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Research paper

A comparative study of parametric and semiparametric autoregressive models

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Abstract

Dynamic linear regression models are used widely in applied econometric research. Most applications employ linear autoregressive (AR) models, distributed lag (DL) models or autoregressive distributed lag (ARDL) models. These models, however, perform poorly for data sets with unknown, complex nonlinear patterns. This paper studies nonlinear and semiparametric extensions of the dynamic linear regression model and explores the autoregressive (AR) extensions of two semiparametric techniques to allow unknown forms of nonlinearities in the regression function. The autoregressive GAM (GAM-AR) and autoregressive multivariate adaptive regression splines (MARS-AR) studied in the paper automatically discover and incorporate nonlinearities in autoregressive (AR) models. Performance comparisons among these semiparametric AR models and the linear AR model are carried out via their application to Australian data on growth in GDP and unemployment using RMSE and GCV measures.

Keywords: *Autoregressive (AR) Models; Semiparametric Autoregressive Models; Autoregressive Generalized Additive Models (GAM-AR)); Autoregressive Multivariate Adaptive Regression Splines (MARS-AR).*

1. Introduction

Linear regression model is the most widely used method in empirical research in economics and other social sciences. Nevertheless, when the functional form and the probability density function of the error term are unknown, linear regression model is not appropriate due to potential misspecifications, which might lead to biased estimates and misleading inference. For time series data, dynamic extensions of the linear regression model in the form of autoregressive (AR) models, distributed lag (DL) models, and autoregressive distributed lags (ADRL) models are deployed in most applications. These dynamic regression models, however, inherit the afore-mentioned weaknesses of the linear regression model. To address these drawbacks of dynamic linear regression models, a promising approach is to consider dynamic semiparametric regression models. While semiparametric regression has found many applications in economics, most applications are limited to cross section data.

Recently, algorithms have been developed in statistics and data science, which are inherently nonlinear and for which, the exact form of the nonlinearity does not need to be specified explicitly prior to model estimation. Rather, these algorithms search for, and discover, nonlinearities and interactions in the data that help maximize predictive accuracy. Two widely used algorithms designed to fit semi-parametric models to data with unknown, complex nonlinear patterns are generalized additive models (GAM) and multivariate adaptive regression splines (MARS). A generalized additive model (GAM) for a continuous response variable is a semi-parametric regression in which part of the regression function is specified in terms of a sum of unknown smooth functions of explanatory variables. GAM is a powerful generalization of linear, logistic, and Poisson regression models. GAMs are very flexible and can provide an excellent fit in the presence of nonlinearities in the link function. While parametric regression models emphasize estimation and inference for the parameters of the model, GAMs focus on exploring data nonparametrically and offer more flexibility in model form than the parametric linear and nonlinear regressions do. The key benefit of GAMs is that each of the individual additive terms is estimated using a univariate smoother instead of a multivariate smoother for a high-dimensional non-parametric term circumventing the curse of dimensionality: the slow convergence of an estimator to the true value in high dimensions. The GAM formulation of the regression model allows us to build a regression surface as a sum of lower-dimensional nonparametric terms. Multivariate adaptive regression splines (MARS) (Friedman, 1991) is an algorithm that automatically creates a piecewise linear model as an approximation to nonlinearity. MARS is a highly adaptive nonlinear algorithm, which falls at the intersection of parametric and non-parametric statistics. The MARS fitting methodology has been adapted for the analysis of time series in TSMARS by Lewis and Stevens (1991). Hardle et al. (1997) provide an excellent survey of nonlinear and nonparametric methods for time series data.

Unfortunately, most applications of semi-parametric and nonparametric extensions of linear regression model have been limited to cross section data, with few exceptions. Some applications of dynamic semi-parametric regression include using TSMARS to model long range dependence (Lewis & Ray, 1997), estimating and forecasting nonlinear structure in weekly exchange rates (De Goojer, Ray & Krager, 1998), and modelling of the annual change in the balance of trade for Ireland from 1970 to 2007 (Keogh, 2015). Despite strong empirical evidence of the superior performance of GAM and MARS documented in empirical studies across several disciplines, there have been few applications of dynamic semiparametric models in applied economic research. This paper attempts to fill this gap by applying

autoregressive extensions of GAM and MARS to macroeconomic data and studying the relative predictive performance of autoregressive (AR) model, autoregressive generalized additive model (GAM-AR), and autoregressive multivariate adaptive regression splines (MARS-AR) model empirically via application of these techniques to data on GDP growth rate and unemployment rate for Australia.
The rest of this paper is organized as follows. Section 2 summarizes the Australian data on unemployment and GDP growth rate used in the paper. Section 3 presents the results of fitting an AR model to change in Australian unemployment rate on its own lags. Sections 4 and 5 present the GAM-AR and MARS-AR extensions respectively of the AR model and fit these models to the data. Section 6 compares the quality of fit performance while section 7 compares the predictive performance of the AR, GAM-AR, and MARS-AR models. Section 8 provides some concluding remarks.

2. Data

The data on unemployment and GDP growth rate for Australia are from Australian Bureau of Statistics and are reproduced in Hill et al (2018). The data used here are from Hill et al. (2018) in which monthly data on the unemployment rate (ABS Series A84423050A) are converted to quarterly data using averages and the growth rate in GDP is taken from ABS Series A2304370T. The data consist of 153 quarterly observations on Australian macro variables GDP growth rate (G) and in unemployment rate (U) from 1978Q2 to 2016Q2. The data summary is displayed in Table 1.
The variables are defined as follows.

G =Quarterly percentage change in Australian. Gross Domestic Product (seasonally adjusted).

U = Quarterly Australian Unemployment Rate (seasonally adjusted)

DU = Change in Quarterly Australian Unemployment Rate (seasonally adjusted)

Table 1: Summary Statistics

Variable	Obs	Mean	Std Dev	Min	Max
G	153	.7908497	.7781016	-1.6	3.1
U	153	6.935294	1.766407	4.1	11.1

3. Autoregressive (AR) linear regression model

Okun's law states that change in unemployment from one period to the next depends on the rate of growth of GDP in the economy. Based on Okun's law, Hill et al. (2018) fitted the following distributed lag (DL) linear regression model to the Australian data on quarterly change in unemployment rate (DU) and GDP growth rate data (G) regressing DU on current and lagged values of G for the past two periods:

$$\mathrm{DU(t)} = \beta_1 + \beta_2 \mathrm{G(t)} + \beta_3 \mathrm{G(t-1)} + \quad \beta_3 \mathrm{G(t-2)} + \mathrm{e(t)}. \tag{1}$$

Here the errors e (t) are i.i.d. $(0, \sigma^2)$.
In contrast with the DL regression model in (1) (Hill et al. (2018)), we fit the following autoregressive (AR) linear regression model to the Australian data on quarterly change in unemployment rate (DU) and GDP growth rate (G), which employs a linear regression of DU (t) on its own lagged values for the past two periods: DU (t-1) and DU (t-2).

$$\mathrm{DU(t)} = \beta_1 + \beta_2 \mathrm{DU(t-1)} + \beta_3 \mathrm{DU(t-2)} + \ \mathrm{e(t)}. \tag{2}$$

The results are displayed in Table 2. Both lagged variables DU (t-1) and DU (t-2) are highly significant and appear linearly in the estimated AR model. However, the maximum likelihood estimation of the AR model fails to account for potential nonlinearities in these predictors in the regression function and as such, the inference based on these results is likely to be unreliable.

Table 2: Linear Regression: GLM Normal with Identity link for DU

Variable	Coefficient	Std. Error	t-statistic	p-value
INTERCEPT	-0.00176	0.02042	-0.086	0.93145
DU(t-1)	0.43876	0.08025	5.468	1.91e-07 ***
DU(t-2)	0.23326	0.08025	2.907	0.00422 **

Signif. Codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2501 on 147 degrees of freedom
Multiple R-squared: 0.3638, Adjusted R-squared: 0.3552
F-statistic: 42.03 on 2 and 147 DF, p-value: 3.656e-15
AIC = 14.92759

4. Generalized additive autoregressive (GAM-AR) models

GAMs are semiparametric generalized linear models. These models extend traditional linear models by allowing for a link between the nonlinear nonparametric predictor and the expected value of y. Generalized additive models (GAMs) have been studied extensively in the regression context by Hastie & Tibshirani (1990). The GAM-AR model in eqn. (3) is a generalization of the first-order nonlinear AR model of Jones (1978). It is very flexible as it encompasses linear AR models and many interesting nonlinear models as special cases. These models naturally generalize the linear regression models and allow interpretation of marginal changes, i.e., the effect of one variable (or lagged variable) on the mean function. They combine flexible nonparametric modeling of many variables with statistical precision that is typical for just one explanatory variable. Accurate estimation can be achieved with moderate sample sizes.

An autoregressive generalized additive (GAM-AR)) model is defined as

$$y(t) = \alpha + \sum_{j=1}^{p} f_j(y(t-j)) + e(t), \tag{3}$$

where $f_1(\cdot), \ldots, f_p(\cdot)$ are smooth nonparametric functions and e(t) are i.i.d. $(0,\sigma^2)$.
The two main approaches to estimation of GAMs are backfitting with local scoring algorithm, which employs smoothed partial residuals Hastie and Tibshirani, (1990) and penalized regression splines (PIRLS) (Wood, 2006).
The backfitting approach employs partial residuals $e(t) = S\left(y(t) - \alpha - \sum_{j\neq i} f_j\,(y(t-j)\right)$ for actual fitting of the GAM model to the data (Hastie and Tibshirani, 1990). The process is iterated until convergence. Chen & Tsay (1993) use backfitting algorithms, such as the Alternating Conditional Expectation (ACE) algorithm and the BRUTO algorithm of Hastie & Tibshirani (1990) to fit the additive model (3). The alternative PIRLS approach (Wood, 2006) fits GAMs to the data by employing basis expansions of smooth functions and penalized likelihood maximization for model estimation in which wiggly models are penalized more heavily than smooth models in a controllable manner, and the degree of smoothness is chosen based on cross validation, or AIC or Mallows' criterion. Backfitting has the advantage that it can be used with any scatterplot smoother while PIRLS has the advantage that estimation of the smoothing parameter using generalized cross validation is integrated into estimation.

Application of GAM-AR to data on unemployment and GDP growth rate for Australia

The following GAM-AR model was fitted to the Australian data on unemployment and GDP growth rate using an identity link for DU(t) and nonparametric smooth terms for DU(t-1) and DU(t-2):

$$DU(t) = \beta_1 + f_2\big(DU(t-1)\big) + f_3\big(DU(t-2)\big) + e(t), \tag{4}$$

where $f_2(\cdot)$ and $f_3(\cdot)$ are smooth nonparametric functions and e (t) are i.i.d. $(0, \sigma^2)$.
All the computations for the GAM-AR model were performed using the R package mgcv (Wood, 2021). The results are displayed in Table 3. Both lagged variables DU (t-1) and DU (t-2) are highly significant and appear nonlinearly in the estimated GAM-AR model.

Table 3: GAM-AR Regression of DU (T) on Its First 2 Lags

Variable	Coefficient	Std. Error	t-statistic	p-value
INTERCEPT	-0.00400	0.01874	-0.213	0.831

Approximate significance of smooth terms:

Variable	Estimated df	Refined df	F	p-value
DU(t-1)	5.842	6.864	8.773	<2e-16 ***
DU(t-2)	3.829	4.705	3.031	0.0129 *

Signif. Codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-sq. (adj) = 0.457 Deviance explained = 49.3%
GCV = 0.056689 Scale est. = 0.052656 n = 150

AIC = -3.641013

5. Multivariate adaptive regression splines autoregressive (MARS-AR) model

Multivariate Adaptive Regression Splines (MARS) is a relatively novel technique that combines classical linear regression, mathematical construction of splines, and binary recursive partitioning to produce a local model where relationships between dependent and independent variables are either linear or non-linear. To do this, MARS approximates the underlying function through a set of adaptive piecewise linear regressions termed basis functions (BF) (Friedman, 1991).
MARS is a nonparametric regression technique that automatically models nonlinearities and interactions among independent variables. The MARS model does not require any a priori assumptions about the underlying functional relationship between dependent and independent variables. Instead, this relation is discovered from a set of coefficients and piecewise polynomials of degree q (basis functions) that are entirely "driven" from the regression data (X, y). The MARS regression model is constructed by fitting basis functions to distinct intervals of the independent variables. Generally, piecewise polynomials, also called splines, have pieces smoothly connected at knots k. For a spline of degree q, each segment is a polynomial function; MARS uses two-sided truncated power functions as spline basis functions (Boehmke and Greenwell, 2020 and Friedman, 1991).
The nonlinear AR model can be represented as

$$y(t) = f(y(t-1), y(t-2), \ldots) + e(t), \tag{5}$$

where e(t) is a series of innovations, which is independent of past y(t). Lewis & Stevens (1991) extended the MARS to the MARS-AR model and proposed the following adaptive spline threshold autoregressive (ASTAR) model built by estimating f as a weighted sum of basis functions:

$$y(t) = \sum_{i=1}^{k} c_i\, B_i\big(y(t-j)\big) + e(t), \tag{6}$$

where each c_i is a constant coefficient and $B_i(y(t-j))$ are product basis functions of truncated splines $T^-(y(t-j)) = (k - y(t-j))_+$ and $T^+(y(t-j)) = (y(t-j) - k)_+$ associated with the subregions Rj, j = 1,2,…s in the domain of the lag variables (y(t-1),, ..., y(t-p)), where $(u)_+ = \max(0,u) = u$ if $u > 0$ and $(u)_+ = 0$ if $u < 0$. Each basis function can take one of three forms: a constant 1, a hinge function $h(y\,(t-j)) = T^-(y(t-j))$ or $T^+(y(t-$

j)), or a product of two or more hinge functions forming a model interaction between two or more variables, where k is a constant called a knot.

MARS automatically produces kinks in the predicted y to take account of non-linearities in the regression function f (.). To fit a MARS model, the following three main steps are applied (Friedman (1991)). In the first step, basis functions are added to the model using a forward stepwise procedure. The predictor and the knot location that contribute significantly to the model are selected. In this stage, interactions are also introduced to examine if they could improve the model's fit. In the second step, the basis functions with the least contribution are eliminated using backward pruning. To overcome overfitting, a generalized cross-validation (GCV) statistic is usually used, where a penalty for model complexity is accounted for. The last step selects the optimum MARS model from a group of recommended models based on the fitting and predictive capability of each.

Application of MARS-AR to data on unemployment and GDP growth rate for Australia

Our hypothesis is that in modelling the distribution of DU (t), the dependent variable is related to its own lags DU(t-1), DU(t-2),… in a non-linear and local fashion. Local nonparametric models are suitable under such a hypothesis since they use a strategy of local variable selection and reduction and are flexible enough to allow non-linear relationships. MARS-AR was applied to the Australian data on unemployment and growth rate of GDP. Using DU (t) as the dependent variable, the results are presented in table 4 for MARS-AR allowing for interactions between DU (t-1) and DU (t-2). Table 4 displays a list of 5 main basis functions and their coefficients. The hinge function h(u) is the bisector of the first quadrant if u is greater than 0 and is 0 if u is less than or equal to 0.

All of the computations for the MARS-AR model were performed using the R package earth (Milborrow, 2018). Table 4 presents the results for the MARS-AR model built allowing interactions between the predictors DU (t-1) and DU (t-2). The basis functions B2 and B3 account for the main nonlinear effect of DU (t-1), the basis functions B4 and B5 account for the main nonlinear effect of DU (t-2), and the basis function B6 accounts for the interaction effect of DU (t-1) and DU (t-2).

Table 4: Basis Functions and Their Coefficients in the MARS-AR Model for DU (t) with Interaction Terms

Basis Function	Definition	Coefficients	Std. Error	t-statistic	p-value
B1	1	0.04479089	0.04870156	0.91970	0.35926691
B2	h(DU(t-1)-0.4)	-2.21841604	0.46988012	-4.72124	5.5034e-06
B3	h(DU(t-1)-0.1)	1.35502167	0.26764325	5.06279	1.2433e-06
B4	h(DU(t-2)-0.4)	-2.17641303	0.63567941	-3.42376	0.00080459
B5	h(0.4-DU(t-2))	-0.28759959	0.09121120	-3.15312	0.00196621
B6	h(DU(t-1)-0) * h(DU(t-2)-0.1)	2.51529906	0.80693241	3.11711	0.00220541

Earth selected 6 of 15 terms, and 2 of 2 predictors
Termination condition: Reached nk 21
Importance: DU (t-1), DU (t-2)
Number of terms at each degree of interaction: 1 4 1
Earth GCV 0.0598185 RSS 7.430355 GRSq 0.3876206 RSq 0.4860589
Interpretation of MARS-AR results
To begin with, for a single knot, our hinge function is h (DU (t-1)-0.1) such that
DU (t) = 0.04479089 + 1.35502167 (DU (t-1) – 0.1) if DU (t-1) > 0.1.
Once the first knot has been found, search continues for a second knot, which is found at 0.4. This results in the following linear models for DU (t):
DU (t) = 0.04479089 -2.21841604 (DU (t-1) – 0.4) if DU (t-1) > 0.4.
and so on. The next knot is found at DU (t-2) = 0.4, resulting in the following linear models for the second last knot:
DU (t) = 0.04479089 -2.17641303 (DU (t-2) - 0.4) if DU (t-2) > 0.4,
= 0.04479089 -0.28759959 (0.4 - DU (t-2)) if DU (t-2) < 0.4.
The last pair of knots are found at DU (t-1) = 0 and DU (t-2) = 0.1, resulting in the following model for DU (t):
DU (t) = 0.04479089 + 2.51529906 (DU (t-1) –0)*(DU (t-2)-0.1) if DU (t-1) > 0 and DU (t-2) > 0.1.
To interpret an interaction term, the main effect variables along with their interaction term should be set together in the basis function. For example, the basis function in the model for the interaction of V9 and V10 can be written as: DU(t) = 0.04479089 + 1.35502167$(DU(t-1)-0.1)_+$ - 2.21841604$(DU(t-1)-0.4)_+$ -2.17641303$(DU(t-2)-0.4)_+$ -0.28759959$(0.4- DU(t-2))_+$ -0.019203989$(DU(t-1)-0)_+$*($DU(t-2)-0.4)_+$.
MARS algorithm is designed to retain only the statistically significant terms and as such only these terms are displayed. Nevertheless, the variable importance measures computed by MARS rank the three predictors in the following order of relevance in predicting DU (t): DU(t-1), DU(t-2). It is also interesting to note that nonlinearities in DU(t-1) and DU(t-2) as well as the interactions between DU(t-1) and DU(t-2) are automatically discovered and incorporated by the models built by MARS-AR, which the nonlinear least squares method fails to detect.

6. Comparing the model fitting performance of AR, GAM-AR and MARS-AR regressions

Table 5 displays the results of fitting the linear AR and the semiparametric AR models: GAM-AR and MARS-AR models to the Australian data for change in unemployment (DU) and growth in GDP (G). The table displays the GCV and AIC for each model. For 2 lagged predictors in each model, GAM-AR has the lowest GCV and AIC and the AR model has the highest GCV and AIC of all models. This indicates that of the three models, the GAM-AR model provides the best fit to the data. Furthermore, a negative AIC for GAM-AR and positive AICs for the AR and MARS-AR models are indicative of a lower degree of information loss for GAM-AR relative to the AR and MARS-AR models. Finally, both the GAM-AR and the MARS-AR provide a better fit than the linear AR model as the latter fails to take account of nonlinearity in the data on DU.

Table 5: Model Fitting Results for AR, GAM-AR, and MARS-AR

MODEL	GCV	AIC
AR	0.06422989	14.92759
GAM-AR	0.056689	-3.641013
MARS-AR	0.0598185	0.05913259

7. Comparing the predictive performance of AR, GAM-AR and MARS-AR regressions

We created a random training and test set (120, 46) split of the 146 observations, trained each method on the training set and evaluated its performance on the test set. The relative prediction performance of the three methods was studied by comparing their training and test set root mean squared errors (RMSEs).

Table 6 presents the training and test set RMSEs of forecasts based on the three techniques. The second column of the table presents the training set RMSEs of forecasts based on the three techniques. The training set RMSE of the AR linear regression of DU (t) on DU (t-1) and DU (t-2), is 0.2228612. The training set RMSE for the GAM-AR with a nonparametric smooth term each for DU (t-1) and DU (t-2), is 0.2091772. In comparison with AR and GAM-AR, the training set RMSE of the MARS-AR regression is 0.2110193. The lowest training set RMSE for GAM-AR forecasts relative to the AR and MARS-AR models suggests that GAM-AR outperforms these techniques in terms of training set forecasting performance.

Table 6: Training and Test Sample Forecast RMSEs

Model	Training Sample Forecast RMSE	Test Sample Forecast RMSE
AR	0.2228612	0.2932992
GAM-AR	0.2091772	0.2133849
MARS-AR	0.2110193	0.241959

The third column of the table presents the test set RMSEs of forecasts based on the three techniques. The test set RMSE of the AR linear regression of DU (t) on DU (t-1) and DU (t-2), is 0.2932992 while the test set RMSE for the GAM-AR with a nonparametric smooth term each for DU (t-1) and DU (t-2), is 0.2133849. In comparison with AR and GAM-AR models, the test set RMSE of the MARS-AR regression is 0.241959. The lowest test set RMSE for GAM-AR forecasts relative to the forecasts based on AR and MARS-AR models suggests that GAM-AR outperforms the AR and MARS-AR techniques in terms of test set performance as well. This finding suggests that GAM-AR models are less likely to overfit than the AR and MARS-AR models. The superior performance of GAM-AR relative to the AR and the MARS-AR models could be attributed to its flexibility and better ability in handling nonlinearity in data relative to the AR model as well as its simplicity relative to the MARS-AR model.

8. Conclusions

The paper studied two dynamic semi-parametric extensions of the AR model: GAM-AR and MARS-AR and compared them to the AR model via their application to Australian data on unemployment and GDP growth rate. A comparison of the model fit performance of these techniques using the GCV and AIC measures suggests that the GAM-AR model provides the best fit of all models compared. A comparison of the predictive performance of these techniques suggests that while the difference between the GAM-AR and MARS-AR techniques is not significant, GAM-AR outperforms both the AR and the MARS-AR models. Nevertheless, the comparison was based on a moderate sample size in the present study. Further studies based on much larger time series data are needed to confirm the validity of this conclusion in large samples. Finally, autoregressive distributed lags (ARDL) extensions of the GAM and MARS models may provide further improvements in the model-fitting and predictive performance with nonlinear data patterns and will be explored in future research by the author.

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