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A study of libration points in CR3BP under albedo effect

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Abstract

In this paper this is investigated how albedo perturbed the libration points from its original position? It is found that there exist five libration points, three collinear and two non-collinear and all the libration points are affected by Albedo. The non-collinear libration points are stable for a critical value of mass parameter $\mu \leq \mu_c$, where $\mu_c = \mu_o - (0.00891747 + 0.222579k)\alpha$ (μ_o is the critical mass parameter for classical case) but collinear libration points are still unstable.

Keywords:*Restricted Three-Body Problem; Radiation Pressure; Albedo Effect; Libration Points; Stability.*

1. Introduction

The restricted three-body problem is one of well known problem in the field of celestial mechanics in which two finite bodies called primaries move around their center of mass in circular or elliptic orbits under the influence of their mutual gravitational attraction and a third body of infinitesimal mass is moving in the plane of the primaries which is attracted by the primaries and influenced by their motion but not influencing them. In classical case there exist five libration points out of which three are collinear and two are non-collinear. The collinear libration points L_1, L_2 and L_3 are unstable for $0 \leq \mu \leq ½$ and the non collinear libration points $L_{4,5}$ are stable for a critical value of mass parameter $\mu < \mu_c = 0.03852….$, Szehebely [2]. Some studies related to the equilibrium points in R3BP or ER3BP, taken into account the oblateness and triaxiality of the primaries, Coriolis and Centrifugal forces, Yarkovsky effect, variation of the masses of the primaries and the infinitesimal mass etc. are discussed by Danby [1]; Vidyakin [5]; Sharma [6]; Subbarao and Sharma [7]; Choudhary R. K. [9]; Cid R. et. al. [14]; El-Shaboury [19]; Bhatnagar et al. [22]; Selaru D. et.al. [24]; Markellos et al. [25]; Subbarao and Sharma [26]; Khanna and Bhatnagar [27, 29]; Roberts G.E. [33]; Oberti and Vienne [34]; Perdiou et. al. [36]; Sosnytskyi [38]; Ershkov [46]; Arredondo et.al. [47]; Idrisi and Taqvi [48]; Idrisi [49]; Idrisi and Shalini [59]; Idrisi and Jain [60].

The photo-gravitational restricted three-body problem arises from the classical problem if one or both primaries is an intense emitter of radiation, formulated by Radzievskii (1950). He has considered only the central forces of gravitation and radiation pressure on the particle of infinitesimal mass without considering the other two components of light pressure field and studied this problem for three specific bodies; the Sun, a planet and a dust particle. The radiation repulsive force F_p exerted on a particle can be represented in terms of gravitational attraction F_g (Radzievskii, 1950) as $F_p = F_g (1 - q)$, where $q = 1 - F_p/F_g$, a constant for a given particle, is a reduction factor expressed in terms of the particle radius a, density δ and radiation-pressure efficiency factor x (in c.g.s. system) as:

$$q = 1 - \frac{5.6\times10^{-3}}{a\,\delta}x.$$

The assumption that 'q' is a constant implies that the fluctuations in the beam of solar radiation and the effect of planet's shadow are neglected. Typical values for diameter of IDP (Interplanetary Dust Particles) are in the range of 50 – 500 μm and their densities range is 1 – 3 g/cm^3 with an average density of 2 g/cm^3. As the size of the particles increases, their density decreases (Grűn et.al. [32]). Some of the notable research in PRTBP are carried by Chernikov [4]; Bhatnagar and Chawla [11]; Schuerman D.W [12]; Kunitsyn and Tureshbaev [15]; Simmons et. al. [16]; Sharma [17]; Lukyanov [18]; Xuetang et.al. [21]; Ammar [40]; Douskos [42]; Singh and Leke [43]; Katour et.al. [50] etc. In 2012, S. V. Ershkov [46] studied the Yarkovsky effect in generalized photogravitational 3-body problem and proved the existence of maximally 256 different non-planar equilibrium points when second primary is non-oblate spheroid. The main contribution of the natural radiation pressure on the satellite is due to the direct solar radiation and the second main contribution of radiation forces is due to the Earth reflected radiation known as the Albedo studied by Anselmo et.al. [13]; Nuss [28]; McInnes [31]; Bhanderi [35]; Pontus [37]; MacDonald [45] etc. Albedo effect is one of the most interesting non-gravitational force having significant effects on the motion of infinitesimal mass. Albedo is the fraction of solar energy reflected diffusely from the planet back into space (Harris and Lyle [3]). It is the measure of the reflectivity of the planet's surface. Therefore, the Albedo can be defined as the fraction of incident solar radiation returned to the space from the surface of the planet (Rocco [41]) as

$$Albedo = \frac{\text{radiation reflected back to the space}}{\text{incident radiation}}.$$

In this paper the Albedo effect on the existence and stability of the libration points when smaller primary is a homogeneous ellipsoid has been studied. This paper is divided into five sections. In section-2, the equations of motion are derived. The existence of non-collinear and collinear libration points is shown in section-3. In section-4, the stability of non-collinear and collinear libration points is discussed. In the last section, all the results are discussed.

2. Equations of motion

Let m_i (i = 1, 2) be point masses such that $m_1 > m_2$ and m_1 is a source of radiation, are moving in the circular orbits around their center of mass O. An infinitesimal mass $m_3 << 1$, is moving in the plane of motion of m_1 and m_2. The distances of m_3 from m_1, m_2 and O are $\boldsymbol{r_1}$, $\boldsymbol{r_2}$ and $\boldsymbol{r}$ respectively. $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are the gravitational forces acting on m_3 due to m_1 and m_2 respectively, $\boldsymbol{F_p}$ is the solar radiation pressure on m_3 due to m_1 and $\boldsymbol{F_A}$ is the Albedo force (solar radiation reflected by m_2 in space) on m_3 due to m_2 (Fig. 1). Let the line joining m_1 and m_2 be taken as X –axis and O their center of mass as origin. Let the line passing through O and perpendicular to OX and lying in the plane of motion m_1 and m_2 be the Y –axis. Let us consider a synodic system of co-ordinates $Oxyz$ initially coincide with the inertial system $OXYZ$, rotating with angular velocity ω about Z –axis (the z –axis is coincide with Z –axis). We wish to find the equations of motion of m_3 using the terminology of Szebehely [3] in the synodic co-ordinate system and dimensionless variables *i.e.* the distance between the primaries is unity, the unit of time t is such that the gravitational constant $G = 1$ and the sum of the masses of the primaries is unity *i.e.* $m_1 + m_2 = 1$.

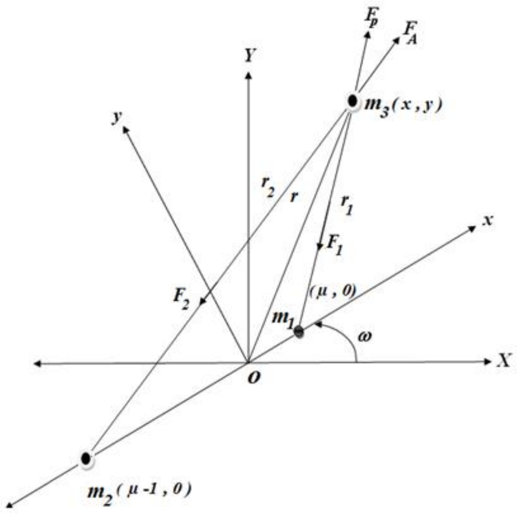

Fig. 1: The Configuration of the R3BP under Albedo effect

The force acting on m_3 due to m_1 and m_2 is $F_1\,(1 - F_p / F_1) = F_1\,(1 - \alpha)$ and $F_2\,(1 - F_A / F_2) = F_2\,(1 - \beta)$ respectively, where $\alpha = F_p / F_1 <<1$ and $\beta = F_A / F_2 << 1$. Also, α and β can be expressed as:

$$\alpha = \frac{L_1}{2\pi G m_1 c\sigma};\ \beta = \frac{L_2}{2\pi G m_2 c\sigma};$$

where L_1 is the luminosity of the large primary m_1, L_2 is the luminosity of small primary m_2, G is the gravitational constant, c is the speed of velocity and σ is mass per unit area.

Now,

$$\frac{\beta}{\alpha} = \frac{m_1}{m_2}\frac{L_2}{L_1} \Rightarrow \beta = \alpha\left(\frac{1-\mu}{\mu}\right)k;$$

$$k = \frac{L_2}{L_1} = \text{constant.} \tag{1}$$

The equations of motion of the infinitesimal mass m_3 in the synodic coordinate system and dimensionless variables are given by

$$\left.\begin{aligned} \ddot{x} - 2n\dot{y} &= \Omega_x \\ \ddot{y} + 2n\dot{x} &= \Omega_y \end{aligned}\right\} \tag{2}$$

where

$$\Omega = \frac{n^2}{2}\left[(1-\mu)r_1^2 + \mu r_2^2\right] + \frac{(1-\mu)(1-\alpha)}{r_1} + \frac{\mu(1-\beta)}{r_2},$$

$$\Omega_x = n^2 x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3},$$

$$\Omega_y = y\left[n^2 - \frac{(1-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(1-\beta)}{r_2^3}\right],$$

$n = 1$ is the mean motion of the primaries, (3)

$$r_1^2 = (x-\mu)^2 + y^2, \tag{4}$$

$$r_2^2 = (x+1-\mu)^2 + y^2, \tag{5}$$

$$0 < \mu = \frac{m_2}{m_1+m_2} < \frac{1}{2} \Rightarrow m_1 = 1-\mu\ ;\ m_2 = \mu.$$

3. Libration Points

At the libration points all the derivatives are zero *i.e.*

$$\dot{x} = 0, \dot{y} = 0, \ddot{x} = 0, \ddot{y} = 0, \Omega_x = 0 \text{ and } \Omega_y = 0.$$

Therefore, the libration points are the solutions of the equations

$$\Omega_x = n^2 x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3} = 0$$

and

$$\Omega_y = y\left[n^2 - \frac{(1-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(1-\beta)}{r_2^3}\right] = 0.$$

3.1 Non-collinear Libration Points

The non-collinear libration points are the solution of the Equations $\Omega_x = 0$ and $\Omega_y = 0$, $y \neq 0$ *i.e.*

$$x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3} = 0, \tag{6}$$

$$1 - \frac{(1-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(1-\beta)}{r_2^3} = 0, \tag{7}$$

On substituting $\alpha = 0$, the solution of Eqns. (6) and (7) is $r_1 = 1$, $r_2 = 1$. Now we assume that the solution of Eqns. (6) and (7) for $\alpha \neq 0$ as $r_1 = 1 + \xi_1$, $r_2 = 1 + \xi_2$, $\xi_1, \xi_2 << 1$.

Substituting these values of r_1 and r_2 in the Eqns. (4) and (5), we get

$$\left.\begin{aligned} x &= \mu - \frac{1}{2} + \xi_2 - \xi_1 \\ y &= \pm\frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}(\xi_2 + \xi_1)\right] \end{aligned}\right\} \tag{8}$$

Now, substituting the values of x, y from Eqns. (8), $r_1 = 1 + \xi_1$, $r_2 = 1 + \xi_2$ in the Eqns. (6) and (7) and neglecting higher order terms, we obtain

$$\xi_1 = -\frac{1}{3}\alpha,\ \xi_2 = -\frac{1}{3}\beta.$$

Thus, the coordinates of the non-collinear libration points $L_{4,5}$ are

$$x = \mu - \frac{1}{2} + \frac{1}{3}(\alpha - \beta),$$

$$y = \pm\frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}\left\{-\frac{1}{3}(\alpha + \beta)\right\}\right]$$

Using the relation (1) *i.e.* $\beta = \alpha\,(1-\mu)\,k / \mu$, we have

$$x = \mu - \frac{1}{2} + \frac{\alpha}{3}\left[1 - \frac{(1-\mu)k}{\mu}\right], \tag{9}$$

$$y = \pm\frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}\left\{-\frac{\alpha}{3}\left[1 + \frac{(1-\mu)k}{\mu}\right]\right\}\right] \tag{10}$$

Thus, we conclude that there exist two non-collinear libration points $L_{4,5}$ and these points are affected by Albedo effect (Fig. 2(*a*) & 2(*b*)), also these points form scalene triangle with the primaries as $r_1 \neq r_2$. For $\mu = 0.1$ and different values of α and k and it is found that the abscissa and ordinate of non-collinear libration points are the decreasing functions of α and k *i.e.* as α and k increases, x and y decreases.

If $k = 0$, the results are in conformity with Bhatnagar & Chawla [10] and for $\alpha = 0$, the classical case of the restricted three body problem is verified (Szebehely [3]).

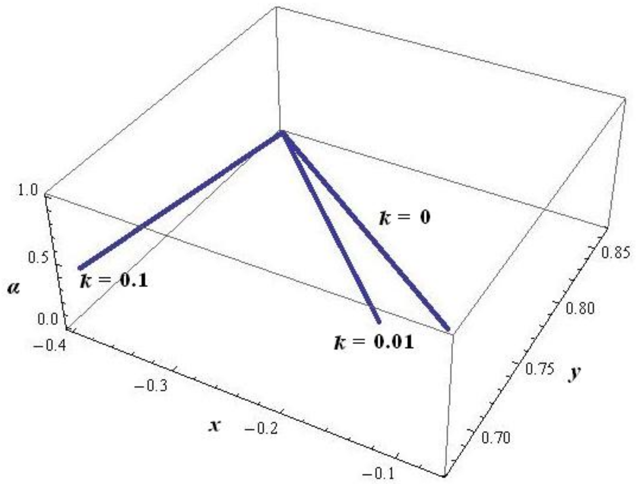

Fig. 2: L_4 versus α; μ = 0.1

3.2 Collinear Libration Points

The collinear libration points are the solution of the Equations $\Omega_x = 0$ and $y = 0$ *i.e.*

$$f(x) = n^2 x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3} = 0 \tag{11}$$

where $r_i = |x - x_i|$, i = 1, 2, is a seventh degree equation in x.

Since $f(x) > 0$ in each of the open intervals $(-\infty, \mu - 1)$, $(\mu - 1, \mu)$ and (μ, ∞), the function $f(x)$ is strictly increasing in each of them. Also, $f(x) \to -\infty$ as $x \to -\infty$, $(\mu - 1) + 0$ or $\mu + 0$ and $f(x) \to \infty$ as $x \to \infty$, $(\mu - 1) - 0$ or $\mu - 0$. Therefore, there exists only one value of x in each of the open intervals $(-\infty, \mu - 1)$, $(\mu - 1, \mu)$ and (μ, ∞) such that $f(x) = 0$. Further, $f(\mu - 2) < 0$, $f(0) \geq 0$ and $f(\mu + 1) > 0$. Therefore, there are only three real roots lying in each of the intervals $(\mu - 2, \mu - 1)$, $(\mu - 1, 0)$ and $(\mu, \mu + 1)$. Thus there are only three collinear libration points.

From the Fig. 3(*a*), this is observed that the first collinear libration point L_1 always lie at the right of the primary m_2, the second libration point L_2 lies between the center of mass of the primaries O and m_1 and the third libration point L_3 lies at the right of the primary m_1.This is also observed that the libration points L_1 and L_3 move away from the center of mass as μ increases while the second libration point L_2 moves toward the center of mass as μ increases.

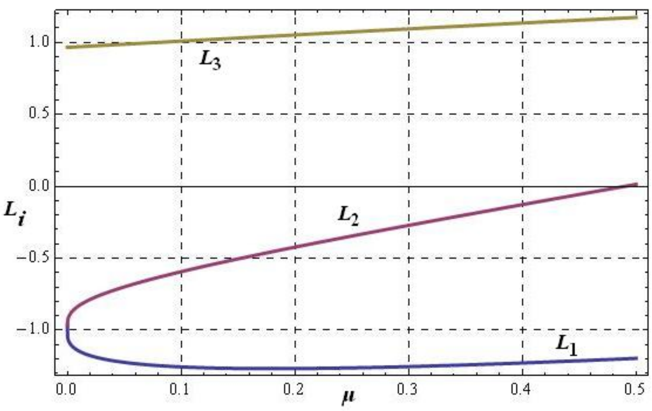

Fig. 3(*a*): μ versus L_i (i = 1, 2, 3); α = 0.1, k = 0.05

The Equation (11) determines the location of the collinear libration points $L_1(x_1, 0)$, L_2 $(x_2, 0)$ and L_3 $(x_3, 0)$ lie in the intervals $(-\infty, \mu - 1)$, $(\mu - 1, \mu)$ and (μ, ∞) respectively, where

$$\begin{aligned} x_1 &= \mu - 1 - \xi_1, \\ x_2 &= \mu - 1 + \xi_2, \\ x_3 &= \mu + \xi_3. \end{aligned} \tag{12}$$

Since the libration point L_1 lies in the interval $(-\infty, \mu - 1)$ *i.e.* left to the smaller primary, we have $r_1 = \mu - x_1$ and $r_2 = \mu - 1 - x_1$ which when substituted in Equation (11), gives

$$n^2 x + \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} + \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3} = 0 \tag{13}$$

Similarly, for L_2 $(x_2, 0)$ and L_3 $(x_3, 0)$ the Equation (11) becomes

$$n^2 x + \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3} = 0 \tag{14}$$

$$n^2 x - \frac{(1-\mu)(x-\mu)(1-\alpha)}{r_1^3} - \frac{\mu(x+1-\mu)(1-\beta)}{r_2^3} = 0 \tag{15}$$

For k = 0, the solutions obtained for the Equations (13), (14) and (15) are the libration points L_i (i = 1, 2, 3) in the photogravitational restricted three-body problem studied by Bhatnagar and Chawla in 1979 but if $k \neq 0$ the libration points L_i (i = 1, 2, 3) are affected by the Albedo effect and this effect displaced the libration points from its actual position as shown in Fig. 3(*b*), 3(*c*) and 3(*d*).

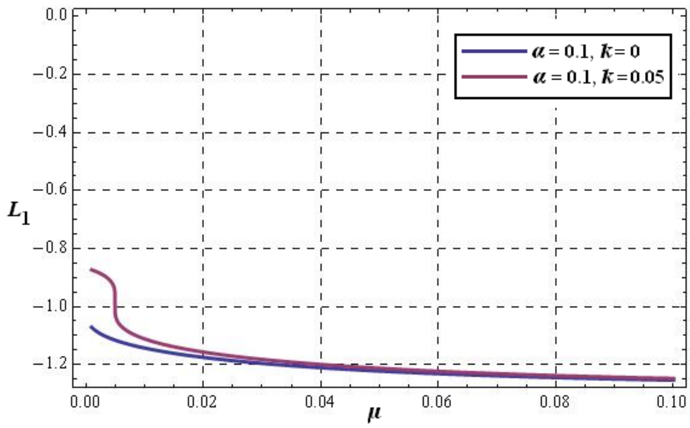

Fig. 3(*b*): μ versus L_1

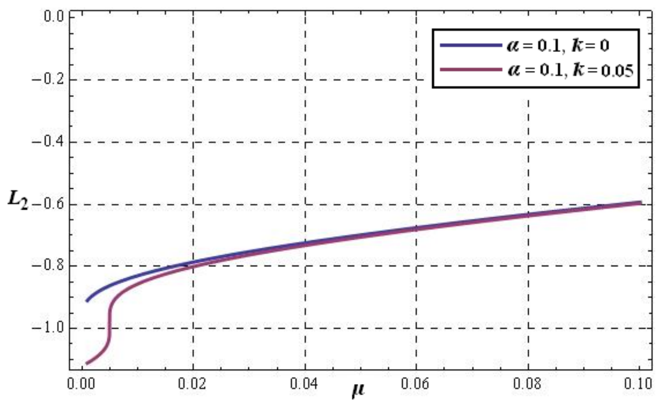

Fig. 3(*c*): μ versus L_2

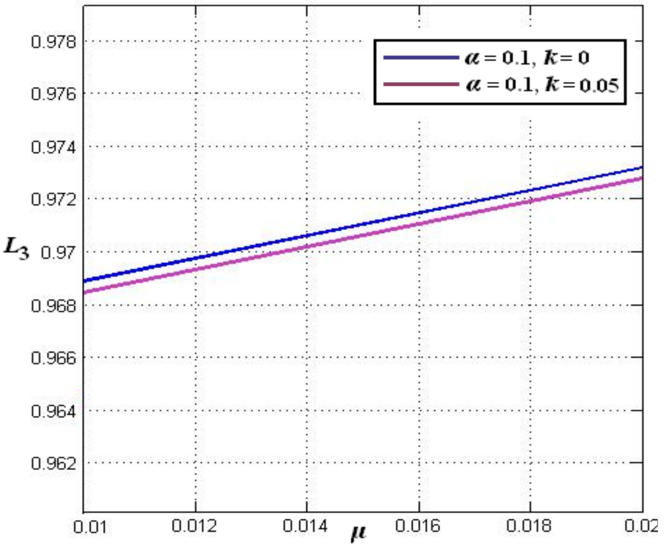

Fig. 3(*d*): *μ* versus L_3

4. Stability of Libration Points

The variational equations are obtained by substituting $x = x_o+\xi$ and $y = y_o+\eta$ in the equations of motion (2), where (x_o, y_o) are the coordinates of L_4 or L_5 and ξ, η <<1 *i.e.*

$$\left.\begin{aligned} \ddot{\xi} - 2n\dot{\eta} &= \xi \overset{o}{\Omega}_{xx} + \eta \overset{o}{\Omega}_{xy}\,, \\ \ddot{\eta} + 2n\dot{\xi} &= \xi \overset{o}{\Omega}_{xy} + \eta \overset{o}{\Omega}_{yy}\,. \end{aligned}\right\} \tag{16}$$

Here we have taken only linear terms in ξ and η. The subscript in Ω indicates the second partial derivative of Ω and superscript *o* indicates that the derivative is to be evaluated at the libration point (x_o, y_o). The characteristic equation corresponding to Eqn. (16) is

$$\lambda^4 + \left(4n^2 - \overset{o}{\Omega}_{xx} - \overset{o}{\Omega}_{yy}\right)\lambda^2 + \overset{o}{\Omega}_{xx}\overset{o}{\Omega}_{yy} - \left(\overset{o}{\Omega}_{xy}\right)^2 = 0 \tag{17}$$

where

$$\overset{o}{\Omega}_{xx} = \frac{3}{4}\left[1 - \frac{2}{3}(1-3\mu)\alpha + \frac{2}{3}(2-3\mu)\beta\right],$$

$$\overset{o}{\Omega}_{xy} = \frac{3\sqrt{3}}{2}\left[\mu - \frac{1}{2} + \frac{1}{9}(1+\mu)\alpha - \frac{1}{9}(2-\mu)\beta\right],$$

$$\overset{o}{\Omega}_{yy} = \frac{9}{4} + \frac{1}{2}(1-3\mu)\alpha - \frac{1}{2}(2-3\mu)\beta.$$

4.1 Stability of Non-collinear Libration points

Let $\lambda^2 = \Pi$, therefore Equation (17) becomes

$$\Pi^2 + q_1\Pi + q_2 = 0 \tag{18}$$

which is a quadratic equation in Π and its roots are given by

$$\Pi_{1,2} = \frac{1}{2}\left(-q_1 \pm \sqrt{D}\right) \tag{19}$$

where

$$q_1 = 4n^2 - \overset{o}{\Omega}_{xx} - \overset{o}{\Omega}_{yy};\ q_2 = \overset{o}{\Omega}_{xx}\overset{o}{\Omega}_{yy} - (\overset{o}{\Omega}_{xy})^2;\ D = q_1^2 - 4q_2.$$

The motion near the Libration point (x_o, y_o) is said to be bounded if all the four roots of Equation (17) are pure imaginary which is possible if and only if the discriminant of Equation (18) is zero *i.e.*

$$1 - 27\mu + 27\mu^2 - 6(1-\mu)\left[\mu + (1-\mu)k\right]\alpha = 0 \tag{20}$$

The Equation (20) is quadratic in μ, on solving it we have

$$\mu_{1,2} = \frac{9 \pm \sqrt{69}}{18} \pm \frac{(2\sqrt{69} \mp 207k + 25\sqrt{69}k)\alpha}{1863} + O(\alpha^2) \tag{21}$$

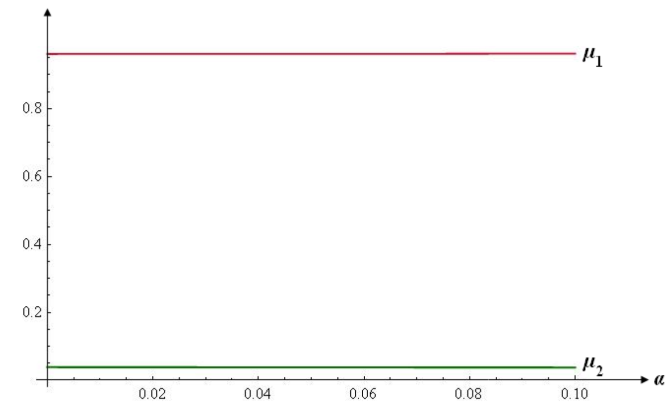

Fig. 4: *μ* versus *α; k* = 0.01

From the Fig. (4), μ_1 > ½ and μ_2 < ½ for all values of α. Thus the critical value of mass parameter μ_c for which the non-collinear libration points $L_{4,5}$ are stable is

$$\mu_c = \frac{9 - \sqrt{69}}{18} - \frac{(2\sqrt{69} + 207k + 25\sqrt{69}k)\alpha}{1863} < \frac{1}{2}.$$

On simplifying we have

$$\mu_c = \mu_o - (0.00891747 + 0.222579\, k)\,\alpha, \tag{22}$$

where μ_o = 0.038520896504551…. is the critical value of mass parameter for classical case. If $\alpha = 0$ then $\mu = \mu_o$ is the solution of the Equation (20) (Szebehely [3]) and for $k = 0$, $\mu = \mu_o - 0.00891747\alpha$ is the solution of the Equation (20) (Bhatnagar and Chawla [10]).

Thus, the non-collinear libration points $L_{4,5}$ are stable for the critical mass parameter $\mu \le \mu_c$, where μ_c is defined in Equation (22).

4.2 Stability of Collinear Libration points

First we consider the point lying in the interval ($\mu - 2$, $\mu - 1$). For this point, $r_2 < 1$, $r_1 > 1$ and

$$\overset{o}{\Omega}_{xx} = n^2 + \frac{2(1-\mu)(1-\alpha)}{r_1^3} + \frac{2\mu(1-\beta)}{r_2^3} > 0,\ \overset{o}{\Omega}_{xy} = 0,$$

$$\overset{o}{\Omega}_{yy} = \mu\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\left(r_2 - \frac{1}{r_2^2}\right) + \frac{\mu}{r_2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)\beta < 0.$$

Similarly, for the points lying in the interval ($\mu - 1$, 0) and (μ, $\mu + 1$), $\overset{o}{\Omega}_{xx} > 0$, $\overset{o}{\Omega}_{xy} = 0$, $\overset{o}{\Omega}_{yy} < 0$. Since the discriminant of Equation (18) is positive and the four roots of the characteristic equation (17) an be written as $\lambda_{1,2} = \pm s$ and $\lambda_{3,4} = \pm it$ (s and t are real). Hence the motion around the collinear libration points is unbounded and consequently the collinear libration points are unstable.

5. Conclusion

In the present paper, the existence and stability of libration points in circular restricted three-body problem has been studied under Albedo effect. The equations of motion in case of Albedo effect are derived, Eqn. (2). For $\beta = 0$, the problem reduces to photogravitational restricted three-body problem. It is found that there exist five libration points, three collinear and two non-collinear. The first collinear libration point L_1 lie at the right of the primary m_2, the second libration point L_2 lies between the center of mass of the primaries O and m_1 and the third libration point L_3 lies at the right of the primary m_1. The libration points L_i (i = 1, 2, 3) are affected by the Albedo effect and this effect displaced the libration points from its actual position as shown in Fig. 3(b), 3(c) and 3(d). Also, there exist two non-collinear libration points $L_{4,5}$ and these points are also affected by Albedo (Fig. 2(a) & 2(b)), these points form scalene triangle with the primaries as $r_1 \neq r_2$. For $k = 0$, the results are agreed with Bhatnagar and Chawla [10] and if we take $\alpha = 0$ and $k = 0$, the classical case of the restricted three body problem is verified (Szebehely [3]). The non-collinear libration points are stable for a critical value of mass parameter $\mu \leq \mu_c$, where $\mu_c = \mu_o - (0.00891747 + 0.222579k)\alpha$ but collinear libration points are still unstable.

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