



Fourier coefficients of a class of ETA quotients of weight 20 with level 12

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Abstract

Williams[18] and later Yao, Xia and Jin[15] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of 126 eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$ and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. Here, we will express the even Fourier coefficients of 570 eta quotients in terms of $\sigma_{19}(n), \sigma_{19}(\frac{n}{2}), \sigma_{19}(\frac{n}{3}), \sigma_{19}(\frac{n}{4}), \sigma_{19}(\frac{n}{6})$ and $\sigma_{19}(\frac{n}{12})$.

Keywords: Dedekind eta function; eta quotients; Fourier series.

1. Introduction

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\sigma_i(n) : = \sum_{d \text{ positive integer}, d|n} d^i, \text{ if } n \text{ is a positive integer, and} \tag{1}$$

$\sigma_i(n) : = 0$, if n is not a positive integer.

The Dedekind eta function is defined by

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \tag{2}$$

where

$$q := e^{2\pi iz}, z \in H = \{x + iy : y > 0\} \tag{3}$$

and an eta quotient of level n is defined by

$$f(z) := \prod_{m|n} \eta(mz)^{a_m}, n, m \in \mathbb{N}, a_m \in \mathbb{Z}. \tag{4}$$

It is interesting and important to determine explicit formulas of the Fourier coefficients of eta quotients, because they are the building blocks of modular forms of level n and weight k . The book of Köhler [13] (Chapter 3, pg.39) describes such expansions by means of Hecke Theta series and develops algorithms for the determination of suitable

eta quotients. One can find more information in [3], [6], [14], [16], [17]. I have determined the Fourier coefficients of the theta series associated to some quadratic forms, see [7], [8], [9][10], [11] and [12].

Recently, Williams, see [18] discovered explicit formulas for the coefficients of Fourier series expansions of a class of 126 eta quotients in terms of $\sigma(n)$, $\sigma(\frac{n}{2})$, $\sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^2(2z)\eta^4(4z)\eta^6(6z)}{\eta^2(z)\eta^2(3z)\eta^4(12z)} = 1 + \sum_{n=1}^{\infty} c(n)q^n,$$

where

$$c(n) = 2\sigma(n) - 3\sigma(n/2) + 4\sigma(n/4) + 9\sigma(n/6) - 36\sigma(n/12).$$

Then Yao, Xia and Jin [15] expressed the even Fourier coefficients of 104 eta quotients in terms of $\sigma_3(n)$, $\sigma_3(\frac{n}{2})$, $\sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)} = 1 + \sum_{n=1}^{\infty} c(n)q^n,$$

where

$$c(2n) = 65\sigma_3(n) - 68\sigma_3(n/2) - 81\sigma_3(n/3) + 324\sigma_3(n/6).$$

After that we find that we can express the even Fourier coefficients of 570 eta quotients in terms of $\sigma_{19}(n)$, $\sigma_{19}(\frac{n}{2})$, $\sigma_{19}(\frac{n}{3})$, $\sigma_{19}(\frac{n}{4})$, $\sigma_{19}(\frac{n}{6})$ and $\sigma_{17}(\frac{n}{12})$, see Table 3. One example is as follows:

$$\frac{\eta^{36}(2z)\eta^{38}(12z)}{\eta^{18}(4z)\eta^{16}(6z)} = 1 + \sum_{n=1}^{\infty} c(n)q^n,$$

where

$$c(2n) = -\frac{1}{7971615}\sigma_{19}(2n) + \frac{174763}{23914845}\sigma_{19}(n) - \frac{524288}{71744535}\sigma_{19}(n/2).$$

We also see that the odd Fourier coefficients of 1208 eta quotients are zero and even coefficients can be expressed by simple formula. Now let

$$f_1 := \sum_{n=0}^{\infty} f_1(n)q^n = \frac{\eta^{43}(4z)\eta^{43}(6z)}{\eta^{29}(2z)\eta^{17}(12z)},$$

$$f_2 = \sum_{n=0}^{\infty} f_2(n)q^n = \frac{\eta^{38}(4z)\eta^{48}(6z)}{\eta^{28}(2z)\eta^{18}(12z)},$$

$$f_3 = \sum_{n=0}^{\infty} f_3(n)q^n = \frac{\eta^{50}(4z)\eta^{24}(6z)}{\eta^{28}(2z)\eta^6(12z)},$$

$$f_4 = \sum_{n=0}^{\infty} f_4(n)q^n = \frac{\eta^{50}(4z)\eta^{48}(6z)}{\eta^{28}(2z)\eta^{30}(12z)},$$

$$f_5 = \sum_{n=0}^{\infty} f_5(n)q^n = \frac{\eta^{45}(4z)\eta^{29}(6z)}{\eta^{27}(2z)\eta^7(12z)},$$

$$f_6 = \sum_{n=0}^{\infty} f_6(n) q^n = \frac{\eta^{40}(4z) \eta^{34}(6z)}{\eta^{26}(2z) \eta^8(12z)},$$

$$f_7 = \sum_{n=0}^{\infty} f_7(n) q^n = \frac{\eta^{44}(4z) \eta^{36}(12z)}{\eta^{22}(2z) \eta^{18}(6z)},$$

$$f_8 = \sum_{n=0}^{\infty} f_8(n) q^n = \frac{\eta^{47}(4z) \eta^{15}(6z) \eta^3(12z)}{\eta^{25}(2z)},$$

$$f_9 = \sum_{n=0}^{\infty} f_9(n) q^n = \frac{\eta^{47}(4z) \eta^{39}(6z)}{\eta^{25}(2z) \eta^{21}(12z)},$$

$$f_{10} = \sum_{n=0}^{\infty} f_{10}(n) q^n = \frac{\eta^{30}(4z) \eta^{44}(6z)}{\eta^{24}(2z) \eta^{10}(12z)},$$

$$f_{11} = \sum_{n=0}^{\infty} f_{11}(n) q^n = \frac{\eta^{44}(4z) \eta^6(6z) \eta^{12}(12z)}{\eta^{22}(2z)},$$

$$f_{12} = \sum_{n=0}^{\infty} f_{12}(n) q^n = \frac{\eta^{15}(2z) \eta^{35}(4z) \eta^{11}(12z)}{\eta^{21}(6z)},$$

$$f_{13} = \sum_{n=0}^{\infty} f_{13}(n) q^n = \frac{\eta^{25}(4z) \eta^{49}(6z)}{\eta^{23}(2z) \eta^{11}(12z)},$$

$$f_{14} = \sum_{n=0}^{\infty} f_{14}(n) q^n = \frac{\eta^{27}(4z) \eta^{11}(6z) \eta^{23}(12z)}{\eta^{21}(2z)},$$

$$f_{15} = \sum_{n=0}^{\infty} f_{15}(n) q^n = \frac{\eta^5(4z) \eta^{45}(6z) \eta^9(12z)}{\eta^{19}(2z)},$$

$$f_{16} = \sum_{n=0}^{\infty} f_{16}(n) q^n = \frac{\eta^{49}(4z) \eta(6z) \eta^{13}(12z)}{\eta^{23}(2z)},$$

$$f_{17} = \sum_{n=0}^{\infty} f_{17}(n) q^n = \frac{\eta^{43}(4z) \eta^{31}(12z)}{\eta^{17}(2z) \eta^{17}(6z)},$$

$$f_{18} = \sum_{n=0}^{\infty} f_{18}(n) q^n = \frac{\eta^{50}(6z) \eta^8(12z)}{\eta^{18}(2z)},$$

$$f_{19} = \sum_{n=0}^{\infty} f_{19}(n) q^n = \frac{\eta^{12}(4z) \eta^{20}(6z) \eta^{20}(12z)}{\eta^{12}(2z)},$$

$$f_{20} = \sum_{n=0}^{\infty} f_{20}(n) q^n = \frac{\eta^{16}(4z) \eta^{16}(6z) \eta^{16}(12z)}{\eta^8(2z)},$$

$$f_{21} = \sum_{n=0}^{\infty} f_{21}(n) q^n = \frac{\eta^{20}(4z) \eta^{12}(6z) \eta^{12}(12z)}{\eta^4(2z)},$$

$$f_{22} = \sum_{n=0}^{\infty} f_{22}(n) q^n = \frac{\eta^{15}(4z) \eta^{17}(6z) \eta^{11}(12z)}{\eta^3(2z)},$$

$$f_{23} = \sum_{n=0}^{\infty} f_{23}(n) q^n = \frac{\eta^{11}(2z) \eta^{17}(4z) \eta^{15}(6z)}{\eta^3(12z)},$$

$$f_{24} = \sum_{n=0}^{\infty} f_{24}(n) q^n = \eta(2z) \eta^7(4z) \eta^{13}(6z) \eta^{19}(12z),$$

$$f_{25} = \sum_{n=0}^{\infty} f_{25}(n) q^n = \eta(2z) \eta^{19}(4z) \eta^{13}(6z) \eta^7(12z),$$

$$f_{26} = \sum_{n=0}^{\infty} f_{26}(n) q^n = \eta^2(2z) \eta^2(4z) \eta^{18}(6z) \eta^{18}(12z),$$

$$f_{27} = \sum_{n=0}^{\infty} f_{27}(n) q^n = \frac{\eta^{12}(2z) \eta^{20}(6z) \eta^{20}(12z)}{\eta^{12}(4z)},$$

$$f_{28} = \sum_{n=0}^{\infty} f_{28}(n) q^n = \eta^4(2z) \eta^{16}(4z) \eta^4(6z) \eta^{16}(12z),$$

$$f_{29} = \sum_{n=0}^{\infty} f_{29}(n) q^n = \frac{\eta^{16}(2z) \eta^{16}(6z) \eta^{16}(12z)}{\eta^8(4z)},$$

$$f_{30} = \sum_{n=0}^{\infty} f_{30}(n) q^n = \frac{\eta^{16}(2z) \eta^{16}(4z) \eta^{16}(12z)}{\eta^8(6z)},$$

$$f_{31} = \sum_{n=0}^{\infty} f_{31}(n) q^n = \eta^6(2z) \eta^{18}(4z) \eta^{14}(6z) \eta^2(12z),$$

$$f_{32} = \sum_{n=0}^{\infty} f_{32}(n) q^n = \frac{\eta^{16}(2z) \eta^{16}(4z) \eta^{16}(6z)}{\eta^8(12z)},$$

$$f_{33} = \sum_{n=0}^{\infty} f_{33}(n) q^n = \frac{\eta^{20}(2z) \eta^{20}(4z) \eta^{12}(12z)}{\eta^{12}(6z)},$$

$$f_{34} = \sum_{n=0}^{\infty} f_{34}(n) q^n = \frac{\eta^{20}(2z) \eta^{20}(4z) \eta^{12}(6z)}{\eta^{12}(12z)},$$

$$f_{35} = \sum_{n=0}^{\infty} f_{35}(n) q^n = \frac{\eta^{30}(2z) \eta^{30}(4z)}{\eta^{10}(6z) \eta^{10}(12z)}.$$

The proof of the following Lemma about these coefficients is obvious.

Lemma 1.1 For $n = 1, 2, \dots$,

$$f_1(2n) = \dots = f_{18}(2n) = 0,$$

$$f_{19}(2n-1) = \dots = f_{35}(2n-1) = 0.$$

2. Main results

Now we can state our main Theorem:

Theorem 2.1 Let b_1, b_2, \dots, b_5 be non-negative integers satisfying

$$b_1 + b_2 + \dots + b_5 \leq 40. \tag{5}$$

Define the integers $a_1, a_2, a_3, a_4, a_6, a_{12}$ by

$$a_1 := -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 40, \tag{6}$$

$$a_2 := 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 100, \tag{7}$$

$$a_3 := 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 120, \tag{8}$$

$$a_4 := -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 40, \tag{9}$$

$$a_6 := -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 300, \tag{10}$$

$$a_{12} := 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 120. \tag{11}$$

Now define the rational numbers $\{k_i : i = 0, \dots, 40\}$ by

$$\frac{1}{2^{b_1+b_5}} x^{b_1} (1-x)^{b_2} (1+x)^{b_3} (1+2x)^{b_4} (2+x)^{b_5} = \sum_{i=0}^{40} k_i x^i. \tag{12}$$

Define the rational numbers

$$c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, \dots, r_{34}$$

and r_{35} as in [www.bariskendirli.com.tr/weight20/Table 1](http://www.bariskendirli.com.tr/weight20/Table%201). Here $\{f_1, \dots, f_{35}\} \setminus \{f_7, f_{12}, f_{14}, f_{15}, f_{17}, f_{18}, f_{35}\} \in S_{20}(\Gamma_0(12))$, $f_7, f_{12}, f_{14}, f_{15}, f_{17}, f_{18}, f_{35} \in M_{20}(\Gamma_0(12)) \setminus S_{20}(\Gamma_0(12))$ and

$$\eta^{a_1}(z) \eta^{a_2}(2z) \eta^{a_3}(3z) \eta^{a_4}(4z) \eta^{a_6}(6z) \eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n)q^n,$$

where for $n \in \mathbb{N}$,

$$c(n) = c_1 \sigma_{19}(n) + c_2 \sigma_{19}\left(\frac{n}{2}\right) + c_3 \sigma_{19}\left(\frac{n}{3}\right) + c_4 \sigma_{19}\left(\frac{n}{4}\right) + c_6 \sigma_{19}\left(\frac{n}{6}\right) + c_{12} \sigma_{19}\left(\frac{n}{12}\right) + r_1 f_1(n) + \dots + r_{35} f_{35}(n).$$

In particular,

$$c(2n) = c_1 \sigma_{19}(2n) + c_2 \sigma_{19}(n) + c_4 \sigma_{19}\left(\frac{n}{2}\right) + (1048577c_3 + c_6) \sigma_{19}\left(\frac{n}{3}\right) + (c_{12} - 1048576c_3) \sigma_{19}\left(\frac{n}{6}\right) + r_1 f_1(2n) + \dots + r_{35} f_{35}(2n),$$

$$c(2n-1) = c_1 \sigma_{19}(2n-1) + c_3 \sigma_{19}\left(\frac{2n-1}{3}\right) + r_{19} f_{19}(2n-1) + \dots + r_{35} f_{35}(2n-1),$$

for $n \in \mathbb{N}$.

Proof: It follows from (6)-(11) that

$$a_1 + 2a_2 + 3a_3 + 4a_4 + 6a_6 + 12a_{12} = 24b_1, \tag{13}$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 40, \tag{14}$$

$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - \frac{2a_4}{3} - \frac{a_6}{3} - \frac{2a_{12}}{3} = -b_1 - b_5.$$

Now we will use (p, k) parametrization of Alaca, Alaca and Williams, see [1]:

$$p(q) := \frac{\varphi^2(q) - \varphi^2(q^3)}{2\varphi^2(q^3)}, k(q) := \frac{\varphi^3(q^3)}{\varphi(q)}, \quad (15)$$

where the theta function $\varphi(q)$ is defined by

$$\varphi(q) = \sum_{n=-\infty}^{\infty} q^{n^2}.$$

Setting $x = p$ in (12), and multiplying both sides by k^{20} , we obtain

$$\begin{aligned} & \frac{k^{20}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5} \\ &= \left(\sum_{i=0}^{40} k_i p^i \right) k^{20}. \end{aligned}$$

Alaca, Alaca and Williams [2] have established the following representations in terms of p and k :

$$\eta(q) = 2^{-1/6} p^{1/24} (1-p)^{1/2} (1+p)^{1/6} (1+2p)^{1/8} (2+p)^{1/8} k^{1/2}, \quad (16)$$

$$\eta(q^2) = 2^{-1/3} p^{1/12} (1-p)^{1/4} (1+p)^{1/12} (1+2p)^{1/4} (2+p)^{1/4} k^{1/2}, \quad (17)$$

$$\eta(q^3) = 2^{-1/6} p^{1/8} (1-p)^{1/6} (1+p)^{1/2} (1+2p)^{1/24} (2+p)^{1/24} k^{1/2}, \quad (18)$$

$$\eta(q^4) = 2^{-2/3} p^{1/6} (1-p)^{1/8} (1+p)^{1/24} (1+2p)^{1/8} (2+p)^{1/2} k^{1/2}, \quad (19)$$

$$\eta(q^6) = 2^{-1/3} p^{1/4} (1-p)^{1/12} (1+p)^{1/4} (1+2p)^{1/12} (2+p)^{1/12} k^{1/2}, \quad (20)$$

$$\eta(q^{12}) = 2^{-2/3} p^{1/2} (1-p)^{1/24} (1+p)^{1/8} (1+2p)^{1/24} (2+p)^{1/6} k^{1/2}, \quad (21)$$

$$\begin{aligned} E_6(q) &:= 1 - 504 \sum_{n=1}^{\infty} \sigma_5(n) q^n \\ &= (1 - 246p - 5532p^2 - 38614p^3 - 135369p^4 - 276084p^5 \\ &\quad - 348024p^6 - 276084p^7 - 135369p^8 - 38614p^9 - 5532p^{10} \\ &\quad - 246p^{11} + p^{12}) k^6, \end{aligned}$$

$$\begin{aligned} E_4(q) &:= 1 + 240 \sum_{n=1}^{\infty} \sigma_3(n) q^n \\ &= (1 + 124p + 964p^2 + 2788p^3 + 3910p^4 + 2788p^5 \\ &\quad + 964p^6 + 124p^7 + p^8) k^4. \end{aligned}$$

Therefore, since

$$E_{20}(q) = \frac{121250}{174611} E_6^2(q) E_4^2(q) + \frac{53361}{174611} E_4^5(q),$$

we have

$$\begin{aligned} E_{20}(q) = & (p^{40} + \frac{3498\ 820}{174\ 611} p^{39} + \frac{1761\ 712\ 380}{174\ 611} p^{38} + \frac{1950\ 771\ 349\ 900}{174\ 611} p^{37} \\ & + \frac{262\ 533\ 149\ 598\ 050}{174\ 611} p^{36} + \frac{12\ 244\ 560\ 329\ 615\ 724}{174\ 611} p^{35} \\ & + \frac{297\ 550\ 209\ 630\ 645\ 660}{174\ 611} p^{34} + \frac{4540\ 576\ 615\ 265\ 413\ 860}{174\ 611} p^{33} \\ & + \frac{48\ 205\ 011\ 029\ 576\ 752\ 215}{174\ 611} p^{32} + \frac{379\ 279\ 886\ 962\ 492\ 426\ 000}{174\ 611} p^{31} \\ & + \frac{2306\ 867\ 772\ 376\ 889\ 866\ 544}{174\ 611} p^{30} + \frac{11\ 172\ 752\ 277\ 687\ 684\ 748\ 080}{174\ 611} p^{29} \\ & + \frac{44\ 032\ 939\ 445\ 831\ 353\ 072\ 280}{174\ 611} p^{28} + \frac{143\ 523\ 588\ 642\ 250\ 460\ 471\ 920}{174\ 611} p^{27} \\ & + \frac{391\ 710\ 935\ 624\ 818\ 929\ 715\ 440}{174\ 611} p^{26} + \frac{903\ 703\ 297\ 519\ 950\ 364\ 759\ 248}{174\ 611} p^{25} \\ & + \frac{1775\ 257\ 667\ 109\ 763\ 890\ 078\ 870}{174\ 611} p^{24} + \frac{2985\ 782\ 926\ 019\ 838\ 802\ 293\ 240}{174\ 611} p^{23} \\ & + \frac{4316\ 791\ 070\ 784\ 960\ 278\ 738\ 760}{174\ 611} p^{22} + \frac{5379\ 734\ 920\ 300\ 102\ 006\ 356\ 840}{174\ 611} p^{21} \\ & + \frac{5788\ 306\ 667\ 816\ 968\ 534\ 061\ 580}{174\ 611} p^{20} + \frac{5379\ 734\ 920\ 300\ 102\ 006\ 356\ 840}{174\ 611} p^{19} \\ & + \frac{4316\ 791\ 070\ 784\ 960\ 278\ 738\ 760}{174\ 611} p^{18} + \frac{2985\ 782\ 926\ 019\ 838\ 802\ 293\ 240}{174\ 611} p^{17} \\ & + \frac{1775\ 257\ 667\ 109\ 763\ 890\ 078\ 870}{174\ 611} p^{16} + \frac{903\ 703\ 297\ 519\ 950\ 364\ 759\ 248}{174\ 611} p^{15} \\ & + \frac{391\ 710\ 935\ 624\ 818\ 929\ 715\ 440}{174\ 611} p^{14} + \frac{143\ 523\ 588\ 642\ 250\ 460\ 471\ 920}{174\ 611} p^{13} \\ & + \frac{44\ 032\ 939\ 445\ 831\ 353\ 072\ 280}{174\ 611} p^{12} + \frac{11\ 172\ 752\ 277\ 687\ 684\ 748\ 080}{174\ 611} p^{11} \\ & + \frac{2306\ 867\ 772\ 376\ 889\ 866\ 544}{174\ 611} p^{10} + \frac{379\ 279\ 886\ 962\ 492\ 426\ 000}{174\ 611} p^9 \\ & + \frac{48\ 205\ 011\ 029\ 576\ 752\ 215}{174\ 611} p^8 + \frac{4540\ 576\ 615\ 265\ 413\ 860}{174\ 611} p^7 \\ & + \frac{297\ 550\ 209\ 630\ 645\ 660}{174\ 611} p^6 + \frac{12\ 244\ 560\ 329\ 615\ 724}{174\ 611} p^5 + \frac{262\ 533\ 149\ 598\ 050}{174\ 611} \\ & p^4 + \frac{1950\ 771\ 349\ 900}{174\ 611} p^3 + \frac{1761\ 712\ 380}{174\ 611} p^2 + \frac{3498\ 820}{174\ 611} p + 1) k^{20}, \end{aligned}$$

$$\begin{aligned}
E_{20}(q^2) = & (p^{40} + 20p^{39} + \frac{31\,433\,280}{174\,611}p^{38} + \frac{165\,943\,150}{174\,611}p^{37} + \frac{987\,462\,125}{174\,611}p^{36} \\
& + \frac{8929\,248\,174}{174\,611}p^{35} + \frac{303\,281\,329\,710}{174\,611}p^{34} + \frac{4364\,341\,784\,610}{174\,611}p^{33} \\
& + \frac{91\,493\,235\,452\,655}{349\,222}p^{32} + \frac{361\,444\,851\,257\,800}{174\,611}p^{31} + \frac{2201\,013\,342\,701\,144}{174\,611}p^{30} \\
& + \frac{10\,656\,464\,415\,766\,680}{174\,611}p^{29} + \frac{41\,990\,748\,446\,379\,980}{174\,611}p^{28} \\
& + \frac{136\,870\,946\,815\,364\,920}{174\,611}p^{27} + \frac{373\,567\,300\,252\,984\,440}{174\,611}p^{26} \\
& + \frac{861\,845\,414\,133\,190\,248}{174\,611}p^{25} + \frac{1693\,017\,415\,776\,001\,995}{174\,611}p^{24} \\
& + \frac{2847\,457\,351\,673\,446\,140}{174\,611}p^{23} + \frac{4116\,809\,048\,699\,351\,460}{174\,611}p^{22} \\
& + \frac{5130\,518\,121\,215\,151\,540}{174\,611}p^{21} + \frac{5520\,165\,078\,218\,375\,430}{174\,611}p^{20} \\
& + \frac{5130\,518\,121\,215\,151\,540}{174\,611}p^{19} + \frac{4116\,809\,048\,699\,351\,460}{174\,611}p^{18} \\
& + \frac{2847\,457\,351\,673\,446\,140}{174\,611}p^{17} + \frac{1693\,017\,415\,776\,001\,995}{174\,611}p^{16} \\
& + \frac{861\,845\,414\,133\,190\,248}{174\,611}p^{15} + \frac{373\,567\,300\,252\,984\,440}{174\,611}p^{14} \\
& + \frac{136\,870\,946\,815\,364\,920}{174\,611}p^{13} + \frac{41\,990\,748\,446\,379\,980}{174\,611}p^{12} \\
& + \frac{10\,656\,464\,415\,766\,680}{174\,611}p^{11} + \frac{2201\,013\,342\,701\,144}{174\,611}p^{10} \\
& + \frac{361\,444\,851\,257\,800}{174\,611}p^9 + \frac{91\,493\,235\,452\,655}{349\,222}p^8 \\
& + \frac{4364\,341\,784\,610}{174\,611}p^7 + \frac{303\,281\,329\,710}{174\,611}p^6 + \frac{8929\,248\,174}{174\,611}p^5 \\
& + \frac{987\,462\,125}{174\,611}p^4 + \frac{165\,943\,150}{174\,611}p^3 + \frac{31\,433\,280}{174\,611}p^2 + 20p + 1)k^{20},
\end{aligned}$$

$$\begin{aligned}
E_{20}(q^3) = & (p^{40} + 20p^{39} + 180p^{38} + \frac{165\,882\,100}{174\,611}p^{37} + \frac{554\,420\,450}{174\,611}p^{36} \\
& + \frac{1141\,160\,724}{174\,611}p^{35} + \frac{1164\,216\,060}{174\,611}p^{34} + \frac{539\,536\,860}{174\,611}p^{33} \\
& + \frac{8009\,545\,815}{174\,611}p^{32} + \frac{82\,124\,642\,800}{174\,611}p^{31} + \frac{622\,305\,005\,744}{174\,611}p^{30} \\
& + \frac{3416\,310\,121\,680}{174\,611}p^{29} + \frac{13\,238\,543\,613\,080}{174\,611}p^{28} + \frac{40\,264\,423\,711\,120}{174\,611}p^{27} \\
& + \frac{108\,624\,031\,190\,640}{174\,611}p^{26} + \frac{261\,168\,091\,468\,848}{174\,611}p^{25} + \frac{521\,667\,517\,163\,670}{174\,611}p^{24} \\
& + \frac{854\,113\,070\,472\,840}{174\,611}p^{23} + \frac{1212\,499\,060\,327\,560}{174\,611}p^{22} + \frac{1543\,771\,577\,414\,040}{174\,611}p^{21} \\
& + \frac{1692\,495\,554\,535\,180}{174\,611}p^{20} + \frac{1543\,771\,577\,414\,040}{174\,611}p^{19} \\
& + \frac{1212\,499\,060\,327\,560}{174\,611}p^{18} + \frac{854\,113\,070\,472\,840}{174\,611}p^{17} + \frac{521\,667\,517\,163\,670}{174\,611}p^{16} \\
& + \frac{261\,168\,091\,468\,848}{174\,611}p^{15} + \frac{108\,624\,031\,190\,640}{174\,611}p^{14} + \frac{40\,264\,423\,711\,120}{174\,611}p^{13} \\
& + \frac{13\,238\,543\,613\,080}{174\,611}p^{12} + \frac{3416\,310\,121\,680}{174\,611}p^{11} + \frac{622\,305\,005\,744}{174\,611}p^{10} \\
& + \frac{82\,124\,642\,800}{174\,611}p^9 + \frac{8009\,545\,815}{174\,611}p^8 + \frac{539\,536\,860}{174\,611}p^7 + \frac{1164\,216\,060}{174\,611}p^6 \\
& + \frac{1141\,160\,724}{174\,611}p^5 + \frac{554\,420\,450}{174\,611}p^4 + \frac{165\,882\,100}{174\,611}p^3 + 180p^2 + 20p + 1)k^{20},
\end{aligned}$$

$$\begin{aligned}
 E_{20}(q^4) = & \left(\frac{1}{1048576}p^{40} + \frac{871405}{45773225984}p^{39} + \frac{440363745}{45773225984}p^{38} \right. \\
 & - \frac{117793359275}{11443306496}p^{37} + \frac{23947134429775}{22886612992}p^{36} \\
 & - \frac{254868233968161}{11443306496}p^{35} + \frac{302463836011515}{2860826624}p^{34} \\
 & + \frac{1160663370518955}{5721653248}p^{33} - \frac{28749261448742445}{22886612992}p^{32} \\
 & - \frac{2100550371547975}{1430413312}p^{31} + \frac{8316909253331623}{1430413312}p^{30} \\
 & + \frac{2649051448593615}{357603328}p^{29} - \frac{9448705577013595}{715206656}p^{28} \\
 & - \frac{3737818023814145}{178801664}p^{27} + \frac{195006967788285}{11175104}p^{26} \\
 & + \frac{3816495781888401}{89400832}p^{25} + \frac{3875892669469395}{715206656}p^{24} \\
 & - \frac{4402258116918165}{178801664}p^{23} + \frac{1872902744061915}{178801664}p^{22} \\
 & + \frac{2346702805920615}{44700416}p^{21} + \frac{4147844506625385}{89400832}p^{20} \\
 & + \frac{951687682156815}{44700416}p^{19} + \frac{147096539206365}{11175104}p^{18} \\
 & + \frac{331640077115295}{22350208}p^{17} + \frac{1052959610032065}{89400832}p^{16} \\
 & + \frac{62680520061747}{11175104}p^{15} + \frac{19650005782785}{11175104}p^{14} \\
 & + \frac{1564837152745}{2793776}p^{13} + \frac{1661596819735}{5587552}p^{12} \\
 & + \frac{24368406705}{174611}p^{11} + \frac{3494456413}{349222}p^{10} - \frac{7531426175}{174611}p^9 \\
 & - \frac{93384029385}{2793776}p^8 - \frac{5207303235}{698444}p^7 + \frac{4219951665}{698444}p^6 \\
 & + \frac{1140923124}{174611}p^5 + \frac{554390750}{174611}p^4 + 950p^3 + 180p^2 + 20p + 1)k^{20},
 \end{aligned}$$

$$\begin{aligned}
 E_{20}(q^6) = & (p^{40} + 20p^{39} + 180p^{38} + 950p^{37} + 3175p^{36} + 6534p^{35} \\
 & + \frac{1054868910}{174611}p^{34} - \frac{1302376290}{174611}p^{33} - \frac{11730204945}{349222}p^{32} \\
 & - \frac{7966512200}{174611}p^{31} - \frac{1308580456}{174611}p^{30} + \frac{12073490280}{174611}p^{29} \\
 & + \frac{18399531980}{174611}p^{28} + \frac{6936773320}{174611}p^{27} - \frac{12990377160}{174611}p^{26} \\
 & - \frac{20113138152}{174611}p^{25} - \frac{7670974005}{174611}p^{24} + \frac{8584483140}{174611}p^{23} \\
 & + \frac{11628731460}{174611}p^{22} + \frac{3054606540}{174611}p^{21} - \frac{2512795770}{174611}p^{20} \\
 & + \frac{3054606540}{174611}p^{19} + \frac{11628731460}{174611}p^{18} + \frac{8584483140}{174611}p^{17} \\
 & - \frac{7670974005}{174611}p^{16} - \frac{20113138152}{174611}p^{15} - \frac{12990377160}{174611}p^{14} \\
 & + \frac{6936773320}{174611}p^{13} + \frac{18399531980}{174611}p^{12} + \frac{12073490280}{174611}p^{11} \\
 & - \frac{1308580456}{174611}p^{10} - \frac{7966512200}{174611}p^9 - \frac{11730204945}{349222}p^8 \\
 & - \frac{1302376290}{174611}p^7 + \frac{1054868910}{174611}p^6 + 6534p^5 + 3175p^4 + 950p^3 \\
 & + 180p^2 + 20p + 1)k^{20},
 \end{aligned}$$

$$\begin{aligned}
E_{20}(q^{12}) = & \left(\frac{1}{1048576}p^{40} + \frac{5}{262144}p^{39} + \frac{45}{262144}p^{38} + \frac{10367425}{11443306496}p^{37} \right. \\
& + \frac{69294925}{22886612992}p^{36} + \frac{71290989}{11443306496}p^{35} + \frac{36305955}{5721653248}p^{34} \\
& + \frac{16633755}{5721653248}p^{33} + \frac{999590355}{22886612992}p^{32} + \frac{173342075}{1430413312}p^{31} \\
& - \frac{2395113227}{1430413312}p^{30} - \frac{5705331435}{357603328}p^{29} - \frac{44179109845}{715206656}p^{28} \\
& - \frac{16087374695}{178801664}p^{27} + \frac{19855618155}{89400832}p^{26} + \frac{124711192701}{89400832}p^{25} \\
& + \frac{1987639824195}{715206656}p^{24} + \frac{99698108985}{178801664}p^{23} - \frac{1697890834635}{178801664}p^{22} \\
& - \frac{935935221585}{44700416}p^{21} - \frac{1056483866565}{89400832}p^{20} + \frac{1341018629565}{44700416}p^{19} \\
& + \frac{1552706681055}{22350208}p^{18} + \frac{982091824695}{22350208}p^{17} - \frac{4430721627135}{89400832}p^{16} \\
& - \frac{1318772631303}{11175104}p^{15} - \frac{840497560965}{11175104}p^{14} + \frac{110625563695}{2793776}p^{13} \\
& + \frac{588727287385}{5587552}p^{12} + \frac{138285}{2}p^{11} - 7496p^{10} - 45625p^9 - \frac{537435}{16}p^8 \\
& - \frac{29835}{4}p^7 + \frac{24165}{4}p^6 + 6534p^5 + 3175p^4 + 950p^3 + 180p^2 + 20p + 1)k^{20}.
\end{aligned}$$

We can also similarly determine f_1, \dots, f_{30} and f_{31} in terms of p and k as in [www.bariskendirli.com.tr/weight20/Table 2](http://www.bariskendirli.com.tr/weight20/Table2). Obviously, f_1, \dots, f_{35} are functions of q , see (3), (15). We see that $\{f_1, \dots, f_{35}\} \setminus \{f_7, f_{12}, f_{14}, f_{15}, f_{17}, f_{18}, f_{35}\} \in S_{20}(\Gamma_0(12))$, $f_7, f_{12}, f_{14}, f_{15}, f_{17}, f_{18}, f_{35} \in M_{20}(\Gamma_0(12)) \setminus S_{20}(\Gamma_0(12))$ by [4]. Now

$$\begin{aligned}
& \eta^{a_1}(z) \eta^{a_2}(2z) \eta^{a_3}(3z) \eta^{a_4}(4z) \eta^{a_6}(6z) \eta^{a_{12}}(12z) \\
= & q^{b_1} \prod_{n=1}^{\infty} (1-q^n)^{a_1} (1-q^{2n})^{a_2} (1-q^{3n})^{a_3} (1-q^{4n})^{a_4} (1-q^{6n})^{a_6} (1-q^{12n})^{a_{12}} \\
= & 2^{-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - \frac{2a_4}{3} - \frac{a_6}{3} - \frac{2a_{12}}{3}} p^{\frac{a_1}{24} + \frac{a_2}{12} + \frac{a_3}{8} + \frac{a_4}{6} + \frac{a_6}{4} + \frac{a_{12}}{2}} (1-p)^{\frac{a_1}{2} + \frac{a_2}{4} + \frac{a_3}{6} + \frac{a_4}{8} + \frac{a_6}{12} + \frac{a_{12}}{24}} \\
& (1+p)^{\frac{a_1}{6} + \frac{a_2}{12} + \frac{a_3}{2} + \frac{a_4}{24} + \frac{a_6}{4} + \frac{a_{12}}{8}} (1+2p)^{\frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{24} + \frac{a_4}{8} + \frac{a_6}{12} + \frac{a_{12}}{24}} (2+p)^{\frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{24} + \frac{a_4}{2} + \frac{a_6}{12} + \frac{a_{12}}{6}} \\
& k^{\frac{a_1+a_2+a_3+a_4+a_6+a_{12}}{2}} = \frac{k^{20}}{2^{b_1+b_5}} p^{b_1} (1-p)^{b_2} (1+p)^{b_3} (1+2p)^{b_4} (2+p)^{b_5}
\end{aligned}$$

$$= \left(\sum_{i=0}^{40} k_i p^i \right) k^{20}$$

$$\begin{aligned}
&= \frac{174611c_1}{13200} \left(1 + \frac{13200}{174611} \sum_{n=1}^{\infty} \sigma_{19}(n) q^n \right) + \frac{174611c_2}{13200} \left(1 + \frac{13200}{174611} \sum_{n=1}^{\infty} \sigma_{19}(n) q^{2n} \right) \\
&+ \frac{174611c_3}{13200} \left(1 + \frac{13200}{174611} \sum_{n=1}^{\infty} \sigma_{19}(n) q^{3n} \right) + \frac{174611c_4}{13200} \left(1 + \frac{13200}{174611} \sum_{n=1}^{\infty} \sigma_{19}(n) q^{4n} \right) \\
&+ \frac{174611c_6}{13200} \left(1 + \frac{13200}{174611} \sum_{n=1}^{\infty} \sigma_{19}(n) q^{6n} \right) + \frac{174611c_{12}}{13200} \left(1 + \frac{13200}{174611} \sum_{n=1}^{\infty} \sigma_{19}(n) q^{12n} \right) \\
&+ r_1 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{43} (1-q^{6n})^{43}}{(1-q^{2n})^{29} (1-q^{12n})^{17}} \\
&+ r_2 q^7 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{38} (1-q^{6n})^{48}}{(1-q^{2n})^{28} (1-q^{12n})^{18}} \\
&+ r_3 q^9 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{50} (1-q^{6n})^{24}}{(1-q^{2n})^{28} (1-q^{12n})^6} \\
&+ r_4 q^3 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{50} (1-q^{6n})^{48}}{(1-q^{2n})^{28} (1-q^{12n})^{30}} \\
&+ r_5 q^9 \prod_{n=1}^{\infty} \frac{((1-q^{4n})^{45} (1-q^{6n})^{29})}{(1-q^{2n})^{27} (1-q^{12n})^7} \\
&+ r_6 q^9 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{40} (1-q^{6n})^{34}}{(1-q^{2n})^{26} (1-q^{12n})^8} \\
&+ r_7 q^{19} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{44} (1-q^{12n})^{36}}{(1-q^{2n})^{22} (1-q^{6n})^{18}} \\
&+ r_8 q^{19} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{47} (1-q^{6n})^{15} (1-q^{12n})^3}{(1-q^{2n})^{25}} \\
&+ r_9 q^5 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{47} (1-q^{6n})^{39}}{(1-q^{2n})^{25} (1-q^{12n})^{21}} \\
&+ r_{10} q^9 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{30} (1-q^{6n})^{44}}{(1-q^{2n})^{24} (1-q^{12n})^{10}}
\end{aligned}$$

$$\begin{aligned}
& +r_{11}q^{13} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{44} (1-q^{6n})^6 (1-q^{12n})^{12}}{(1-q^{2n})^{22}} \\
& +r_{12}q^{15} \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{15} (1-q^{4n})^{35} (1-q^{12n})^{11}}{(1-q^{6n})^{21}} \\
& +r_{13}q^9 \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{25} (1-q^{6n})^{49}}{(1-q^{2n})^{23} (1-q^{12n})^{11}} \\
& +r_{14}q^{17} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{27} (1-q^{6n})^{11} (1-q^{12n})^{23}}{(1-q^{2n})^{21}} \\
& +r_{15}q^{15} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^5 (1-q^{6n})^{45} (1-q^{12n})^9}{(1-q^{2n})^{19}} \\
& +r_{16}q^{13} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{49} (1-q^{6n}) (1-q^{12n})^{13}}{(1-q^{2n})^{23}} \\
& +r_{17}q^{17} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{43} (1-q^{12n})^{31}}{(1-q^{2n})^{17} (1-q^{6n})^{17}} \\
& +r_{18}q^{15} \prod_{n=1}^{\infty} \frac{(1-q^{6n})^{50} (1-q^{12n})^8}{(1-q^{2n})^{18}} \\
& +r_{19}q^{16} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{12} (1-q^{6n})^{20} (1-q^{12n})^{20}}{(1-q^{2n})^{12}} \\
& +r_{20}q^{14} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{16} (1-q^{6n})^{16} (1-q^{12n})^{16}}{(1-q^{2n})^8} \\
& +r_{21}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{20} (1-q^{6n})^{12} (1-q^{12n})^{12}}{(1-q^{2n})^4} \\
& +r_{22}q^{12} \prod_{n=1}^{\infty} \frac{(1-q^{4n})^{15} (1-q^{6n})^{17} (1-q^{12n})^{11}}{(1-q^{2n})^3} \\
& +r_{23}q^6 \prod_{n=1}^{\infty} \frac{(1-q^{2n})^{11} (1-q^{4n})^{17} (1-q^{6n})^{15}}{(1-q^{12n})^3} \\
& +r_{24}q^{14} \prod_{n=1}^{\infty} (1-q^{2n}) (1-q^{4n})^7 (1-q^{6n})^{13} (1-q^{12n})^{19} \\
& +r_{25}q^{10} \prod_{n=1}^{\infty} (1-q^{2n}) (1-q^{4n})^{19} (1-q^{6n})^{13} (1-q^{12n})^{17}
\end{aligned}$$

$$\begin{aligned}
 &+r_{26}q^{14} \prod_{n=1}^{\infty} (1 - q^{2n})^2 (1 - q^{4n})^2 (1 - q^{6n})^{18} (1 - q^{12n})^{18} \\
 &+r_{27}q^{14} \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^{12} (1 - q^{6n})^{20} (1 - q^{12n})^{20}}{(1 - q^{4n})^{12}} \\
 &+r_{28}q^{12} \prod_{n=1}^{\infty} (1 - q^{2n})^4 (1 - q^{4n})^{16} (1 - q^{6n})^4 (1 - q^{12n})^{16} \\
 &+r_{29}q^{12} \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^{16} (1 - q^{6n})^{16} (1 - q^{12n})^{16}}{(1 - q^{4n})^8} \\
 &+r_{30}q^{10} \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^{16} (1 - q^{4n})^{16} (1 - q^{12n})^{16}}{(1 - q^{6n})^8} \\
 &+r_{31}q^8 \prod_{n=1}^{\infty} (1 - q^{2n})^6 (1 - q^{4n})^{18} (1 - q^{6n})^{14} (1 - q^{12n})^2 \\
 &+r_{32}q^4 \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^{16} (1 - q^{4n})^{16} (1 - q^{6n})^{16}}{(1 - q^{12n})^8} \\
 &+r_{33}q^8 \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^{20} (1 - q^{4n})^{20} (1 - q^{12n})^{12}}{(1 - q^{6n})^{12}} \\
 &+r_{34}q^2 \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^{20} (1 - q^{4n})^{20} (1 - q^{6n})^{12}}{(1 - q^{12n})^{12}} \\
 &+r_{35} \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^{30} (1 - q^{4n})^{30}}{(1 - q^{6n})^{10} (1 - q^{12n})^{10}} \\
 &= \delta(b_1) + \sum_{n=1}^{\infty} (c_1\sigma_{19}(n) + c_2\sigma_{19}\left(\frac{n}{2}\right) + c_3\sigma_{19}\left(\frac{n}{3}\right) + c_4\sigma_{19}\left(\frac{n}{4}\right) \\
 &\quad + c_6\sigma_{19}\left(\frac{n}{6}\right) + c_{12}\sigma_{19}\left(\frac{n}{12}\right)) + r_1f_1(n) + \dots + r_{35}f_{35}(n),
 \end{aligned}$$

where

$$\delta(b_1) = \begin{cases} 0 & \text{if } b_1 \neq 0 \\ 1 & \text{if } b_1 = 0 \end{cases} .$$

So

$$\begin{aligned}
 c(n) &= (c_1\sigma_{19}(n) + c_2\sigma_{19}\left(\frac{n}{2}\right) + c_3\sigma_{19}\left(\frac{n}{3}\right) + c_4\sigma_{19}\left(\frac{n}{4}\right) \\
 &\quad + c_6\sigma_{19}\left(\frac{n}{6}\right) + c_{12}\sigma_{19}\left(\frac{n}{12}\right)) + r_1f_1(n) + \dots + r_{35}f_{35}(n).
 \end{aligned}$$

Therefore, for $n=1,2,\dots$,

$$\begin{aligned}
 c(2n) &= c_1\sigma_{19}(2n) + c_2\sigma_{19}(n) + c_4\sigma_{19}\left(\frac{n}{2}\right) + (1048\,577c_3 + c_6)\sigma_{19}\left(\frac{n}{3}\right) \\
 &\quad + (c_{12} - 1048\,576c_3)\sigma_{19}\left(\frac{n}{6}\right) + r_{19}f_{19}(2n) + \dots + r_{35}f_{35}(2n),
 \end{aligned}$$

$$\begin{aligned}
 c(2n - 1) &= c_1\sigma_{19}(2n - 1) + c_3\sigma_{19}\left(\frac{2n - 1}{3}\right) \\
 &\quad + r_1f_1(2n - 1) + \dots + r_{18}f_{18}(2n - 1),
 \end{aligned}$$

since it is easy to see that

$$\sigma_k \left(\frac{2n}{3} \right) = (2^k + 1) \sigma_k \left(\frac{n}{3} \right) - 2^k \sigma_k \left(\frac{n}{6} \right)$$

hence,

$$\sigma_{19} \left(\frac{2n}{3} \right) = 1048577 \sigma_{19} \left(\frac{n}{3} \right) - 1048576 \sigma_{19} \left(\frac{n}{6} \right)$$

and use the Lemma before the Theorem.

3. Conclusion

We have found 570 eta quotients, see Table 4, such that, for $n = 1, 2, \dots$,

$$\begin{aligned} c(2n) &= c_1 \sigma_{19}(2n) + c_2 \sigma_{19}(n) + c_4 \sigma_{19} \left(\frac{n}{2} \right) + (1048577c_3 + c_6) \sigma_{19} \left(\frac{n}{3} \right) \\ &\quad + (c_{12} - 1048576c_3) \sigma_{19} \left(\frac{n}{6} \right) \\ c(2n-1) &= c_1 \sigma_{19}(2n-1) + c_3 \sigma_{19} \left(\frac{2n-1}{3} \right) + r_1 f_1(2n-1) + \dots + r_{18} f_{18}(2n-1). \end{aligned}$$

and 1208 eta quotients, such that for $n = 1, 2, \dots$,

$$\begin{aligned} c(2n) &= c_1 \sigma_{19}(2n) + c_2 \sigma_{19}(n) + c_4 \sigma_{19} \left(\frac{n}{2} \right) + c_6 \sigma_{19} \left(\frac{n}{3} \right) \\ &\quad + c_{12} \sigma_{19} \left(\frac{n}{6} \right) + r_{19} f_{19}(2n) + \dots + r_{35} f_{35}(2n), \\ c(2n-1) &= 0. \end{aligned}$$

Moreover, if f is an eta quotient, then the coefficients of $\frac{1}{2}(f(q) + f(-q))$ are exactly the even coefficients of f . In particular, it means that we have obtained all coefficients of some sum of 570 eta quotients.

Table 4

THE ETA QUOTIENTS WHOSE EVEN COEFFICIENTS CAN BE EXPLICITLY CALCULATED

t	HAM	HPM	a_{12}	a_6	a_4	a_2	C_1	C_2	C_3	C_4	C_6	C_{12}
1	-41	31	-53	103	31	-41	1	174 763	0	524 288	0	0
2	-40	26	-54	108	26	-40	38 127 987 424 935	-12 709 329 141 645	0	38 127 987 424 935	0	0
3	-39	21	-55	113	21	-39	38 127 987 424 935	-12 709 329 141 645	0	38 127 987 424 935	0	0
4	-33	27	-49	95	27	-33	114 383 962 274 805	38 127 987 424 935	19 683	114 383 962 274 805	174 763	524 288
5	-32	22	-50	100	22	-32	4236 443 047 215	-1412 147 682 405	0	4236 443 047 215	0	0
6	-31	17	-51	105	17	-31	7625 597 484 987	1922 333	2187	7625 597 484 987	-174 763	524 288
7	-25	23	-45	87	23	-25	4236 443 047 215	-1412 147 682 405	0	4236 443 047 215	0	0
8	-24	18	-46	92	18	-24	7625 597 484 987	1922 333	0	7625 597 484 987	0	0
9	-23	13	-47	97	13	-23	1114 160 034 927 448 993	1114 160 034 927 448 993	1	1114 160 034 927 448 993	174 763	524 288
10	-17	19	-41	79	19	-17	52 301 766 015	52 301 766 015	243	52 301 766 015	81	243
11	-16	14	-42	84	14	-16	17 433 922 005	-17 433 922 005	0	17 433 922 005	0	0
12	-15	9	-43	89	9	-15	1114 160 034 927 448 993	1114 160 034 927 448 993	1	1114 160 034 927 448 993	-174 763	524 288
13	-9	15	-37	71	15	-9	470 715 894 135	470 715 894 135	27	470 715 894 135	9	27
14	-8	10	-38	76	10	-8	1937 102 445	-1937 102 445	0	1937 102 445	0	0
15	-7	5	-39	81	5	-7	52 301 766 015	52 301 766 015	3	52 301 766 015	-174 763	524 288
16	-1	11	-33	63	11	-1	645 790 815	-645 790 815	0	645 790 815	0	0
17	0	6	-34	68	6	0	19 125 788 095 241	-19 125 788 095 241	0	19 125 788 095 241	0	0
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21	9	-3	-31	65	-3	9	129 140 163	-129 140 163	27	129 140 163	-14 155 803	14 155 776
22	15	3	-25	47	3	15	797 1615	-797 1615	0	797 1615	0	0
23	16	-2	-26	52	-2	16	43 046 721	-43 046 721	0	43 046 721	0	0
24	17	-7	-27	57	-7	17	387 420 489	-387 420 489	0	387 420 489	0	0
25	23	-1	-39	39	-1	23	581 307 335	-581 307 335	0	581 307 335	0	0
26	24	-6	-44	44	-6	24	52 301 766 015	52 301 766 015	1	52 301 766 015	-174 763	524 288
27	25	-11	-49	49	-11	25	156 905 298 045	-156 905 298 045	3	156 905 298 045	0	0
28	31	-5	-17	31	-5	28	645 790 815	-645 790 815	0	645 790 815	0	0
29	32	-10	-18	36	-10	29	19 125 788 095 241	-19 125 788 095 241	0	19 125 788 095 241	0	0
30	33	-15	-19	41	-15	30	3342 480 104 888 602 883	3342 480 104 888 602 883	243	3342 480 104 888 602 883	0	0
31	39	-9	-23	13	-9	31	174 763	-174 763	0	174 763	0	0
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35	48	-18	-20	20	-18	35	371 386 678 890 800 531	371 386 678 890 800 531	0	371 386 678 890 800 531	0	0
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37	55	-17	-7	5	-17	37	41 265 186 479 933 603	41 265 186 479 933 603	0	41 265 186 479 933 603	0	0
38	56	-22	-6	6	-22	38	3945	-3945	0	3945	0	0
39	57	-27	-7	7	-27	39	87305	-87305	0	87305	0	0
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42	65	-31	9	-3	-31	42	1922 393	-1922 393	0	1922 393	0	0
43	71	-25	-9	3	-25	43	222 832 006 987 726 765	222 832 006 987 726 765	1594 323	222 832 006 987 726 765	0	0

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45	-35	1	1	<u>19 125 788 094 233</u>	<u>3342 480 104 712 441 779</u>	1162 261 467	-609 360 902 271 963	<u>10 027 421 188 349 231 104</u>	0	0	
46	-29	-17	7	<u>27</u>	<u>14 155 803</u>	0	0	<u>135</u>	0	0	
47	-34	-12	6	<u>571</u>	<u>89 653 419</u>	0	0	<u>5</u>	0	0	
48	-39	-7	5	<u>6375 263 697 947</u>	<u>3342 480 104 712 441 779</u>	10 460 353 203	-5484 248 120 447 667	<u>3342 473 729 381 236 736</u>	0	0	5484 237 660 094 464
49	-33	-25	11	<u>243</u>	<u>127 402 227</u>	0	0	<u>5</u>	0	0	
50	-38	-20	10	-351	184 025 439	0	0	-184 025 088	0	0	
51	-43	-15	9	6016 464 188 215 636 959	94 143 178 827	94 143 178 827	-49 358 233 084 029 003	-6016 452 712 742 780 928	0	0	49 358 138 940 850 176
52	-37	-33	15	2187	1146 620 043	0	0	<u>1146 617 856</u>	0	0	
53	-42	-28	14	<u>5</u>	<u>9300 362 571</u>	0	0	<u>5</u>	0	0	
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56	-51	-31	17	<u>5</u>	<u>21 930 011 964 037 118 400 219</u>	68 630 377 364 883	-35 982 151 918 257 143 187	<u>21 929 970 135 938 561 998 848</u>	0	0	35 982 083 287 879 778 33
57	-55	-39	21	<u>5</u>	<u>174 763</u>	0	0	<u>524 288</u>	0	0	
58	-38	94	-44	<u>38 127 987 424 935</u>	<u>12 709 329 141 645</u>	0	0	<u>38 127 987 424 935</u>	0	0	
59	-37	99	-45	<u>108 363 851 984 5</u>	<u>123 765 459 076 791</u>	0	0	<u>1048 576</u>	0	0	
60	-36	18	104	114 383 962 274 805	<u>123 765 459 076 791</u>	-19 683	174 763	<u>371 385 063 695 419 392</u>	0	0	524 288
61	-30	24	86	-4236 443 047 215	38 127 987 424 935	0	0	<u>114 383 962 274 805</u>	0	0	-19 683
62	-29	91	-41	4236 443 047 215	1412 147 682 405	0	0	<u>4236 443 047 215</u>	0	0	
63	-28	14	96	38 127 987 424 935	12 709 329 141 645	0	0	<u>24 117 248</u>	0	0	
64	-22	20	78	2541 865 828 329	24 759 117 886 457 200	-2187	174 763	<u>38 127 987 424 935</u>	0	0	524 288
65	-21	15	83	470 713 894 135	847 288 609 443	0	0	<u>2541 865 828 329</u>	0	0	2187
66	-20	10	88	156 905 298 045	156 905 298 045	0	0	<u>470 713 894 135</u>	0	0	
67	-14	16	70	4936 143 017 315	14 127 987 424 935	-243	174 763	<u>14 127 987 424 935</u>	0	0	524 288
68	-13	11	75	6375 262 698 224	3342 480 104 712 441 779	0	0	<u>38 127 987 424 935</u>	0	0	243
69	-12	6	80	1412 147 682 405	17 433 922 005	0	0	<u>52 301 766 015</u>	0	0	
70	-6	12	62	1984 076 345	349 526	0	0	<u>1048 576</u>	0	0	
71	-5	7	67	52 301 766 015	10 460 353 203	0	0	<u>31 381 059 609</u>	0	0	
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73	3	54	-24	156 905 298 045	470 713 894 135	0	0	<u>1412 147 682 405</u>	0	0	
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75	4	-2	64	52 301 766 015	17 433 922 005	-3	1572 867	<u>52 301 766 015</u>	0	0	
76	10	4	46	645 700 815	215 233 605	0	0	<u>645 700 815</u>	0	0	
77	11	-1	51	58 11 307 335	1937 102 445	0	0	<u>58 11 307 335</u>	0	0	
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80	19	-5	43	797 1 615	2657 205	0	0	<u>52 301 766 015</u>	0	0	
81	20	-10	48	6375 262 698 224	23 914 845	-243	127 402 227	<u>71 444 535</u>	0	0	
82	26	-4	30	885 735	349 526	0	0	<u>1048 576</u>	0	0	
83	27	-9	35	215 233 605	1114 160 034 963 275 408	0	0	<u>215 233 605</u>	0	0	
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85	34	-8	22	71 744 535	23 914 845	-2187	1146 620 043	<u>71 744 535</u>	0	0	1146 617 856
86	35	-13	27	98 415	32 805	0	0	<u>98 415</u>	0	0	
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92	51	-21	11	1215	405	0	0	<u>98 415</u>	0	0	
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157	85	-35	-35	25	0	<u>5</u> 14 155 803	5	0	<u>5</u> 23 592 960	0	0
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164	-31	17	81	-27	0	<u>4</u> 38 127 987 424 935	4	0	<u>4</u> 38 127 987 424 935	0	0
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166	-24	18	68	-22	0	<u>805</u> 114 383 967 274 805	805	0	<u>805</u> 114 383 967 274 805	6561	-19 683
167	-23	13	73	-23	0	<u>28</u> 4236 443 047 215	28	0	<u>28</u> 4236 443 047 215	0	0
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172	-8	10	52	-14	0	<u>1052</u> 38 127 987 424 935	1052	0	<u>1052</u> 38 127 987 424 935	81	-243
173	-7	5	57	-15	0	<u>15</u> 52 301 766 015	15	0	<u>15</u> 52 301 766 015	0	0
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175	0	6	44	-10	0	<u>335</u> 581 1307 335	335	0	<u>335</u> 581 1307 335	0	0
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177	2	-4	54	-12	-1	<u>15</u> 632 303 688 915	15	0	<u>15</u> 632 303 688 915	174 763	524 288
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179	9	-3	41	-7	0	<u>815</u> 645 700 815	815	0	<u>815</u> 645 700 815	0	0
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188	33	-15	17	5	0	<u>735</u> 883 735	735	0	<u>735</u> 883 735	0	0
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191	41	-19	9	9	0	<u>415</u> 98 415	415	0	<u>415</u> 98 415	0	0
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193	48	-18	-4	14	0	<u>735</u> 883 735	735	0	<u>735</u> 883 735	0	0
194	49	-23	1	13	0	<u>935</u> 10 935	935	0	<u>935</u> 10 935	0	0
195	50	-28	6	12	0	<u>1052</u> 52 301 766 015	1052	0	<u>1052</u> 52 301 766 015	92 876 223 483	-92 876 046 336
196	56	-22	-12	18	0	<u>805</u> 32 805	805	0	<u>805</u> 32 805	0	0
197	57	-27	-7	17	0	<u>15</u> 1215	15	0	<u>15</u> 1215	0	0
198	58	-32	-2	16	-1594 323	<u>922</u> 3342 480 104 758 054 922	922	0	<u>922</u> 3342 480 104 758 054 922	835 886 011 347	-835 884 417 024
199	64	-26	-20	22	0	<u>763</u> 174 763	763	0	<u>763</u> 174 763	0	0
200	65	-31	-15	21	0	<u>35</u> 2097 152	35	0	<u>35</u> 2097 152	0	0
201	66	-36	-10	20	-14 348 907	<u>176</u> 1114 160 034 921 332 288	176	0	<u>176</u> 1114 160 034 921 332 288	7522 974 102 123	-7522 959 753 216
202	72	-30	-28	26	0	<u>15</u> 524 288	15	0	<u>15</u> 524 288	0	0

203	73	-35	-23	25	68	-	0	0	35 651 584	0	0	0
204	74	-40	-18	24	<u>135</u>	-	-129 140 163	67 706 766 919 107	3342 473 29 507 065 856	0	0	-67 706 637 778 944
205	80	-34	-36	30	<u>405</u>	-	0	0	1572 867	0	0	0
206	81	-39	-31	29	<u>76</u>	-	0	0	39 845 888	0	0	0
207	82	-44	-26	28	<u>19 125 783 093 792</u>	-	-11 62 261 467	609 360 902 271 963	10 027 421 188 118 020 096	0	0	-609 359 740 010 496
208	89	-43	-39	33	<u>355</u>	-	0	0	<u>132 130 576</u>	0	0	0
209	90	-48	-34	32	<u>6375 262 697 776</u>	-	-10 460 353 203	5484 248 120 447 667	3342 473 29 507 065 856	0	0	-5484 237 660 094 46
210	98	-52	-42	36	<u>57 377 364 276 402</u>	-	-94 143 178 827	49 358 233 084 029 003	30 082 263 561 746 251 776	0	0	-49 358 138 940 850 1
211	-29	19	67	-17	<u>1</u>	-	0	0	<u>524 288</u>	0	0	0
212	-28	14	72	-18	<u>38 127 987 424 935</u>	-	0	0	<u>38 127 987 424 935</u>	0	0	0
213	-27	9	77	-19	<u>12 708 329 141 645</u>	-	0	0	<u>524 288</u>	0	0	0
214	-21	15	59	-13	<u>123 765 39 335 561 71</u>	-	19 683	0	<u>37 498 669 43 948 896</u>	-174 763	0	524 288
215	-20	10	64	-14	<u>38 127 987 424 935</u>	-	0	0	<u>524 288</u>	6561	0	0
216	-19	5	69	-15	<u>4236 443 047 215</u>	-	0	0	<u>4236 443 047 215</u>	0	0	0
217	-13	11	51	-9	<u>38 127 987 424 935</u>	-	2187	0	<u>41 265 107 772 637 184</u>	-174 763	0	524 288
218	-12	6	56	-10	<u>470 715 894 135</u>	-	0	0	<u>1412 147 682 405</u>	729	0	0
219	-11	1	61	-11	<u>156 905 298 045</u>	-	0	0	<u>470 715 894 135</u>	0	0	0
220	-5	7	43	-5	<u>3707 108</u>	-	243	0	<u>4236 443 047 215</u>	0	0	0
221	-4	2	48	-6	<u>12 709 329 141 645</u>	-	0	0	<u>10 027 421 188 118 020 096</u>	-174 763	0	524 288
222	-3	-3	53	-7	<u>52 301 766 015</u>	-	0	0	<u>38 127 987 424 935</u>	81	0	243
223	3	3	35	-1	<u>17 433 922 005</u>	-	0	0	<u>52 301 766 015</u>	0	0	0
224	4	-2	40	-2	<u>156 905 298 045</u>	-	0	0	<u>156 905 298 045</u>	0	0	0
225	5	-7	45	-3	<u>470 715 894 135</u>	-	1	0	<u>1412 147 682 405</u>	-174 763	0	524 288
226	11	-1	27	3	<u>3187 784 01</u>	-	0	0	<u>524 288</u>	0	0	0
227	12	-6	32	2	<u>6375 262 698 107</u>	-	0	0	<u>5811 307 335</u>	0	0	0
228	13	-11	37	1	<u>156 905 298 045</u>	-	3	0	<u>645 700 815</u>	0	0	0
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244	59	-25	-21	27	<u>9266 439</u>	-	0	0	<u>3342 473 29 507 065 856</u>	0	0	0
245	60	-30	-16	26	<u>32 805</u>	-	0	0	<u>3342 473 29 507 065 856</u>	0	0	0
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248	68	-34	-24	30	<u>4019 549</u>	-	0	0	<u>3342 473 29 507 065 856</u>	0	0	0
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376	-8	-2	28	22	114 383 962 274 805	38 127 987 424 935	0	114 383 962 274 805	0	0
377	-7	-7	33	21	4236 443 047 215	4236 443 047 215	0	4236 443 047 215	0	0
378	-1	-1	15	27	38 127 987 424 935	38 127 987 424 935	0	38 127 987 424 935	0	0
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383	9	-15	17	29	19 125 788 094 494	38 127 987 424 935	0	10 027 421 188 876 009 984	-174 763	524 288
384	15	-9	-1	35	1275 052 359 655	12 709 329 141 645	243	38 127 987 424 935	81	243
385	16	-14	4	34	749	174 763	0	52 301 766 015	0	0
386	17	-19	9	33	2125 087 566 112	17 433 922 005	0	52 301 766 015	0	0
387	23	-13	-9	39	98 415	32 805	0	52 301 766 015	0	0
388	24	-18	-4	38	19 125 788 095 241	17 433 922 005	0	52 301 766 015	0	0
389	25	-23	1	37	71 744 535	23 914 845	0	3342 473 729 628 176 384	-174 763	524 288
390	31	-17	-17	43	3320 497	3320 497	0	141 524 288 2405	9	27
391	32	-22	-12	42	71 744 535	23 914 845	0	141 524 288 2405	0	0
392	33	-27	-7	47	215 233 605	215 233 605	0	581 307 335	0	0
393	39	-21	-25	41	6375 262 698 418	645 700 815	0	52 301 766 015	0	0
394	40	-26	-20	46	1937 102 445	1937 102 445	0	645 700 815	0	0
395	41	-31	-15	45	797 1615	797 1615	0	25 690 112	0	0
396	47	-25	-33	51	113 987 333 81	113 987 333 81	0	797 1615	0	0
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398	49	-35	-23	49	215 233 605	215 233 605	0	141 524 288 2405	0	0
399	55	-29	-41	55	885 735	885 735	0	52 301 766 015	-1572 867	1572 864
400	56	-34	-36	54	797 1615	797 1615	0	71 744 535	0	0
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407	-12	-6	32	26	71 744 535	71 744 535	0	524 288	-127 402 227	127 401 984
408	-6	0	14	32	2125 087 566 137	2125 087 566 137	0	524 288	0	0
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411	2	-4	6	36	2125 087 566 112	2125 087 566 112	0	524 288	0	0
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415	11	-13	3	39	14	156 905 298 045	2446 682	0	73 40 032	0	0
416	12	-18	8	38	52 301 766 015	52 301 766 015	52 301 766 015	-27	156 905 298 045	174 763	524 288
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418	19	-17	-5	43	5811 307 335	1937 102 445	1747 630	0	581 307 335	0	0
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423	34	-20	-26	52	19 125 788 094 881	17 433 922 005	23 914 845	0	10 027 421 188 688 969 728	1572 867	-1572 864
424	35	-25	-21	51	52 301 766 015	52 301 766 015	3844 786	0	52 301 766 015	0	0
425	36	-30	-16	50	1114 160 034 633 624 934	645 700 815	11 534 336	-27	11 534 336	0	0
426	42	-24	-34	56	645 700 815	2657 205	12 744 535	0	1937 102 445	14 155 803	-14 155 776
427	43	-29	-29	55	6375 262 698 416	1937 102 445	13 932 462	0	7971 615	0	0
428	44	-34	-24	54	215 233 605	174 763	23 914 845	-243	71 744 535	127 402 227	-127 401 984
429	50	-28	-42	60	885 735	3645	293 265 366	0	215 233 605	0	0
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432	59	-37	-45	63	2125 087 566 137	65 651	371 386 678 320 800 531	0	1114 157 909 874 835 456	0	0
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435	-11	1	13	37	38 127 987 424 935	12 709 329 41 645	12 709 329 41 645	0	524 288	0	0
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441	5	-7	-3	45	2541 865 828 329	847 288 609 443	847 288 609 443	0	2541 865 828 329	729	2187
442	6	-12	2	44	470 715 894 135	156 905 298 045	470 715 894 135	0	470 715 894 135	0	0
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445	14	-16	-6	48	52 301 766 015	17 433 922 005	17 433 922 005	0	52 301 766 015	81	243
446	15	-21	-1	47	156 905 298 045	2970 971	2970 971	0	156 905 298 045	0	0
447	21	-15	-19	53	6375 262 698 107	1412 147 682 405	1412 147 682 405	27	6375 262 698 107	-174 763	524 288
448	22	-20	-14	52	5811 307 335	1937 102 445	1937 102 445	0	5811 307 335	9	27
449	23	-25	-9	51	6375 262 698 211	1937 102 445	1937 102 445	0	6375 262 698 211	0	0
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451	30	-24	-22	56	645 700 815	215 233 605	215 233 605	0	645 700 815	0	0
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453	37	-23	-35	61	19 125 788 094 881	17 433 922 005	17 433 922 005	3	19 125 788 094 881	-1572 867	1572 864
454	38	-28	-30	60	52 301 766 015	52 301 766 015	52 301 766 015	0	52 301 766 015	0	0
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456	45	-27	-43	65	156 905 298 045	52 301 766 015	52 301 766 015	0	156 905 298 045	0	0
457	46	-32	-38	64	645 700 815	215 233 605	215 233 605	0	645 700 815	0	0
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461	63	-45	-49	71	52 301 766 015	52 301 766 015	52 301 766 015	0	52 301 766 015	0	0
462	-8	-2	4	46	645 700 815	215 233 605	215 233 605	0	645 700 815	0	0
463	-7	-7	9	45	5811 307 335	1937 102 445	1937 102 445	0	5811 307 335	0	0
464	-6	-12	14	44	708 362 520 174 805	114 383 962 274 805	114 383 962 274 805	1	708 362 520 174 805	127 402 227	127 401 984
465	0	-6	-4	50	4236 443 047 215	4236 443 047 215	4236 443 047 215	0	4236 443 047 215	0	0
466	1	-11	1	49	38 127 987 424 935	38 127 987 424 935	38 127 987 424 935	0	38 127 987 424 935	0	0
467	2	-16	6	48	7625 597 484 987	2541 865 828 329	2541 865 828 329	-2187	7625 597 484 987	174 763	524 288

468	8	-10	-12	54	$\frac{1}{-470715894135}$	$\frac{174763}{156905298045}$	0	$\frac{524288}{-470715894135}$	0
469	9	-15	-7	53	$\frac{4236443047215}{19125788024368}$	$\frac{9087676}{3342480104736034784}$	0	$\frac{524288}{-470715894135}$	0
470	10	-20	-2	52	$\frac{38127987424935}{52301766015}$	$\frac{174763}{12709329141645}$	-243	$\frac{174763}{81}$	$\frac{524288}{-243}$
471	16	-14	-20	58	$\frac{31381059600}{6375262698107}$	$\frac{174763}{17436975905}$	0	0	0
472	17	-19	-15	57	$\frac{1412147682405}{5811307335}$	$\frac{10460353203}{470715894135}$	0	0	0
473	18	-24	-10	56	$\frac{6375262698107}{1412147682405}$	$\frac{10460353203}{470715894135}$	-27	$\frac{174763}{9}$	$\frac{524288}{-27}$
474	24	-18	-28	62	$\frac{5811307335}{52301766015}$	$\frac{1937102445}{11883884}$	0	0	0
475	25	-23	-23	61	$\frac{52301766015}{6375262698107}$	$\frac{174763}{11883884}$	0	0	0
476	26	-28	-18	60	$\frac{156905298045}{645700815}$	$\frac{52301766015}{11883884}$	-1	174763	$\frac{524288}{3}$
477	32	-22	-36	66	$\frac{645700815}{5811307335}$	$\frac{174763}{1937102445}$	0	0	0
478	33	-27	-31	65	$\frac{5811307335}{19125788024368}$	$\frac{1937102445}{152331083}$	0	0	0
479	34	-32	-26	64	$\frac{19125788024368}{52301766015}$	$\frac{174763}{17436975905}$	-3	1572867	$\frac{524288}{-3}$
480	40	-26	-44	70	$\frac{52301766015}{71744535}$	$\frac{174763}{17436975905}$	0	0	0
481	41	-31	-39	69	$\frac{71744535}{215233605}$	$\frac{23914845}{174763}$	0	0	0
482	42	-36	-34	68	$\frac{215233605}{1275052539671}$	$\frac{174763}{4893364}$	0	0	0
483	49	-35	-47	73	$\frac{387420489}{6375262698107}$	$\frac{129140163}{152331083}$	-27	14155803	$\frac{524288}{-27}$
484	50	-40	-42	73	$\frac{6375262698107}{19125788024368}$	$\frac{174763}{17436975905}$	0	0	0
485	58	-44	-50	76	$\frac{19125788024368}{71744535}$	$\frac{3342480104888602883}{23914845}$	-243	127402227	$\frac{524288}{-243}$
486	-5	-5	-5	55	$\frac{38127987424935}{470715894135}$	$\frac{174763}{17436975905}$	-2187	1146620043	$\frac{524288}{-2187}$
487	-4	-10	0	54	$\frac{7625597484987}{7083652520177}$	$\frac{174763}{17436975905}$	0	0	0
488	-3	-15	5	53	$\frac{7083652520177}{114383962274805}$	$\frac{19683}{38127987424935}$	1	0	0
489	3	-9	-13	59	$\frac{4236443047215}{38127987424935}$	$\frac{19683}{38127987424935}$	19683	0	0
490	4	-14	-8	58	$\frac{38127987424935}{2125087566046}$	$\frac{4236443047215}{38127987424935}$	0	0	0
491	5	-19	-3	57	$\frac{2125087566046}{38127987424935}$	$\frac{371386678304897098}{12709329141645}$	0	0	0
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493	12	-18	-16	62	$\frac{4236443047215}{3825157618942}$	$\frac{1412147682405}{10660543}$	0	0	0
494	13	-23	-11	61	$\frac{3825157618942}{7625597484987}$	$\frac{608496020959160746}{2541869828329}$	0	0	0
495	19	-17	-29	67	$\frac{52301766015}{25}$	$\frac{17436975905}{4018345}$	0	0	0
496	20	-22	-24	66	$\frac{156905298045}{6375262698107}$	$\frac{52301766015}{1114160034969273641}$	0	0	0
497	21	-27	-19	65	$\frac{6375262698107}{1412147682405}$	$\frac{470715894135}{174763}$	0	0	0
498	27	-21	-37	71	$\frac{5811307335}{77}$	$\frac{1937102445}{13456751}$	0	0	0
499	28	-26	-32	70	$\frac{1523052539674}{31381059609}$	$\frac{433922005}{222832006991047262}$	0	0	0
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507	-2	-8	-14	64	$\frac{38127987424935}{12709329141645}$	$\frac{12709329141645}{4236443047215}$	0	0	0
508	-1	-13	-9	63	$\frac{12709329141645}{7083652520177}$	$\frac{4236443047215}{114383962274805}$	0	0	0
509	0	-18	-2	62	$\frac{7083652520177}{114383962274805}$	$\frac{38127987424935}{174763}$	-1	0	0
510	6	-12	-22	68	$\frac{4236443047215}{62}$	$\frac{1412147682405}{10835306}$	0	0	0
511	7	-17	-17	67	$\frac{38127987424935}{23612985401602}$	$\frac{1412147682405}{4236443047215}$	0	0	0
512	8	-22	-12	66	$\frac{23612985401602}{4236443047215}$	$\frac{1412147682405}{156905298045}$	-1	0	0
513	14	-16	-30	72	$\frac{470715894135}{14}$	$\frac{156905298045}{2446682}$	0	0	0
514	15	-21	-25	71	$\frac{847288609443}{18125788024368}$	$\frac{282429536481}{3342480104736034784}$	0	0	0
515	16	-26	-20	70	$\frac{18125788024368}{38127987424935}$	$\frac{12709329141645}{174763}$	-1	0	0
516	22	-20	-38	76	$\frac{52301766015}{26}$	$\frac{17436975905}{4543838}$	0	0	0
517	23	-25	-33	75	$\frac{156905298045}{6319252980167}$	$\frac{156905298045}{1412147682405}$	0	0	0
518	24	-30	-28	74	$\frac{6319252980167}{1412147682405}$	$\frac{1412147682405}{470715894135}$	-1	0	0
519	30	-24	-46	80	$\frac{5811307335}{86}$	$\frac{1937102445}{15028618}$	0	0	0
520	31	-29	-41	79	$\frac{52301766015}{17436975905}$	$\frac{17436975905}{17433922005}$	0	0	0

521	32	-34	78	6375 262 698 211	-	1114 160 034 927 448 993	-	1	3342 473 729 519 648 768	174 763	524 288
522	39	-33	-49	156 905 298 045	94	52 301 766 015	3	0	156 905 298 045	0	3
523	40	-38	83	581 1307 335	-	1937 102 445	-	0	581 1307 335	0	0
524	48	-42	86	19 125 788 091 881	-	3342 480 104 825 688 203	-3	0	10 027 421 188 688 969 728	1572 867	-1572 864
525	1	-11	-23	52 301 766 015	-	1114 160 034 927 448 993	-27	0	3342 473 729 519 648 768	14 155 803	-14 155 776
526	2	-16	73	1937 102 445	-	645 700 815	-27	0	1937 102 445	0	0
527	3	-21	-13	12 709 329 141 645	-	12 709 329 141 645	0	0	38 127 987 424 935	0	0
528	9	-15	71	38 127 987 424 935	-	123 795 559 435 256 971	19 683	0	37 1385 969 943 248 896	174 763	524 288
529	10	-20	76	708 362 522 017	-	38 127 987 424 935	0	0	114 383 962 274 805	6561	19 683
530	11	-25	75	38 127 987 424 935	-	1412 147 682 405	2187	0	4236 443 047 215	0	524 288
531	17	-19	-39	38 127 987 424 935	-	12 709 329 141 645	0	0	38 127 987 424 935	0	2187
532	18	-24	80	156 905 298 045	-	12 709 329 141 645	0	0	470 715 894 135	0	0
533	19	-29	79	19 125 788 091 881	-	1412 147 682 405	0	0	4236 443 047 215	0	0
534	25	-23	-47	38 127 987 424 935	-	12 709 329 141 645	243	0	10 027 421 188 430 009 984	174 763	524 288
535	26	-28	-42	52 301 766 015	-	17 433 922 005	0	0	38 127 987 424 935	81	243
536	27	-33	83	156 905 298 045	-	5068 127	0	0	52 301 766 015	0	0
537	34	-32	-50	6375 262 698 107	-	1114 160 034 927 448 993	1	0	156 905 298 045	0	524 288
538	35	-37	88	1412 147 682 405	-	470 715 894 135	27	0	3342 473 729 519 648 768	6561	19 683
539	43	-41	-53	6375 262 698 211	-	1114 160 034 927 448 993	0	0	114 383 962 274 805	0	0
540	4	-14	-32	19 125 788 091 881	-	17 433 922 005	3	0	10 027 421 188 688 969 728	3	524 288
541	5	-19	-27	38 127 987 424 935	-	17 433 922 005	0	0	52 301 766 015	0	1572 864
542	6	-24	80	38 127 987 424 935	-	12 709 329 141 645	0	0	38 127 987 424 935	0	0
543	12	-18	-40	708 362 522 017	-	1398 104	0	0	4194 304	0	0
544	13	-23	85	114 383 962 274 805	-	123 795 559 435 256 971	-1	19 683	37 1385 969 943 248 896	174 763	524 288
545	14	-28	84	38 127 987 424 935	-	470 715 894 135	0	0	114 383 962 274 805	6561	19 683
546	20	-22	-48	4236 443 047 215	-	1412 147 682 405	0	0	4236 443 047 215	0	0
547	21	-27	-43	7625 597 484 987	-	74 277 831 663 915 438	0	0	7625 597 484 987	0	0
548	22	-32	88	7625 597 484 987	-	254 1 865 828 329	-2187	0	7625 597 484 987	729	2187
549	29	-31	-51	156 905 298 045	-	156 905 298 045	0	0	470 715 894 135	0	0
550	30	-36	-46	6375 262 698 107	-	15 379 144	0	0	46 137 344	0	0
551	38	-40	-54	6375 262 698 211	-	668 106 026 087 405 746	-243	1	2005 239 343 016 243 296	81	524 288
552	7	-17	-41	156 905 298 045	-	254 1 865 828 329	0	0	7625 597 484 987	0	243
553	8	-22	-36	38 127 987 424 935	-	5893 316	0	0	16 977 516	0	0
554	9	-27	-31	4236 443 047 215	-	52 301 766 015	0	0	470 715 894 135	0	0
555	15	-21	-49	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
556	16	-26	-44	4236 443 047 215	-	1114 160 034 927 448 993	0	0	470 715 894 135	0	0
557	17	-31	-39	4236 443 047 215	-	1114 160 034 927 448 993	0	0	470 715 894 135	0	0
558	24	-30	-52	4236 443 047 215	-	52 301 766 015	0	0	470 715 894 135	0	0
559	25	-35	-47	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
560	33	-39	-55	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
561	10	-20	-50	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
562	11	-25	-45	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
563	12	-30	-40	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
564	19	-29	-53	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
565	20	-34	-48	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
566	28	-38	-56	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
567	14	-28	-54	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
568	15	-33	-49	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
569	23	-37	-57	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0
570	18	-36	-58	4236 443 047 215	-	174 763	0	0	470 715 894 135	0	0

References

- [1] A. Alaca, S. Alaca and K. S. Williams, *On the two-dimensional theta functions of Borweins*, *Acta Arith.* 124 (2006), 177-195.
- [2] A. Alaca, S. Alaca and K. S. Williams, *Evaluation of the convolution sums $\sum_{l+12m=n} \sigma(l)\sigma(m)$ and $\sum_{3l+4m=n} \sigma(l)\sigma(m)$* , *Adv. Theor. Appl. Math.* 1(2006), 27-48.
- [3] B. Gordon, *Some identities in combinatorial analysis*, *Quart. J. Math. Oxford Ser.* 12 (1961), 285-290.
- [4] B. Gordon and S. Robins, *Lacunarity of Dedekind η -products*, *Glasgow Math. J.* 37 (1995), 1-14.
- [5] F. Diamond, J. Shurman, *A First Course in Modular Forms*, *Springer Graduate Texts in Mathematics* 228, (2005).
- [6] V. G. Kac, *Infinite-dimensional algebras, Dedekind's η -function, classical Möbius function and the very strange formula*, *Adv. Math.* 30 (1978), 85-136.
- [7] B. Kendirli, *Evaluation of Some Convolution Sums by Quasimodular Forms*, *European Journal of Pure and Applied Mathematics* ISSN 13075543 Vol.8., No. 1, (2015), 81-110.
- [8] B. Kendirli, *Evaluation of Some Convolution Sums and Representation Numbers of Quadratic Forms of Discriminant 135*, *British Journal of Mathematics and Computer Science*, Vol 6/6, (2015), 494-531.
- [9] B. Kendirli, *Evaluation of Some Convolution Sums and the Representation numbers*, *Ars Combinatoria* Volume CXVI, (2014), 65-91.
- [10] B. Kendirli, *Cusp Forms in $S_4(\Gamma_0(79))$ and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant -79*, *Bulletin of the Korean Mathematical Society*, Vol 49/3, (2012), 529-572.
- [11] B. Kendirli, *Cusp Forms in $S_4(\Gamma_0(47))$ and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant -47*, *International Journal of Mathematics and Mathematical Sciences* Vol. (2012), Article ID 303492, 10 pages.
- [12] B. Kendirli, *The Bases of $M_4(\Gamma_0(71))$, $M_6(\Gamma_0(71))$ and the Number of Representations of Integers*, *Mathematical Problems in Engineering* Vol (2013), Article ID 695265, 34 pages
- [13] G. Köhler, *Eta Products and Theta Series Identities* Springer-Verlag, Berlin, (2011).
- [14] I. G. Macdonald, *Affine root systems and Dedekind's η -function*, *Invent. Math.* 15 (1972), 91-143.
- [15] Olivia X. M. Yao, Ernest X. W. Xia and J. Jin, *Explicit Formulas for the Fourier coefficients of a class of eta quotients*, *International Journal of Number Theory* Vol. 9, No. 2 (2013), 487-503.
- [16] I. J. Zucker, *A systematic way of converting infinite series into infinite products*, *J. Phys. A* 20 (1987) L13-L17.
- [17] I. J. Zucker, *Further relations amongst infinite series and products:II. The evaluation of three-dimensional lattice sums*, *J. Phys. A* 23 (1990), 117-132.
- [18] K. S. Williams, *Fourier series of a class of eta quotients*, *Int. J. Number Theory* 8 (2012), 993-1004.