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Research paper

Normalisation of Hamiltonian in photogravitational elliptic restricted three body problem with Poynting-Robertson drag

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Abstract

In this paper, we have performed first order normalization in the photogravitational elliptic restricted three body problem with Poynting-Robertson drag. We suppose that bigger primary as radiating and smaller primary is an oblate spheroid. We have found the Lagrangian and the Hamiltonian of the problem. Then, we have expanded the Lagrangian function in power series of x and y, where (x, y) are the coordinates of the triangular equilibrium points. Using Whittaker (1965) method, we have found that the second order part H_2 of the Hamiltonian is transformed into the normal form $H_2 = \omega_1 I_1 - \omega_2 I_2$.

Keywords: *First-Order Normalization/ERTBP/Photogravitational/P-R Drag.*

1. Introduction

The elliptic restricted three body problem (ERTBP) is a generalization of the classical restricted three body problem (RTBP). It describes the three dimensional motion of a small particle, called the third body (infinitesimal mass) under the gravitational attraction force of two finite bodies, called the primaries, which revolve in elliptic orbit in a plane around their common centre of mass. The infinitesimal mass moves in the plane of motion of the primaries and does not influence the motion of the primaries.

Radiation, oblateness and a drag force known as Poynting-Robertson (P-R) drag also affects the motion of infinitesimal mass. Hence, many researchers studied the effect of these forces. Wyatt and Whipple (1950) have shown that P-R effect has been of very little significance. Chernikov (1970) has dealt with the Sun-Planet-Particle model and concludes that due to P-R drag, triangular points are unstable. Schuerman (1980) studied the classical RTBP by including the radiation pressure and P-R effect. Murray (1994) investigated location and stability of the five Lagrangian points in the CRTBP when infinitesimal mass is acted by a variety of drag forces. Kushvah and Ishwar (2006) examined the linear stability of generalized photogravitational RTBP with P-R drag.

We have already studied the linear stability of the given problem. We have found that in linear case, triangular equilibrium points are unstable. So our aim is to study the non-linear stability of triangular points. Liapunov's theorem (1956), Arnold's theorem (1961) and Moser's conditions (1962) played a significant role in deciding the non-linear stability of an equilibrium point. Moser (1962) gave some modification in Arnol's (1961) theorem. Then Deprit and Deprit-Bartholome (1962) investigated the non-linear stability of triangular points by applying Moser's modified version of Arnold's theorem (1961). Bhatnagar and Hallan (1983) studied the effect of perturbations on the non-linear stability of triangular points. Maciejewski and Gozdziewski (1991) described the normalization algorithms of Hamiltonian near an equilibrium point. Further, Bhatnagar, Gupta and Bhardwaj (1994) examined the non-linear stability of L_4 for perturbed potential. Mishra and Ishwar (1995) studied second order normalization in the generalized restricted problem of three bodies, smaller primary being an oblate spheroid. Ishwar (1997) studied non linear stability in the generalized restricted three body problem. J.Singh (2010) studied the combined effect of perturbations, radiation and oblateness on the non linear stability of triangular points in the restricted three body problem. Kushvah and Ishwar (2006) studied the second order normalization in the generalized photogravitational restricted three body problem with Poynting-Robertson drag.

The present study aims to perform first order normalization in the photogravitational elliptic restricted three body problem with P-R drag. We suppose that bigger primary is radiating and smaller an oblate spheroid. Using Whittaker (1965) method the second order part H_2 of the Hamiltonian is transformed into the normal form $H_2 = \omega_1 I_1 - \omega_2 I_2$ .This paper is divided in four sections. Section 2 contains equations of motion and location of triangular equilibrium points. In section 3, we have performed the first order normalization of H_2, while section 4 concludes the result.

2. Equations of motion and location of triangular equilibrium points

We consider two bodies (primaries) of masses m_1 and m_2 with $m_1 > m_2$moving in a plane around their common center of mass in elliptic orbit and a third body (infinitesimal mass) of mass m is moving in a plane of motion of the primaries. Equations of motion of our problem in rotating and pulsating co-ordinate system are given by (Sahoo and Ishwar 2000):

$$x'' - 2y' = \frac{\partial\Omega}{\partial x} - \frac{W_1 N_1}{n^2 r_1^2\sqrt{1-e^2}} = U_x \quad (2.1)$$

$$y'' + 2x' = \frac{\partial\Omega}{\partial y} - \frac{W_1 N_2}{n^2 r_1^2\sqrt{1-e^2}} = U_y\ , \quad (2.2)$$

where the force function

$$U=\frac{1}{\sqrt{1-e^2}}\left[\frac{x^2+y^2}{2}+\frac{1}{n^2}\left\{\frac{(1-\mu)q_1}{r_1}+\frac{\mu}{r_2}+\frac{\mu A_2}{2r_2^3}+W_1\left(\frac{(x+\mu)x'+yy'}{2r_1^2}-n\ \arctan\left(\frac{y}{x+\mu}\right)\right)\right\}\right] \quad (2.3)$$

$$\Omega=\frac{1}{\sqrt{1-e^2}}\left[\frac{x^2+y^2}{2}+\frac{1}{n^2}\left\{\frac{(1-\mu)q_1}{r_1}+\frac{\mu}{r_2}+\frac{\mu A_2}{2r_2^3}\right\}\right]$$

$$N_1=\frac{(x+\mu)N}{r_1^2}+x'-ny$$

$$N_2=\frac{yN}{r_1^2}+y'+n(x+\mu)$$

$$N=(x+\mu)x'+yy'$$

$$W_1=\frac{(1-\mu)(1-q_1)}{c_d}.$$

Here, dash (‘) represents differentiation with respect to eccentric anomaly (E). The mean motion of our problem is given by:

$$n^2=\frac{1}{a}\left(1+\frac{3e^2}{2}+\frac{3A_2}{2}\right) \quad (2.4)$$

$$r_i=(x+x_i)^2+y^2+z^2\ (i=1,2)$$

$$x_1=-\mu,\ x_2=1-\mu,\ \mu=\frac{m_2}{m_1+m_2}.$$

Here, m_1, m_2 are the masses of the bigger and smaller primaries. $(x_1,0,0)$ and $(x_2,0,0)$ are the coordinate of m_1 and m_2 respectively. q_1 is mass reduction factor and W_1 is P-R drag due to bigger primary m_1. $A_2=\frac{r_e^2-r_p^2}{5r^2}$ is oblateness coefficient due to smaller primary m_2, where r_e, r_p represents equatorial radii and polar radii respectively. $r_i (i=1,2)$ are the distances of the infinitesimal mass from m_1 and m_2 respectively. Semi-major axis and eccentricity of orbit is denoted by a and e respectively. c_d is dimensionless velocity of light.

Using perturbation method, we have found location of triangular equilibrium point. For triangular equilibrium points $U_x=0, U_y=0, y\neq 0$ and $z=0$ then we have

$$x=\frac{1}{2}-\mu+\frac{1}{2}\left[(aq_1)^{2/3}(1-A_2-e^2)-a^{2/3}(1-A_2-e^2+A_2a^{-2/3})\right]+\frac{W_1a^{1/2}}{3y_0(1-\mu)\mu}\left\{\left(1+\frac{A_2}{4}-\frac{3e^2}{4}\right)\frac{\mu}{2}-\left(a^{2/3}+(aq_1)^{2/3}\right)\left(1+\frac{A_2}{4}-\frac{7e^2}{4}\right)\frac{\mu}{2}-(1-\mu)a^{2/3}\left(1+\frac{A_2}{4}-2A_2a^{-2/3}-\frac{7e^2}{4}\right)\right\} \quad (2.5)$$

$$y=\pm\left[(aq_1)^{2/3}(1-A_2-e^2)-\frac{1}{4}\left\{1+2\left((aq_1)^{2/3}-a^{2/3}\right)(1-e^2)+\left((aq_1)^{2/3}-a^{2/3}\right)^2(1-2e^2)-2A_2(1+(aq_1)^{2/3}-a^{2/3})^2\right\}+\frac{W_1a^{1/2}}{3y_0(1-\mu)\mu}\left\{\left(1+\frac{A_2}{4}-\frac{3e^2}{4}\right)\frac{\mu}{2}-\mu(aq_1)^{2/3}\left(1+\frac{A_2}{4}-\frac{7e^2}{4}\right)+\left((aq_1)^{4/3}-a^{4/3}\right)\left(1+\frac{A_2}{4}-\frac{11e^2}{4}\right)\frac{\mu}{2}+(1-\mu)a^{2/3}\left(1+\frac{A_2}{4}-2A_2a^{-2/3}-\frac{7e^2}{4}\right)+(1-\mu)a^{2/3}\left((aq_1)^{2/3}-a^{2/3}\right)\left(1+\frac{A_2}{4}-2A_2a^{-2/3}-\frac{11e^2}{4}\right)-\frac{\mu}{2}A_2\left(1+(aq_1)^{2/3}-a^{2/3}\right)\left((aq_1)^{2/3}+a^{2/3}-1\right)-(1-\mu)a^{2/3}\left(1+(aq_1)^{2/3}-a^{2/3}\right)A_2\right\}\right]^{1/2} \quad (2.6)$$

where $y_0=\pm\left[\delta^2(1-e^2)-\frac{1}{4}\left\{1+2(\delta^2-a^{2/3})(1-e^2)\right\}\right]$,
$\delta=(aq_1)^{1/3}$

3. First order normalization

The Lagrangian function of our problem is written as:

$$L=\frac{1}{2}(\dot{x}^2+\dot{y}^2)+\frac{\dot{v}^2}{2}(x^2+y^2)+\dot{v}(x\dot{y}-\dot{x}y)+\frac{(1-\mu)q_1}{r_1}+\frac{\mu}{r_2}+\frac{\mu A_2}{2r_2^3}+W_1\left(\frac{(x+\mu)\dot{x}+y\dot{y}+z\dot{z}}{2r_1^2}-n\ \arctan\left(\frac{y}{x+\mu}\right)\right) \quad (3.1)$$

where v is true anomaly and the Hamiltonian $H=-L+p_x\dot{x}+p_y\dot{y}$, p_x, p_y are the momenta coordinates given by $p_x=\frac{\partial L}{\partial \dot{x}}=\dot{x}-\dot{v}y+\frac{W_1(x+\mu)}{2r_1^2}$, $p_y=\frac{\partial L}{\partial \dot{y}}=\dot{y}-\dot{v}x+\frac{W_1 y}{2r_1^2}$. For simplicity, we suppose $q_1=1-\epsilon$ with $\epsilon\ll 1$. Then triangular equilibrium points takes the form

$$x=\frac{1}{2}-\mu-\frac{a^{2/3}\epsilon}{3}+\frac{A_2a^{2/3}\epsilon}{3}+\frac{a^{2/3}\epsilon e^2}{3}-\frac{A_2}{2}+\frac{2\sqrt{3}W_1a^{1/2}}{9\mu(1-\mu)}\left[\frac{\mu}{6}\left(1-\frac{3e^2}{4}+\frac{A_2}{4}\right)+\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)\left(\frac{\mu\epsilon a^{2/3}}{36}-\frac{a^{2/3}}{3}\right)-\frac{a^{2/3}}{3}\left(1-\frac{11e^2}{4}-\frac{3A_2}{4}\right)+\left(1-\frac{7e^2}{4}-\frac{3A_2}{4}\right)\left(\frac{\mu\epsilon}{6}-\frac{a^{2/3}\mu}{4}\right)+2A_2(1-\mu)\left(\frac{1}{3}-\frac{a^{2/3}}{2}-\frac{\epsilon}{3}+\frac{a^{2/3}\epsilon}{6}\right)\right] \quad (3.2)$$

$$y=\frac{\sqrt{3}}{2}\left[1+\frac{a^{2/3}}{2}(1-e^2)\left(1-\frac{\epsilon}{3}\right)-\frac{A_2}{2}\left(1+a^{2/3}\right)+\frac{4}{3}\left(A_2a^{2/3}\epsilon-1\right)-\frac{A_2}{2}+\frac{4W_1a^{1/2}}{9\sqrt{3}\mu(1-\mu)}\left\{\frac{\mu}{6}\left(1-\frac{3e^2}{4}+\frac{A_2}{4}\right)-\frac{\mu a^{2/3}}{3}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)+\frac{2\epsilon a^{2/3}}{9}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)+(1-\mu)\frac{a^{2/3}}{3}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}-2A_2a^{-2/3}\right)+\frac{\mu A_2}{6}\left(1+a^{2/3}-\frac{2\epsilon a^{2/3}}{3}+\frac{2\epsilon}{3}\right)-\frac{A_2a^{2/3}(1-\mu)}{3}-\frac{\mu a^{2/3}}{4}\left(1-\frac{7e^2}{4}-\frac{3A_2}{4}\right)-\frac{\mu A_2a^{2/3}}{4}\left(1+\frac{2\epsilon}{3}\right)+\frac{\mu\epsilon}{6}\left(1-\frac{7e^2}{4}-\frac{3A_2}{4}\right)-\frac{\mu\epsilon a^{2/3}}{3}\left(1-\frac{11e^2}{4}-\frac{3A_2}{4}\right)+\frac{\epsilon a^{2/3}(1-\mu)}{3}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}-2A_2a^{-2/3}\right)+\frac{\mu\epsilon A_2}{6}(1-e^2-A_2)\left(1+a^{2/3}-\frac{2\epsilon a^{2/3}}{3}+\frac{2\epsilon}{3}\right)-\frac{A_2\epsilon a^{2/3}}{3}(1-\mu)(1-e^2-A_2)+\frac{\mu\epsilon a^{2/3}}{12}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)+\frac{\mu A_2\epsilon a^{2/3}}{12}\right\}\right]. \quad (3.3)$$

We shift the origin to L_4 (triangular equilibrium point). For that, we change $x\to x_*+x$ and $y\to y_*+y$. We suppose $a_*=x_*+\mu$ and $b_*=y_*$, so that

$$a_*=\frac{1}{2}-\frac{a^{2/3}\epsilon}{3}+\frac{A_2a^{2/3}\epsilon}{3}+\frac{a^{2/3}\epsilon e^2}{3}-\frac{A_2}{2}+\frac{2\sqrt{3}W_1a^{1/2}}{9\mu(1-\mu)}\left[\frac{\mu}{6}\left(1-\frac{3e^2}{4}+\frac{A_2}{4}\right)+\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)\left(\frac{\mu\epsilon a^{2/3}}{36}-\frac{a^{2/3}}{3}\right)-\frac{a^{2/3}}{3}\left(1-\frac{11e^2}{4}-\frac{3A_2}{4}\right)+\left(1-\frac{7e^2}{4}-\frac{3A_2}{4}\right)\left(\frac{\mu\epsilon}{6}-\frac{a^{2/3}\mu}{4}\right)+2A_2(1-\mu)\left(\frac{1}{3}-\frac{a^{2/3}}{2}-\frac{\epsilon}{3}+\frac{a^{2/3}\epsilon}{6}\right)\right]$$

$$b_*=\frac{\sqrt{3}}{2}\left[1+\frac{a^{2/3}}{2}(1-e^2)\left(1-\frac{\epsilon}{3}\right)-\frac{A_2}{2}\left(1+a^{2/3}\right)+\frac{4}{3}\left(A_2a^{2/3}\epsilon-1\right)-\frac{A_2}{2}+\frac{4W_1a^{1/2}}{9\sqrt{3}\mu(1-\mu)}\left\{\frac{\mu}{6}\left(1-\frac{3e^2}{4}+\frac{A_2}{4}\right)-\frac{\mu a^{2/3}}{3}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)+\frac{2\epsilon a^{2/3}}{9}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)+(1-\mu)\frac{a^{2/3}}{3}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}-2A_2a^{-2/3}\right)+\frac{\mu A_2}{6}\left(1+a^{2/3}-\frac{2\epsilon a^{2/3}}{3}+\frac{2\epsilon}{3}\right)-\frac{A_2a^{2/3}(1-\mu)}{3}-\frac{\mu a^{2/3}}{4}\left(1-\frac{7e^2}{4}-\frac{3A_2}{4}\right)-\frac{\mu A_2a^{2/3}}{4}\left(1+\frac{2\epsilon}{3}\right)+\frac{\mu\epsilon}{6}\left(1-\frac{7e^2}{4}-\frac{3A_2}{4}\right)-\frac{\mu\epsilon a^{2/3}}{3}\left(1-\frac{11e^2}{4}-\frac{3A_2}{4}\right)+\frac{\epsilon a^{2/3}(1-\mu)}{3}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}-2A_2a^{-2/3}\right)+\frac{\mu\epsilon A_2}{6}(1-e^2-A_2)\left(1+a^{2/3}-\frac{2\epsilon a^{2/3}}{3}+\frac{2\epsilon}{3}\right)-\frac{A_2\epsilon a^{2/3}}{3}(1-\mu)(1-e^2-A_2)+\frac{\mu\epsilon a^{2/3}}{12}\left(1-\frac{7e^2}{4}+\frac{A_2}{4}\right)+\frac{\mu A_2\epsilon a^{2/3}}{12}\right\}\right].$$

Expanding L in power series of x and y, we get

$$L=L_0+L_1+L_2+L_3-----------$$

$$H=H_0+H_1+H_2+H_3-----=-L+p_x\dot{x}+p_y\dot{y},$$

where L_0, L_1, L_2, L_3 are constant, first order term, second order term respectively. Second order term H_2 of Hamiltonian is written as:

$$H_2=\frac{p_x^2+p_y^2}{2}+\dot{v}\left(yp_x-xp_y\right)+Ex^2+Gxy+Fy^2 \quad (3.4)$$

where

$$E=\frac{a^{2/3}}{16}\Big[-1+3a^{2/3}-3\epsilon-3\epsilon a^{2/3}-2A_2-A_2a^{2/3}-9A_2\epsilon a^{2/3}-\frac{13A_2\epsilon}{2}-\frac{5}{4}(1+\mu)e^2a^{2/3}+\frac{e^2\epsilon}{4}-\frac{W_1a^{2/3}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{\frac{17}{2}+\frac{\gamma}{2}-\frac{207\epsilon}{9}-\frac{23\gamma\epsilon}{3}+\frac{a^{2/3}}{3}\Big(24+\frac{3\gamma}{2}-\frac{211\epsilon}{3}-22\gamma\epsilon\Big)+\frac{a^{4/3}}{3}\Big(54-\frac{3\gamma}{2}-\frac{196\epsilon}{3}-5\gamma\epsilon\Big)\Big\}+\gamma\Big\{2\epsilon a^{2/3}-10A_2+22A_2a^{2/3}+\frac{A_2\epsilon a^{2/3}}{3}+\frac{W_1a^{2/3}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{54+\frac{9\gamma}{2}-\frac{197\epsilon}{9}-7\gamma\epsilon+\frac{a^{2/3}}{3}\Big(\frac{201}{2}+12\gamma-\frac{203\epsilon}{3}-\frac{100\gamma\epsilon}{3}\Big)+\frac{a^{4/3}}{3}\Big(36-\frac{52\epsilon}{3}-\frac{35\gamma\epsilon}{3}\Big)\Big\}\Big\}\Big]$$

$$F=-\frac{a^{2/3}}{16}\Big[4+6a^{2/3}-4\epsilon+2\epsilon a^{2/3}+15A_2+6A_2a^{2/3}-39A_2\epsilon a^{2/3}-\frac{5}{4}(1+2\mu)e^2a^{2/3}+\frac{e^2\epsilon a^{2/3}}{4}-\frac{W_1a^{2/3}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{\frac{37}{2}+\frac{11\gamma}{2}-\frac{347\epsilon}{9}+\frac{13\gamma\epsilon}{3}+\frac{a^{2/3}}{3}\Big(57+\frac{15\gamma}{2}-\frac{278\epsilon}{3}+8\gamma\epsilon\Big)+\frac{a^{4/3}}{3}\Big(\frac{33}{2}+\frac{9\gamma}{2}-\frac{201\epsilon}{3}+5\gamma\epsilon\Big)\Big\}+\gamma\Big\{3\epsilon+3\epsilon a^{2/3}-16A_2\epsilon-\frac{5A_2\epsilon a^{2/3}}{18}+\frac{W_1a^{1/2}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{\frac{95}{2}+\frac{15\gamma}{2}-\frac{514\epsilon}{9}-\frac{\gamma\epsilon}{3}+\frac{a^{2/3}}{3}\Big(\frac{243}{2}+\frac{25\gamma}{2}-\frac{412\epsilon}{3}-2\gamma\epsilon\Big)+\frac{a^{4/3}}{3}\Big(\frac{33}{2}-\frac{62\epsilon}{3}+\frac{11\gamma}{2}-\gamma\epsilon\Big)\Big\}\Big\}\Big]$$

$$G=\frac{\sqrt{3}a^{2/3}}{8}\Big[3\epsilon-\epsilon a^{2/3}+5A_2+A_2a^{2/3}-\frac{15A_2\epsilon}{2}-11A_2\epsilon a^{2/3}-\frac{3}{4}(1+\mu)e^2a^{2/3}+\frac{e^2\epsilon a^{2/3}}{4}-\frac{W_1a^{2/3}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{\frac{9}{2}+\frac{5\gamma}{6}-\frac{88\epsilon}{9}+\frac{8\gamma\epsilon}{9}+\frac{a^{2/3}}{3}\Big(33+\frac{2\gamma}{2}-\frac{40\epsilon}{3}+\frac{5\gamma\epsilon}{3}\Big)+\frac{a^{4/3}}{3}\Big(12+\gamma-10\epsilon+\frac{\gamma\epsilon}{3}\Big)\Big\}+\gamma\Big\{\frac{15A_2\epsilon}{2}+9A_2\epsilon a^{2/3}-13A_2-\frac{11\epsilon}{3}-2\epsilon a^{2/3}+\frac{W_1a^{1/2}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{-\frac{15}{2}+\frac{3\gamma}{6}-\frac{98\epsilon}{3}+\frac{2\gamma\epsilon}{9}+\frac{a^{2/3}}{3}\Big(-18+\frac{5\gamma}{2}+45\epsilon+\frac{2\gamma\epsilon}{9}\Big)+\frac{a^{4/3}}{3}\Big(-9+43\epsilon+\frac{\gamma\epsilon}{9}+\frac{\gamma}{2}\Big)\Big\}\Big\}\Big]$$

and $\gamma=1-2\mu$.

Now, we perform transformation from the phase space (x,y,p_xp_y) into the phase space $(\emptyset_1,\emptyset_2,I_1,I_2)$, with the help of Whittaker (1965) method. We suppose the following set of linear equations of variables x and y

$$-\lambda p_x=\frac{\partial H_2}{\partial x}\qquad \lambda_x=\frac{\partial H_2}{\partial p_x}$$

$$-\lambda p_y=\frac{\partial H_2}{\partial y}\qquad \lambda_y=\frac{\partial H_2}{\partial p_y}$$

that is AX=0

$$X=\begin{bmatrix}x\\y\\p_x\\p_y\end{bmatrix},A=\begin{bmatrix}2E&G&\lambda&-n\\G&2F&n&\lambda\\-\lambda&n&1&0\\-n&-\lambda&0&1\end{bmatrix}$$

Clearly $|A|=0$ implies that the characterstic equation corresponding to Hamiltonian is given by

$\lambda^4+2(E+F+n^2)\lambda^2+4EF-G^2+n^4-2n^2(E+F)=0.$

Discriminant is given by

$D=4(E+F+n^2)^2-4\{4EF-G^2+n^4-2n^2(E+F)\}.$

Stability is assured only when D>0.WhenD>0

the roots $\pm i\omega_1 and\ \pm i\omega_2$are related to each other as

$$\omega_1^2+\omega_2^2=\frac{a^{2/3}}{8}\Big[-5-3a^{2/3}+\epsilon-5\epsilon a^{2/3}-17A_2-7A_2a^{2/3}+\frac{127A_2\epsilon a^{2/3}}{6}+\frac{5}{4}\mu e^2a^{2/3}+\frac{e^2\epsilon}{4}(1-a^{2/3})-\frac{W_1a^{2/3}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{-10-5\gamma-\frac{140\epsilon}{9}-12\gamma\epsilon+\frac{a^{2/3}}{3}\Big(-33-6\gamma-\frac{67\epsilon}{3}-30\gamma\epsilon\Big)+\frac{a^{4/3}}{3}\Big(66-3\gamma-\frac{5\epsilon}{3}-10\gamma\epsilon\Big)\Big\}+\gamma\Big\{-\epsilon a^{2/3}-10A_2-3\epsilon+16A_2\epsilon+22A_2a^{2/3}+\frac{11A_2\epsilon a^{2/3}}{18}+\frac{W_1a^{2/3}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{3-3\gamma+\frac{514\epsilon}{9}-\frac{20\gamma\epsilon}{3}+\frac{a^{2/3}}{3}\Big(-21-\frac{\gamma}{2}+\frac{168\epsilon}{3}-\frac{94\gamma\epsilon}{3}\Big)+\frac{a^{4/3}}{3}\Big(\frac{39}{2}+\frac{10\epsilon}{3}-\frac{11\gamma}{2}-\frac{32\gamma\epsilon}{3}\Big)\Big\}\Big\}\Big]+\frac{2}{a}\Big(1+\frac{3e^2}{2}+\frac{3A_2}{2}\Big)\qquad(3.5)$$

$$\omega_1^2\omega_2^2=\frac{19a^{2/3}}{16}-\frac{1}{2}-\frac{5\gamma^2}{8}-\frac{17\gamma^2a^{2/3}}{16}+\frac{9\epsilon}{8}+\frac{9\gamma\epsilon a^{2/3}}{8}-\frac{\gamma^2\epsilon}{4}-\frac{\gamma^2\epsilon a^{2/3}}{8}+\frac{97\gamma A_2}{16}+\frac{5\gamma A_2a^{2/3}}{4}-3\gamma A_2a^{2/3}-\frac{177A_2\epsilon}{32}-2A_2\epsilon a^{2/3}+\frac{2485\gamma\epsilon A_2a^{2/3}}{192}-\frac{5000\gamma\epsilon A_2}{192}+\frac{25e^2\gamma\epsilon}{16}+\frac{7e^2\gamma\epsilon a^{2/3}}{16}-\frac{34e^2\gamma^2\epsilon}{16}+\frac{W_1a^{2/3}}{3\sqrt{3}}\Big(1+\frac{3e^2}{4}+\frac{3A_2}{4}\Big)\Big\{\frac{15}{16}-\frac{216\gamma}{16}-\frac{27\gamma^2}{8}-\frac{2225\epsilon}{192}-\frac{5\epsilon}{96}-\frac{3260\gamma\epsilon}{96}+\frac{900\gamma^2\epsilon}{16}+\frac{a^{2/3}}{3}\Big(\frac{45}{8}-\frac{315\gamma}{16}-\frac{60\gamma^2}{8}-\frac{525\epsilon}{192}-\frac{2700\gamma\epsilon}{96}+\frac{99\gamma^2\epsilon}{16}\Big)+\frac{a^{4/3}}{3}\Big(\frac{90}{8}-\frac{432\epsilon}{192}-\frac{522\gamma}{16}-\frac{129\gamma^2}{8}-\frac{2583\gamma\epsilon}{96}+\frac{99\gamma^2\epsilon}{16}\Big)\Big\}+\frac{1}{a^2}(1+3A_2+3e^2)\ .\qquad(3.6)$$

Following the method for reducing H_2 to the normal form, as in Whittaker (1965) , we use the transformation X=JT where

$$X=\begin{bmatrix}x\\y\\p_x\\p_y\end{bmatrix},T=\begin{bmatrix}Q_1\\Q_2\\P_1\\P_2\end{bmatrix}$$

and $J=[J_{ij}]_{1\le i,j\le 4}$

$P_i=(2I_i\omega_i)^{1/2}\,Cos\,\emptyset_i\,,Q_i=\left(\frac{2I_i}{\omega_i}\right)^{1/2}Sin\,\emptyset_i$, i = 1,2.

We follow the approach of Breakwell and Pringle (1966) and choose $J_{11}=J_{12}=0$, H_2 takes the form

$$H_2=\frac{1}{2}(P_1^2-P_2^2+\omega_1^2Q_1^2-\omega_2^2Q_2^2)\qquad(3.7)$$

and

$$J=\begin{bmatrix}0&0&\frac{-M_1}{\omega_1M_{1*}}&\frac{iM_2}{\omega_2M_{2*}}\\\frac{-2n\omega_1}{M_1M_{1*}}&\frac{2n\omega_2}{M_2M_{2*}}&\frac{G}{\omega_1M_1M_{1*}}&\frac{iG}{\omega_2M_2M_{2*}}\\\frac{-\omega_1(M_1^2-2n^2)}{M_1M_{1*}}&\frac{i\omega_2(M_2^2-2n^2)}{M_2M_{2*}}&\frac{-nG}{\omega_1M_1M_{1*}}&\frac{niG}{\omega_2M_2M_{2*}}\\\frac{-\omega_1G}{M_1M_{1*}}&\frac{iG\omega_2}{\omega_2M_2M_{2*}}&\frac{n(\omega_1^2-M_1^2)}{\omega_1M_1M_{1*}}&\frac{-ni(2\omega_2^2-M_2^2)}{\omega_2M_2M_{2*}}\end{bmatrix}$$

where

$M_j=(\omega_j^2-2F+n^2)^{1/2}\,and\ \ M_{j*}=\sqrt{2}(\omega_j^2-2E+n^2)^{1/2}$ j=1,2

Applying a contact transformation from Q_1,Q_2,P_1,P_2 to Q_1'',Q_2'',P_1'',P_2'' defined by Whittaker (1965)

$P_j''=\frac{\partial W}{\partial Q_j},Q_j''=\frac{\partial W}{\partial P_j}j=1,2\,and$

$$W=\sum_{j=1}^{2}\left[Q_j''\sin^{-1}\left(\frac{P_j}{\sqrt{2\omega_jQ_j''}}\right)+\frac{P_j}{2\omega_j}\sqrt{2\omega_jQ_j''-P_j^2}\right]$$

that is

$$Q_j=\sqrt{\frac{2Q_j''}{\omega_j}}\cos P_j'',\ P_j=\sqrt{2\omega_jQ_j''}\ \sin P_j'',\ j=1,2.$$

Hamiltonian H_2 is transformed into the form $H_2=\omega_1Q_1''-\omega_2Q_2''$. We denote the angular variables $P_1''$and P_2'' by $\emptyset_1$and $\emptyset_2$ and the actions $Q_1''and\ Q_2''\ by\ I_1,I_2$ respectively. The second order part H_2 takes the form $H_2=\omega_1I_1-\omega_2I_2$.

4. Conclusion

Using Whittaker (1965) method, we have found that the second order part H_2 of the Hamiltonian is transformed into the normal form. $H_2 = \omega_1 I_1 - \omega_2 I_2$. We conclude that the values of first and second order components are affected by radiation pressure, oblateness and P-R drag.

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