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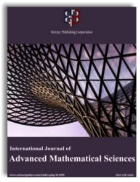

Comparative view of the modified Vogel's approximation methods in transportation problem

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Abstract

Recently, the whole world is experiencing economic hardship and one fast road to ply is to cut out some expenses. Xlife group is equally affected and so the study proposes a means to minimize its transportation cost if effected. A comparative surgery on Vogel's approximation with its modified versions were conducted on minimizing the cost of transporting manufactured goods from (Xlife group's) points of supply to certain destinations was undertaken. The Vogel's Approximation Method (VAM) and the Modified Vogel's Approximation Method based on standard deviation, distribution indicators and average cost were applied to determine an initial basic feasible solution and optimal solution for certain transportation problems. All methods yield identical minimum transportation cost except at an instance.

Keywords: *Optimization; Vogel's Approximation Method; Standard Deviation; Transportation Cost.*

1. Introduction

Companies, firms, groups, families, individuals often have a pattern for moving and getting goods and services to intended destinations. But the need do arise to cut transportation cost at an event of poor economic trend. This problem of reducing cost of moving from one point can be alleviated by employing the transportation models in operation research. Hitchcock [1] pioneered the work on transportation problems when he considered the rudiments of transportation notion. Gass [2] investigated transportation problem with consideration to a distribution method which takes solace in simplex method. Juman et al. [3] employed the Vogel's approximation method for solving the unbalanced form of transportation problem. Resolving transportation problem having degeneracy was examined by Sarbjit Singh [4] where the trio of Least cost, North West and Vogel's method were used. Genetic algorithm was employed to minimize the problem of transportation cost for a beverage producing company in Nigeria by Edokpia and Amiolemhen [5]. Lakshimi [6] obtained the preliminary feasible solution that is leading to the minimum transportation cost using various methods. Using the data obtained from BUA group, CCCNN transport, Aliyu et al. [7] obtained a lessened transportation cost using the Least Cost, North West and Vogel's approximation methods. However, each of these three methods was enough to obtain optimal solution. Rafi et al. [8] compared the performance of some existing methods in solving transportation problems where they inferred that the Vogel's approximation method gave the best optimal result. Victor-Edema and Akehwe [9] employed the basic four transportation methods to diminish the high transport cost experienced by Dangote Cement, Port Harcourt.

Later, modification for some of these methods were suggested. Das et al. [10] modified Vogel's approximation method by proposing a new approach to obtaining the penalty cost of the transportation table. A standard deviation based hybrid version of Vogel's method was proposed by Akpan et al. [11]. Ullah et al. [12] developed a newer version of Vogel's approximation method which relies on distribution indicators. The new method performed excellently well when compared with Vogel's approximation. Agaie et al. [13] postulated an improved Vogel's approximation method that replaces the conventional taking of cost difference with average cost. Karagu and Sahin [14] developed a new method called Karagu – Sahin Approximation method which performs reasonably well when compared with North West Corner and Vogel's method. A modified version of Least Cost method was postulated by Prasad and Singh [15]. Nopiyana et al.[16] developed a modified version of the ASM method for solving transportation problems. The modified Vogel's method suggested by Akpan (2015) was employied by Crankson et al. [17] to obtain the optimal solution for the transportation problem encountered by a mining company, which will save the company a sum of GH ¢996,315 within 8 months when utilized.

2. Methodology

It is important to first decipher that a transportation problem is balanced or not. If it is balanced then a decision need to be made as to what method is to be used. For an unbalanced problem a dummy variable need to be added to apportion with the deficiency. However, all the problems considered here are balanced and the comparison is on two different techniques of same big class together with the conventional one (which is Vogel).

Conventional Vogel's Approximation Method [18]

- Ensure the problem is balanced (total demand=total supply). Initiate the next process when it is balanced.
- Obtain the penalty for each row and column by finding the difference in the two lowest-cost cells,
- Select the biggest penalty (regardless of the row or column it can be found) then, apportion the minimum value between the demand and supply to the cell with the least cost in the corresponding row or column.
- Continue the second and third steps above, until both the demand and supply are completely exhausted.

Distribution Indicator Based Modified Vogel's Approximation Method [12]

- Ensure the problem is balanced (total demand=total supply) and if not, introduce a dummy to deficient side. Initiate the next process when it is balanced.
- Take the positive difference of the largest element and each element of every row, then place this difference on the left top corner of that element.
- Take the positive difference of the largest element and each element of every column then place this difference on the left bottom corner of that element.
- A new matrix should be produced with each element being a sum of items in the left top and left bottom element of the above two procedures.
- Obtain the distribution indicators for each row and column by taking the difference between the two largest element of each row and column of the new matrix formed.
- Identify the highest distribution indicator (even if such highest indicator are more than one), then select the highest indicator along which the cell with the biggest value is, and assign value to that cell based on the minimum between the demand and the supply that correspond to where that indicator is.
- Cancel out any row or column that either its corresponding demand or supply has been exhausted.
- Steps 5 to 7 are to be rehashed until all the demand and supply have been drained.

Standard Deviation Based Modified Vogel's Approximation Method [11]

- Ensure the problem is balanced (total demand=total supply), if not, introduce a dummy to deficient side. Initiate the next process when it is balanced.
- Obtain the cost standard deviation $\left(\sigma = \sqrt{\frac{(x_i-\mu)^2}{n}}\right)$ for each row and column.
- Detect the row or column with the biggest cost standard deviation and allot the minimum between the demand and supply that can corresponds to the least cost cell in that row or column.
- Revoke any row or column that either its corresponding demand or supply has been exhausted.
- Repeat 3 and 4 until all the demand and supply have been depleted.

Average Cost Based Modified Vogel's Approximation Method [13]

- Ensure the problem is balanced (total demand=total supply), if not, introduce a dummy to deficient side. Initiate the next process when it is balanced.
- Obtain the average cost for each row and column by taking the average of the highest cost and least cost within that row or column based on $\left(\bar{x} = \frac{\text{highest cost}+\text{lowest cost}}{2}\right)$ for each row and column.
- Allocate the $\min(\text{demand}, \text{supply})$ to the cell with the least cost located in the least average cost row or column. In a case where there is more than one least cost row or column then select the next level average having less average.
- Dismiss the rows and columns that their supply and demand have been met.
- Repeat these steps till all demand and supply have been worn out.

3. Data analysis

- Problem 1

Xlife company has three production facilities A, B, C with production capabilities of 280, 320 and 400 (100s) per month of product, respectively. These units are to be moved to destinations (Ile Ife, Ede, Osogbo, Imesi Ile) based on a respective demand for 200, 240,360 and 200 units. The transportation cost (in naira) per unit from factories to destinations are below above.

Table 1: Transportation Tableau from Xlife Company

Source	Destination				Supply
	Ile Ife	Ede	Osogbo	Imesi Ile	
A	5	12	17	15	280
B	16	18	14	10	320
C	22	25	13	9	400
Demand	200	240	360	200	1000

- Model Formulation and Solution

Let x_{ij}= units of product to be moved from production $i(i = 1,2,3)$to destinations $j(j = 1,2,3,4)$

The transportation problem as a Linear Programming model takes the form:

Minimize (total transportation cost) $Z = 5x_{11} + 12x_{12} + 17x_{13} + 15x_{14} + 16x_{21} + 18x_{22} + 14x_{23} + 10x_{24} + 22x_{31} + 25x_{32} + 13x_{33} + 9x_{34}$

Subject to the constraints

$x_{11} + x_{12} + x_{13} + x_{14} = 280, x_{21} + x_{22} + x_{23} + x_{24} = 320, x_{31} + x_{32} + x_{33} + x_{34} = 400$ (supply)

$x_{11} + x_{21} + x_{31} = 200, x_{12} + x_{22} + x_{32} = 240, x_{13} + x_{23} + x_{33} = 360, x_{14} + x_{24} + x_{34} = 200$ (demand)

and $x_{ij} \geq 0$ for $i = 1,2,3$ and $j = 1,2,3,4$.

Table 2: Tables Showing Procedures for the Basic Feasible Solution to Problem 1 Using the Vogel's Approximation Method

Source	Destination				Supply	RD	RD	RD	RD
	Ile Ife	Ede	Osogbo	Imesi Ile					
A	5	12	17	15	280	7	3		
B	16	18	14	10	320	4	4	4	4
C	22	25	13	9	400	4	4	4	4
Demand	200	240	360	200					
CD	11	6	1	1					
CD		6	1	1					
CD		7	1	1					

Initial basic feasible solution by Vogel $=5 \times 200 + 12 \times 80 + 18 \times 160 + 9 \times 200 + 13 \times 200 + 14 \times 160 = 11480$

Table 3: Tables Showing Procedures for the Basic Feasible Solution to Problem 1 Using the Distribution Indicator Based Modified Vogel's Approximation Method

Iteration 1	Destination				Supply
Source	Ile Ife	Ede	Osogbo	Imesi Ile	
A	${}^{12}_{17}5$	${}^{5}_{13}12$	${}^{0}_{0}17$	${}^{2}_{0}15$	280
B	${}^{2}_{6}16$	${}^{0}_{7}18$	${}^{4}_{3}14$	${}^{8}_{5}10$	320
C	${}^{3}_{0}22$	${}^{0}_{0}25$	${}^{12}_{4}13$	${}^{16}_{6}9$	400
Demand	200	240	360	200	

Adding up the numbers in subscripts and superscripts gives a reduced matrix below. The row and column distribution indicators are obtainable by subtracting the two biggest element in each row and column. The $min(demand, supply)$ is then allocated to the largest cell in the row or column with the highest distribution indicator.

Iteration 2	Destination				Supply	RD	RD	RD	RD	RD
Source	Ile Ife	Ede	Osogbo	Imesi Ile						
A	29	18	0	2	280	11	16			
B	8	7	7	13	320	5	6	6	0	
C	3	0	16	22	400	6	6	6	16	
Demand	200	240	360	200						
CD	21	11	9	9						
CD		11	9	9						
CD		7	9	9						
		7	9							
		7	9							

The basic feasible solution based on this problem based on this above table according to the Distribution Indicator based Modified Vogel's Approximation $= 5 \times 200 + 12 \times 80 + 9 \times 200 + 13 \times 200 + 18 \times 160 + 14 \times 160 = 11480$

Table 4: Table for Obtaining the Initial Basic Feasible Solution to Problem 1 Via the Standard Deviation Based Modified Vogel's Approximation Method

Source	Destination				Supply	SD	SD	SD	SD	SD
	Ile Ife	Ede	Osogbo	Imesi Ile						
A	5	12	17	15	280	4.6	2.1	2.5	2.5	
B	16	18	14	10	320	3.0	3.3	2	2	2
C	22	25	13	9	400	6.5	6.8	6		
Demand	200	240	360	200						
	7.0	5.3	1.7	2.6						
	5.3	1.9								
	3.0	1.5								

Based on this above table on standard deviation based Vogel modification, the initial basic feasible solution $= 5 \times 200 + 9 \times 200 + 13 \times 200 + 12 \times 80 + 18 \times 160 + 14 \times 160 = 11480$.

Table 5: Table for Obtaining the Initial Basic Feasible Solution to Problem 1 Via the Average Cost Based Modified Vogel's Approximation Method

Source	Destination				Supply	RA	RA	RA	RA	RA
	Ile Ife	Ede	Osogbo	Imesi Ile						
A	5	12	17	15	280	11	14.5	14.5		
B	16	18	14	10	320	14	14	16	16	16
C	22	25	13	9	400	17	17	19	19	
Demand	200	240	360	200						
CA	13.5	18.5	15	12						
CA		18.5	15	12						
CA		18.5	15							
CA		18	14							

Based on this above table on the average cost based Vogel modification, the initial basic feasible solution $= 5 \times 200 + 9 \times 200 + 12 \times 80 + 13 \times 200 + 14 \times 160 + 18 \times 160 = 11480$.

- Problem 2:

The attempt is to minimize the cost of transportation from xlife business based on data given in Table 6 using the methods earlier explained.

Table 6: Transportation Tableau from Xlife Business

Source	Destination				Supply
	P	Q	R	T	
A	2	2	2	4	1000
B	4	6	4	3	700
C	3	2	1	7	800
Demand	900	700	500	400	

The overview to the basic feasible solution for problem 2 above is drafted in Tables 7, 8, 9, 10 using aforementioned techniques.

Table 7: Framework for the Basic Feasible Solution to Problem 2 Via the Vogel's Approximation Method

Source	Destination				Supply	RD	RD
	P	Q	R	T			
A	2	2	2	4	1000	0	1
B	4	6	4	3	700	1	1
C	3	2	1	7	800		
Demand	900	700	500	400			
CD	1	0	1	1			
CD	1	0		1			

The basic feasible solution based on table 6 via the Vogel's $= 1 \times 500 + 2 \times 300 + 2 \times 600 + 4 \times 300 + 3 \times 400 + 2 \times 400 = 5500$

Table 8: Outline for the Basic Feasible Solution to Problem 2 Using the Distribution Indicator Based Modified Vogel's Approximation Method

Iteration 1	Destination				Supply					
Source	P	Q	R	T						
A	${}^{2}_{2}2$	${}^{2}_{4}2$	${}^{2}_{2}2$	${}^{0}_{3}4$	1000					
B	${}^{2}_{0}4$	${}^{0}_{0}6$	${}^{2}_{0}4$	${}^{3}_{4}3$	700					
C	${}^{4}_{1}3$	${}^{5}_{4}2$	${}^{6}_{3}1$	${}^{0}_{0}7$	800					
Demand	900	700	500	400						
Iteration 2	Destination				Supply	RD	RD	RD	RD	RD
Source	P	Q	R	T						
A	4	6	4	3	1000	2	2	2	2	4
B	2	0	2	7	700	5	5	2	2	2
C	5	9	9	0	800					
Demand	900	700	500	400						
	1	3	5	4						
	1	3		4						
	1	3								
	2	6								

The basic feasible solution based on this problem based on table 8 above according to the Distribution Indicator based Modified Vogel's Approximation $= 1 \times 500 + 3 \times 400 + 2 \times 300 + 2 \times 400 + 2 \times 600 + 4 \times 300 = 5500$

Table 9: Table for Obtaining the Initial Basic Feasible Solution to Problem 2 Via the Standard Deviation Based Modified Vogel's Approximation Method

Source	Destination				Supply	SD	SD	SD	SD	SD
	P	Q	R	T						
A	2	2	2	4	1000	0.9	0.9	1.0	2.0	
B	4	6	4	3	700	1.1	1.3	1.3	0.5	0.5
C	5	2	1	7	800	2.3	2.2			
Demand	900	700	500	400						
	0.8	1.9	1.3	1.7						
	0.8	1.9		1.7						
	1.0	2.0		0.5						
	1.0			0.5						

Based on Table 9, on standard deviation based Vogel modification, the initial basic feasible solution $= 1 \times 500 + 2 \times 300 + 2 \times 400 + 2 \times 600 + 4 \times 300 + 3 \times 400 = 5500$

Table 10: Table for Obtaining the Initial Basic Feasible Solution to Problem 2 Via the Average Cost Based Modified Vogel's Approximation Method

Source	Destination				Supply	RA	RA	RA	RA	RA
	P	Q	R	T						
A	2	2	2	4	1000	3	3	3		
B	4	6	4	3	700	4.5	4.5	4.5	4.5	3
C	5	2	1	7	800	4	4.5	4.5	4.5	
Demand	900	700	500	400						
	3	4	2.5	5						
	3	4		5						
		4		5						
		4		5						

Based on Table 10, on the average cost based Vogel modification, the initial basic feasible solution $= 1 \times 500 + 2 \times 900 + 2 \times 100 + 2 \times 300 + 6 \times 300 + 3 \times 400 = 6100$

- Problem 3

Obtain the initial basic feasible solution for the transportation problem sourced from Xlife Exports put on Table 11 using aforementioned methods.

Table 11: Transportation Tableau from Xlife Exports

Source	Destination				Supply
	P	Q	R	T	
A	14	19	6	24	200
B	3	17	10	18	450
C	11	7	11	12	350
Demand	150	350	80	420	

The overview to the basic feasible solution for problem 3 above is drafted in Tables 12, 13, 14 and 15 through earlier introduced procedures.

Table 12: Framework for the Basic Feasible Solution to Problem 3 Via the Vogel's Approximation Method

Source	Destination				Supply	RD	RD	RD
	P	Q	R	T				
A	14	19	6	24	8	8	8	18
B	3	17	10	18	450	7	7	8
C	11	7	11	12	350	4		
Demand	150	350	80	420				
CD	8	10	4	6				
CD	11		4	6				
CD			4	6				

The basic feasible solution based on Table 12 via the Vogel's $= 7 \times 350 + 3 \times 150 + 6 \times 80 + 24 \times 120 + 18 \times 300 = 11{,}660$.

Table 13: Outline for the Basic Feasible Solution to Problem 3 Using the Distribution Indicator Based Modified Vogel's Approximation Method

Iteration 1	Destination				Supply
Source	P	Q	R	T	
A	${}^{10}_{0}14$	${}^{5}_{0}19$	${}^{18}_{5}6$	${}^{0}_{0}24$	200
B	${}^{15}_{11}3$	${}^{1}_{2}17$	${}^{8}_{1}10$	${}^{0}_{6}18$	450
C	${}^{1}_{3}11$	${}^{5}_{12}7$	${}^{1}_{0}11$	${}^{0}_{12}12$	350
Demand	150	350	80	420	

Iteration 2	Destination				Supply	RD	RD	RD	RD
Source	P	Q	R	T					
A	10_{14}	5_{19}	23_{6}	0_{24}	200	13	18	5	0
B	26_{3}	3_{17}	9_{10}	6_{18}	450	17	3	3	6
C	4_{11}	17_{7}	1_{11}	12_{12}	350	5	5	5	12
Demand	150	350	80	420					
CD	16	12	14	6					
CD		12	4	6					
CD		12		6					
CD				6					

The basic feasible solution based on this problem based on table 13 which is according to the Distribution Indicator based Modified Vogel's Approximation $= 3 \times 150 + 6 \times 80 + 7 \times 350 + 18 \times 300 + 24 \times 120 = 11660$

Table 14: Table for Obtaining the Initial Basic Feasible Solution to Problem 3 Via the Standard Deviation Based Modified Vogel's Approximation Method

Source	Destination				Supply	SD	SD	SD
	P	Q	R	T				
A	14	19	6	24	200	6.7	4.1	2.5
B	3	17	10	18	450	6.0	6.9	0.5
C	11	7	11	12	350	1.92	2.2	2.5
Demand	150	350	80	420				
	4.6	5.3	2.2	4.9				
	4.6	5.3		4.9				
		5.3		4.9				

Based on Table 14, on standard deviation based Vogel modification, the initial basic feasible solution $= 6 \times 80 + 3 \times 150 + 7 \times 350 + 24 \times 120 + 18 \times 300 = 11660$.

Table 15: Table for Obtaining the Initial Basic Feasible Solution to Problem 3 Via the Average Cost Based Modified Vogel's Approximation Method

Source	Destination				Supply	RA	RA	RA
	P	Q	R	T				
A	14	19	6	24	200	15	15	21.5
B	3	17	10	18	450	10.5	14	17.5
C	11	7	11	12	350	9.5	9.5	9.5
Demand	150	350	80	420				
CA	8.5	13	8.5	18				
CA		13	8.5	18				
CA		13		18				

Based on Table 15, on average cost based Vogel modification, the initial basic feasible solution $= 3 \times 150 + 6 \times 80 + 7 \times 350 + 18 \times 300 + 24 \times 120 = 11660$

4. Result and discussion

The three transportation problems have been examined using the four techniques and all produced the result except the average cost modification for the case involving the second problem.

Test for Optimality of the Basic Feasible Solution

All the above obtained initial basic feasible solutions were examined for optimal solution via the modified distribution (MODI) method. Using $Ev_{ij} = C - (u_i + v_j)$ where C is the cost at each cell, u_{ij} and v_{ij} are the cells values to be determined at cells that were used. Using an extract of the data obtained from table 2 as an illustration,

	$v_1 = 5$	$v_1 = 12$	$v_3 = 8$	$v_4 = 4$
$u_1 = 0$	5	12	17	15
$u_2 = 6$	16	18	14	10
$u_3 = 5$	22	25	13	9

By default, let $u_1 = 0$, using $C_{ij} = u_i + v_j$ for allocated cells then $c_1 = 5$ then $5 = 0 + v_1$ so $v_1 = 5$. Also $C_{12} = u_1 + v_2$ then $12 = 0 + v_2$ and so $v_2 = 12$. Going on and on, we have other values.

The test proper for the optimality can now be conducted using $Ev_{ij} = C - (u_i + v_{ij})$. First, $Ev_{13} = C_{13} - (u_1 + v_3) = 17 - (0 + 5) = 12 > 0$;

$Ev_{14} = \big(15 - (0 + 4)\big) > 0$; $Ev_{21} = \big(16 - (6 + 5)\big) > 0$; $Ev_{24} = \big(10 - (4 + 6)\big) \geq 0$;

$Ev_{31} = \big(22 - (5 + 5)\big) > 0$; $Ev_{32} = \big(25 - (12 + 5)\big) > 0$

All the Ev_{ij}s are positive for the above problem. Also, for the other problems for which the test were not shown in this write up, all $\big(Ev_{ij}\big)s \geq 0$ which implies that each of the obtained basic feasible solutions is optimal solution. The only exception is in basic feasible solution obtained using the average cost idea where the basic feasible solution is higher than that of others which invariably implies that this method didn't give an optimal solution for that problem. Table 16 below summarized the optimal solution for each of the problems. Based on this result, the least and most realistic cost to each problem is shown which will save the group a lot of money if followed or executed. From, the result, virtually all the methods or the techniques utilized produce an optimal solution.

Table 16: Optimal Solution to the Problems Considered

Problem	Methods	Transportation Cost
1	Vogel's, modified versions based on average cost, distribution indicator, standard deviation	11480
2	Vogel's, modified versions based on distribution indicator, standard deviation	5500
3	Vogel's, modified versions based on average cost, distribution indicator, standard deviation	11660

5. Conclusion

Here, the performance of Vogel's Approximation method with its some modified companions have been examined and the following were inferred:

- The Vogel's method together with all the advanced versions (except the average cost) all gave the same initial basic feasible solution and all passed the MOD's test for optimal, which invariably gave the same optimal solution.
- The average cost method need to be modified so that it can be guaranteed to give optimal solution at any instance.
- The distribution indicator based method is easier and simpler but goes through more iterations and timing than the standard deviation based method.

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