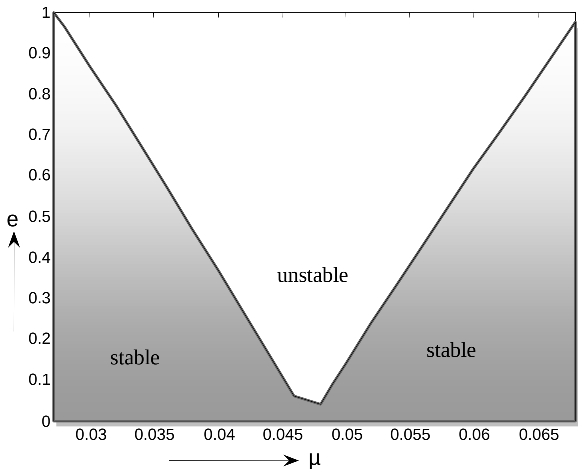

Fig. 12: Transition Curve of Kruger 60-III

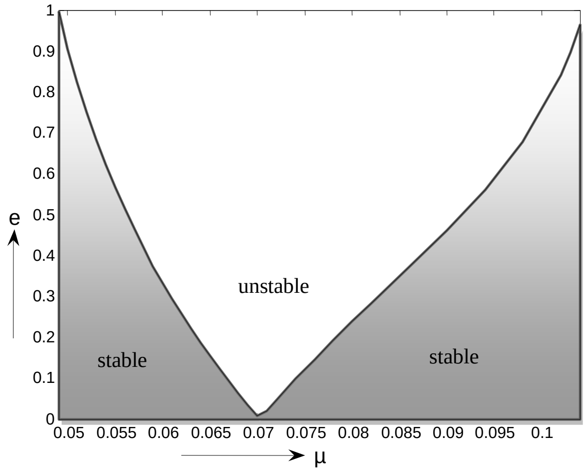

Fig. 13: Transition Curve of Xi Booties-I

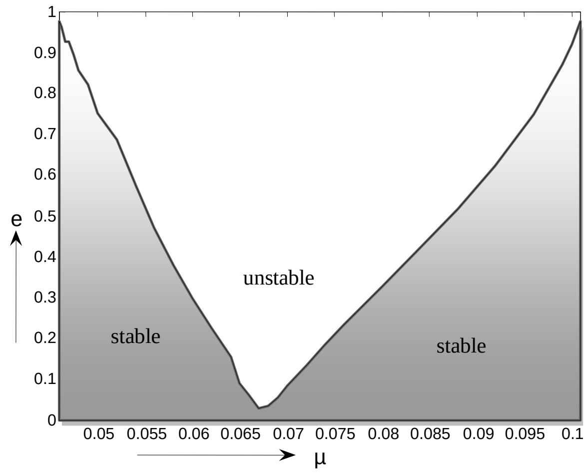

Fig. 14: Transition Curve of Xi Booties-II

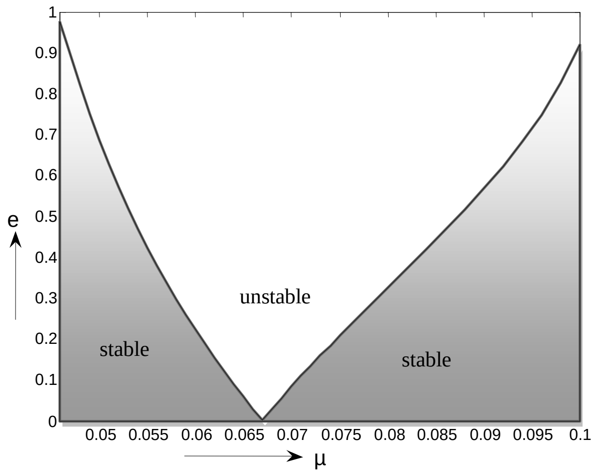

Fig. 15: Transition Curve of Xi Booties-III

5. Discussion and conclusion

The effects of oblateness and radiation of the primaries on the stability of the triangular equilibrium points around the binary system in the elliptical restricted three body problem has been studied. The problem is studied under the assumption that the eccentricity of the orbit of the gravitating bodies is small. The oblateness and radiation of the more massive primary does not affect the motion of the smaller primary due its large mass, whereas affects the motion of infinitesimal body.

The stability of triangular equilibrium points in the elliptical restricted three body problem under the oblate and radiating primaries around the binary system has been investigated. We have exploited an analytical method for determining of characteristic exponent based on the Floquet's theory. The stability of the triangular equilibrium points under the effects of the radiation and oblateness of both the primaries around the binary system Achird, Luyten, Alpha Cen-AB, Kruger 60 and Xi-Bootis, has been studied, using simulation technique by drawing transition curves.
It is observed that the triangular equilibrium points remains stable in the shaded region of the traced transition curves and unstable in the region outside the shaded portion, around the binary system.

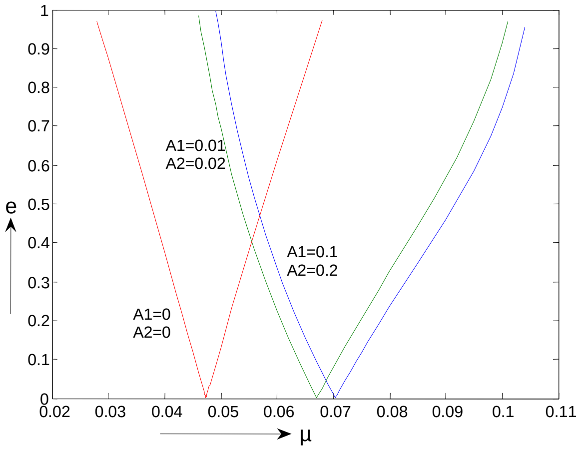

Fig. 16: Shifting Of Bifercation Point for Achird

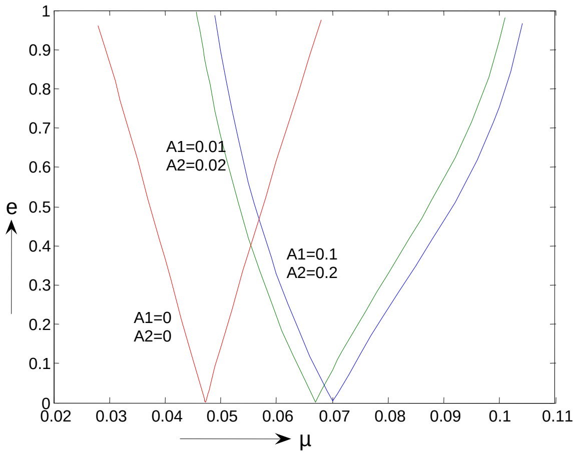

Fig. 17: Shifting of Bifurcation Point for Luyten

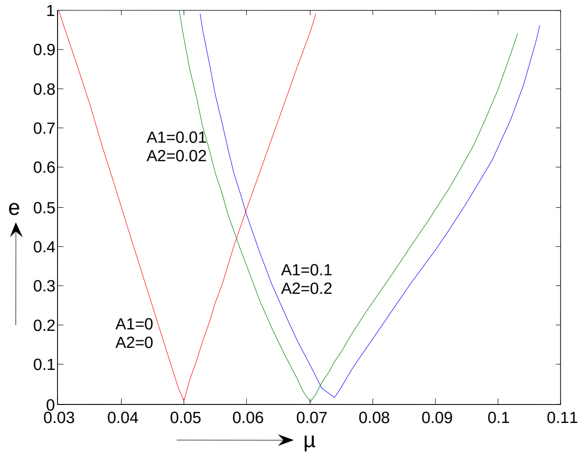

Fig. 18: Shiftinf of Bifurcation Point Foralpha Cen-AB

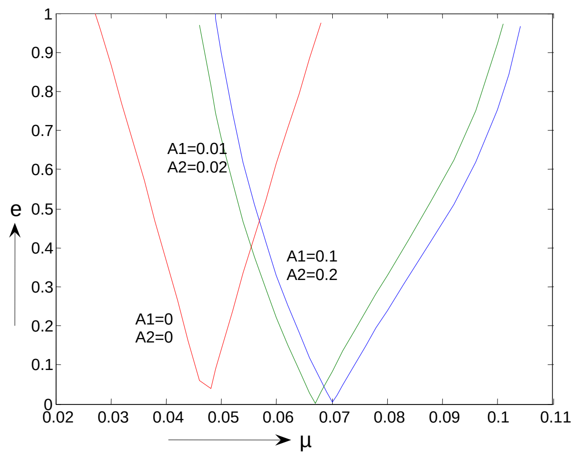

Fig. 19: Shifting of Bifurcation Point for Kruger-60

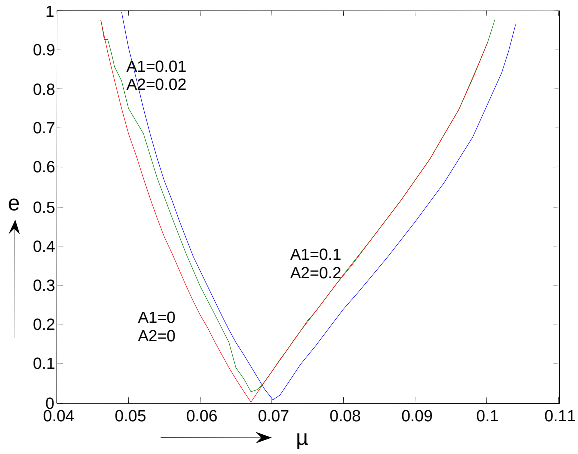

Fig. 20: Shifting of Bifurcation Point for xi-Booties

It is also observed that there is a visible left shift in the bifurcation points in each of the binary system and visible right shift is due to the effects of oblateness of the primaries around the binary system, which is obvious from the fig.16, fig.17, fig.18, fig.19 and fig.20. Hence we arrived at the conclusion that the effects of oblateness and radiation of the primaries around the binary system responsible for right shift of bifurcation points.

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