International Journal of Advanced Mathematical Sciences, 3 (2) (2015) 161-171
www.sciencepubco.com/index.php/IJAMS
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doi: 10.14419/ijams.v3i2.5410
Research Paper

Computational modeling of hypersingular integral equations for 2D pre-cantor scattering structure

Kateryna Nesvit*, Yuriy Gandel

Department of Applied Mathematics, Karazin Kharkiv National University, Ukraine
**Corresponding author E-mail: nesvit@karazin.ua*

Abstract

This paper presents the investigative study to derive a computational model based on hypersingular integral equations for the pre-Cantor plane-parallel diffraction structure. Such structure consists of finite numbers of the thin impedance strips located in the XY plane. A plane transverse magnetic wave is incident from infinity on considered diffraction structure at an angle and need to find the total field resulting from the scattering. The model which is considered in this work is an approximation of real fractal antennas in two-dimensional case. Pre-fractal properties of grating allow producing the newest antennas for modern mobile devices due to their compact size and broadband properties. The purpose of this work is to develop computer model their structure using parametric representation of hypersingular integral operator, Nyström method with specific quadrature formulas. The numerical results have been obtained and investigated for pre-Cantor structures for calculating physics characteristics. These results have been compared and analyzed in different mathematical models and softwares.

Keywords: *Computational Model; Hypersingular Integral Equation; Parametric Representation; Pre-Cantor Grating.*

1. Introduction

The electromagnetic (EM) force is one of the four fundamental interactions in nature. Today, it is important to study the interaction of EM waves with pre-fractal structures. Because this feature allows to design the best compact and powerful antennas in the world and to produce them for today's variety mobile devices embodying the newest innovative computer simulation. The analysis of various EM phenomena can be a very complicated task from the experimental point of view. The use of mathematical modeling, however, allows simulation of complex cases and sometimes is the only tool at hand of the engineer. Each real-life problem should be solved correctly, timely, productivity and the decision should be available for existing computing softwares and suitable for actual use, and of course the accuracy of the solution should be consistent. Therefore it's important to apply a high level of knowledge for new problems in mathematical modeling of physical processes.

One of the important features of the diffraction problem lies in calculating the scattered and total fields generated due to the incidence of the EM waves on the diffraction structures. These fields are critically important for difficult structures, objects and for the antennas which are used in modern mobile communication devices. Two-dimensional (2-D) scattered field analysis of transverse magnetic (TM) wave diffraction problem based on the mathematical model using hypersingular integral equations (IEs), is an important and crucial research of its kind. The model discussed here is derived from the 2-D model of real fractal antennas. Such types of antennas are very useful, as they fulfill the requirements of recent wireless communication needs due to their compact size, low-profile design and broadband properties.

The main contribution in the EM wave scattering and diffraction the follows authors A. Sommerfeld [1], J. M. Cowley [2], R. King [3], J. Kong [4], in fractal electrodynamics D. Jaggard [5], in fractal multiband and small antennas N. Cohen [6], C. Puente [7]. As for mathematical and computational sides of this type of problem we should note authors D. Colton, R. Kress, R. Chapko, L. Monch who have important results for the numerical solutions of hypersingular IEs (HSIEs) in scattering theory [8] using a fully discrete quadrature method by combining a collocation and a quadrature method [9], and by trigonometric interpolation [10]. HSIEs are using in different important fields and one of these fields is ocean currents and circulation, physical oceanography. The most efficient works in this topic are from P. A. Martin, L. Farina, V. Peron where were developed an appropriate spectral method, using Fourier expansions in the azimuthal direction and Jacobi polynomials in the radial direction [11]. In [12] was illustrated the spectral method by choosing the problem a submerged disc is perturbed out of it original plane. Other problems with applying Newton's method and its modified version to solve the equations obtained by applying a collocation method to a nonlinear HSIEs of Prandtl's type were considered in [13]. That's why the considering of fractal and hypersingular properties are modern in last ten years and interesting for investigation.

The early research carried out in this field investigation are [14], [15], [16] which the boundary-value problems of mathematical theory of diffraction have been solved based on the singular and hypersingular boundary IEs. In work [17] was demonstrated that the mathematical models for periodic and finite plane-parallel pre-Cantor structures that may have a multilayer diffraction structure and considering the impedance of material and the dielectric inserts, based on HSIEs better than on the base of singular IEs (SIEs).

In [18], the discrete mathematical model and computational results of the diffraction problem on pre-fractal impedance grating was obtained. For case with perfectly electrically conducting (PEC) strips some numerical results were presented in [19]. The research in [20] aimed at deriving of the HSIE and the Fredholm IE of 2nd kind with logarithmic singularities. Current article describes a computational modeling of hypersingular integral equations and is continuation of the paper [20], where were consider the diffraction structure with impedance strips. Mathematical model more difficult structures with PEC thin strips above reflector have been considered in [21], with shielded dielectric layer based on singular integral equations in paper [22]. Some compared results for different structures which are resolve by discrete singularities method were consider in [23].

2. Formulation diffraction problem

To solve a 2D diffraction problem for the TM case we calculate the total field which satisfies Maxwell's equations, supplemented with the Shchukin-Leontovich impedance boundary conditions. Besides, the total field must also satisfy the Sommerfeld radiation conditions and the Meixner edge condition.

In this paper the diffraction structures, which consist of finite numbers of the impedance pre-Cantor thin strips are shown in Fig 1. In the TM case the length of the strips have a pre-Cantor set.

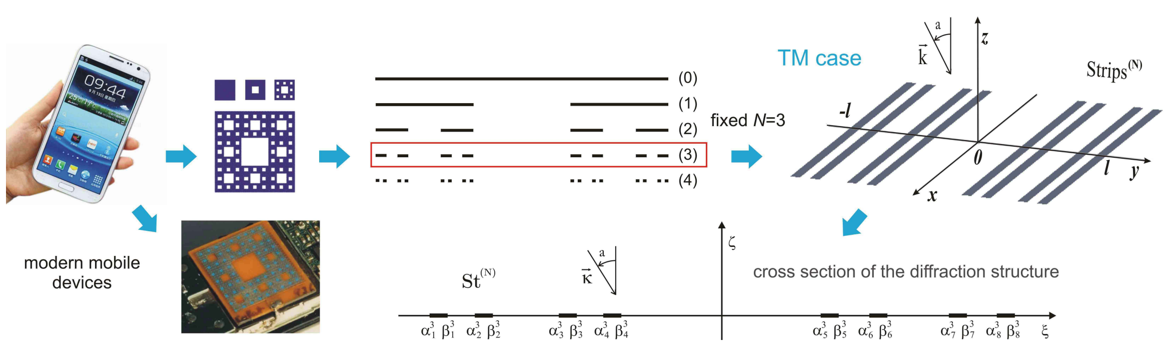

Figure 1: Schematic of the considered diffraction structure.

Usually we use the theory potential or Green formula for reduce the problem from differential to integral form.

Then it is possible to obtain the corresponding integral equations. Another theory if the total field is taken as

$$u^{(N)}(\xi,\zeta)=\begin{cases} u^N_{inc}(\xi,\zeta)+u^N_{+}(\xi,\zeta), & \zeta>0,\\ u^N_{inc}(\xi,\zeta)+u^N_{-}(\xi,\zeta), & \zeta<0, \end{cases} \tag{1}$$

where

$$u^N_{\pm}(\xi,\zeta)=\int\limits_{-\infty}^{+\infty} C^N_{\pm}(\lambda)e^{i\lambda\xi\mp\gamma(\lambda)\zeta}d\lambda,\ \zeta>0,(\zeta<0). \tag{2}$$

These Fourier series in the integral form satisfies the Helmholtz equation, the boundary conditions, the Sommerfeld and Meixner conditions, and the conditions of conjugation in the slits, as described in [14], [24]. The radiation condition will be satisfied if $\gamma(\lambda)$ is given by $Re(\gamma(\lambda))\geq 0,\ Im(\gamma(\lambda)\geq 0,\ \lambda\in\Re$.

We need to find the total field for calculation electromagnetic characteristics.

3. Computational model

3.1. Mathematical model

To derive the hypersingular IEs [25] we need to write down two coupled IEs using all the aforementioned conditions as it done in [14], [15] and [17]:

$$\begin{cases} \int\limits_{-\infty}^{+\infty}\left(C^N_{+}(\lambda)-C^N_{-}(\lambda)\right)e^{i\lambda\xi}d\lambda=0,\ \xi\in CSt^{(N)},\\ \int\limits_{-\infty}^{+\infty}(\gamma(\lambda)+B)\left(C^N_{+}(\lambda)-C^N_{-}(\lambda)\right)e^{i\lambda\xi}d\lambda=f^N_1(\xi),\ \ \xi\in St^{(N)}, \end{cases} \tag{3}$$

$$\begin{cases} \int\limits_{-\infty}^{+\infty}\gamma(\lambda)\left(C^N_{+}(\lambda)+C^N_{-}(\lambda)\right)e^{i\lambda\xi}d\lambda=0,\ \xi\in CSt^{(N)},\\ \int\limits_{-\infty}^{+\infty}(\gamma(\lambda)+B)\left(C^N_{+}(\lambda)+C^N_{-}(\lambda)\right)e^{i\lambda\xi}d\lambda=f^N_2(\xi),\ \xi\in St^{(N)}. \end{cases} \tag{4}$$

It should be noted that everywhere below the relations between unknown coefficients and unknown functions are written in the form

$$\begin{aligned} &B^N_1(\lambda)=\left(C^N_{+}(\lambda)-C^N_{-}(\lambda)\right),\ \ B^N_2(\lambda)=\gamma(\lambda)\left(C^N_{+}(\lambda)+C^N_{-}(\lambda)\right),\\ &F^N_i(\xi)=\int\limits_{-\infty}^{+\infty}B^N_i(\lambda)e^{i\lambda\xi}d\lambda,\ \ B^N_i(\lambda)=\tfrac{1}{2\pi}\int\limits_{-\infty}^{+\infty}F^N_i(\eta)e^{-i\lambda\eta}d\eta,\ \ i=1,2,\ \ \xi\in\Re. \end{aligned} \tag{5}$$

To obtain the mathematical model with hypersingular IEs we using the parametric representations specific form as shown in [26], [27]. As were derived in [20] the mathematical model based on HSIEs takes the following form:

$$\begin{cases} BF^N_1(\xi)-\frac{1}{\pi}\int\limits_{St^{(N)}}\frac{F^N_1(\eta)}{(\eta-\xi)^2}d\eta+\frac{\kappa^2}{2\pi}\int\limits_{St^{(N)}}\ln|\eta-\xi|\,F^N_1(\eta)d\eta+\frac{1}{\pi}\int\limits_{St^{(N)}}Q^N_1(\eta,\xi)F^N_1(\eta)d\eta=f^N_1(\xi),\ \xi\in St^{(N)},\\ F^N_2(\xi)-\frac{B}{\pi}\int\limits_{St^{(N)}}\ln|\eta-\xi|\,F^N_2(\eta)d\eta+\frac{1}{\pi}\int\limits_{St^N}Q^N_2(\eta,\xi)F^N_2(\eta)d\eta=f^N_2(\xi),\ \xi\in St^{(N)}, \end{cases} \tag{6}$$

where $B = i\kappa\frac{Z_c}{Z_0}$ defines impedance of the material strips, $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ is the free space impedance, μ_0, ε_0 are magnetic and dielectric constants, respectively;

$$K(\eta,\xi) = \frac{\kappa^4}{4}\int\limits_{-\infty}^{\infty}\frac{\exp^{i\lambda(\xi-\eta)}}{\gamma(\lambda)(|\lambda|+\gamma(\lambda))^2}d\lambda,\ \ KQ(\xi,\eta) = H_0^{(1)}(\kappa\mid\eta-\xi\mid) - \frac{2i}{\pi}\ln\mid\eta-\xi\mid,$$

$$Q_1(\eta,\xi) = K(\eta,\xi) - \frac{\kappa^2 i\pi}{4}KQ(\eta,\xi),\ \ Q_2(\eta,\xi) = B\frac{i\pi}{2}KQ(\eta,\xi), \tag{7}$$

$$f_1^N(\xi) = 2\left.\frac{\partial u_{inc}^N(\xi,\zeta)}{\partial\zeta}\right|_{\zeta=0},\ \ f_2^N(\xi) = -2Bu_{inc}^N(\xi,+0),\ \ u_{inc}^N(\xi,\zeta) = e^{i\kappa(\xi\sin\alpha-\zeta\cos\alpha)}.$$

Term $Q_2^N(\eta,\xi)$ in equation (7) have a singularity in logarithm when $\eta=\xi$. For this reason we need to take advantage and to expand to series a Hankel function using Bessel functions of the first kind. Thus, the remainders of the series have been obtained which doesn't have a singularity.

The solutions of the HSIEs (7) of the second kind were proposed in [28] where was considered the regularization these IEs. The theorem on existence and uniqueness of this type of IEs have been proved there.

For numerical calculation we should to prepare our mathematical model for discretization. Introduce restrictive conditions to the functions:

$$F_{i,q}^N(\eta) = \left.F_i^N(\eta)\right|_{\eta\in St_q^{(N)}},\ \ f_{i,p}^N(\xi) = \left.f_i^N(\xi)\right|_{\xi\in St_p^{(N)}},\ \ St_q^{(N)} = (\alpha_q^N,\beta_q^N),\ \ i=1,2,\ p,q=\overline{1,2^N}. \tag{8}$$

It is essential to reduce the HSIE of the second kind on a set of intervals and the Fredholm IE of the second kind at the same set of interval, namely - $St^{(N)}$, for all strips to equations on following intervals - $St_q^{(N)} = (\alpha_q^N,\beta_q^N)$, $q=\overline{1,2^N}$. Next, the Meixner condition will be satisfied if unknown functions represented by equation (8) will be written as:

$$F_{i,q}^N(\eta) = w_{i,q}^N(\eta)\sqrt{(\beta_q^N-\eta)(\eta-\alpha_q^N)},\ \ i=1,2,\ \ q=\overline{1,2^N}, \tag{9}$$

where the functions $w_{i,q}^{(N)}(\eta)$, $i=1,2$, $q=\overline{1,2^N}$, are Holder continuous.

By choosing a normalized interval as:

$$g_q^{(N)}\ :\ (-1,1)\longmapsto(\alpha_q^N,\beta_q^N)\ :\ t\longmapsto g_q^{(N)}(t) = \frac{\beta_q^N-\alpha_q^N}{2}t+\frac{\beta_q^N+\alpha_q^N}{2},\ |t|<1, \tag{10}$$

the variables will be transformed to the following form

$$\eta = g_q^{(N)}(t),\ \ \xi = g_p^{(N)}(t_0),\ \ |t|<1,\ \ |t_0|<1,\ \ \eta\in St_q^{(N)},\ \ \xi\in St_p^{(N)},\ \ p,q=\overline{1,2^N}. \tag{11}$$

We have the next view for required functions:

$$F_{i,q}^N(g_q^{(N)}(t)) = v_{i,q}^N(t)\frac{\beta_q^N-\alpha_q^N}{2}\sqrt{1-t^2},\ \ i=1,2,\ \ q=\overline{1,2^N}. \tag{12}$$

Introduce the notations using previous notes in formulas (7):

$$Q^N_{1,qp}(g^{(N)}_q(t), g^{(N)}_p(t_0)) \overset{p=q}{=} \left(\frac{\beta^N_p-\alpha^N_p}{2}\right)^2 \left[\frac{\kappa^2}{2}\ln\left|\frac{\beta^N_p-\alpha^N_p}{2}\right| + Q_1(g^{(N)}_q(t), g^{(N)}_p(t_0))\right],$$

$$Q^N_{1,qp}(g^{(N)}_q(t), g^{(N)}_p(t_0)) \overset{p\neq q}{=} \tag{13}$$

$$\left(\frac{\beta^N_q-\alpha^N_q}{2}\right)^2 \left[\frac{(-1)}{\left(g^{(N)}_q(t)-g^{(N)}_p(t_0)\right)^2} + \frac{\kappa^2}{2}\ln\left|g^{(N)}_q(t)-g^{(N)}_p(t_0)\right| + Q_1(g^{(N)}_q(t), g^{(N)}_p(t_0))\right].$$

$$Q^N_{2,qp}(g^{(N)}_q(t), g^{(N)}_p(t_0)) \overset{p=q}{=} \left(\frac{\beta^N_q-\alpha^N_q}{2}\right)^2 \left[(-B)\ln\left|\frac{\beta^N_p-\alpha^N_p}{2}\right| + Q_2(g^{(N)}_q(t), g^{(N)}_p(t_0))\right],$$

$$Q^N_{2,qp}(g^{(N)}_q(t), g^{(N)}_p(t_0)) \overset{p\neq q}{=} \left(\frac{\beta^N_q-\alpha^N_q}{2}\right)^2 \left[(-B)\ln\left|g^{(N)}_q(t)-g^{(N)}_p(t_0)\right| + Q_2(g^{(N)}_q(t), g^{(N)}_p(t_0))\right]. \tag{14}$$

After doing some detailed analytical transforms, not mentioned here, using the relations in (8) - (14) and excluding the logarithmic and hypersingular singularities at p=q, we obtain from system (6) following its form as:

$$\begin{cases} B\frac{\beta^N_p-\alpha^N_p}{2}v^N_{1,p}(t_0)\sqrt{1-t_0^2} - \frac{1}{\pi}\int\limits_{-1}^{1}\frac{v^N_{1,p}(t)\sqrt{1-t^2}dt}{(t-t_0)^2} + \frac{\kappa^2}{2\pi}\left(\frac{\beta^N_p-\alpha^N_p}{2}\right)^2\int\limits_{-1}^{1}\ln|t-t_0|\,v^N_{1,p}(t)\sqrt{1-t^2}dt+ \\ \frac{1}{\pi}\sum\limits_{q=1}^{2^N}\int\limits_{-1}^{1}Q^N_{1,qp}(g^{(N)}_q(t), g^{(N)}_p(t_0))v^N_{1,q}(t)\sqrt{1-t^2}dt = f^N_{1,p}(g^{(N)}_p(t_0)),\ |t_0|<1,\ p=\overline{1,2^N}, \\ \frac{\beta^N_p-\alpha^N_p}{2}v^N_{2,p}(t_0)\sqrt{1-t_0^2} - \frac{B}{\pi}\left(\frac{\beta^N_q-\alpha^N_q}{2}\right)^2\int\limits_{-1}^{1}\ln|t-t_0|\,v^N_{2,p}(t)\sqrt{1-t^2}dt+ \\ \frac{1}{\pi}\sum\limits_{q=1}^{2^N}\int\limits_{-1}^{1}Q^N_{2,qp}(g^{(N)}_q(t), g^{(N)}_p(t_0))v^N_{2,q}(t)\sqrt{1-t^2}dt = f^N_{2,p}(g^{(N)}_p(t_0)),\quad |t_0|<1, p=\overline{1,2^N}. \end{cases} \tag{15}$$

where $Q^N_{i,qp}(g^{(N)}_q(t_0), g^{(N)}_p(t_0)), f^N_{i,p}(g^{(N)}_p(t_0)),\ \ i=1,2$ are known smooth functions from (13) and (7).

3.2. Discretization

For numerical solution in this research was proposed discretization by specific quadrature formulas [29]. This type is known as Nystrom type method for calculations IEs. Some of modification of this method was proposed in [30] for solving the boundary singular IEs. More different efficient numerical methods for specific EM scattering problems was considering by Jin Au Kong in his book [31].

The discretization of problem based on HSIE was performed as follows way. The unknown and smooth functions are interpolated by their Lagrange polynomial of a corresponding degree in the nodes which are the nulls of

Chebyshev polynomials of the second kind. Replacing the unknown function at its interpolation polynomial seems correct. These polynomials in corresponding points was defined as:

$$v_{i,p}^{N,n-2}(t_{0k}^n)=v_{i,p}^{N}(t_{0k}^n),\;\; Q_{i,qp}^{N,n-2,n-2}\left(g_q^{(N)}(t_{0k}^n),g_p^{(N)}(t_{0j}^n)\right)=Q_{i,qp}^{N}\left(g_q^{(N)}(t_{0k}^n),g_p^{(N)}(t_{0j}^n)\right),$$
$$f_{i,p}^{N,n-2}(t_{0k}^n)=f_{i,p}^{N}(t_{0k}^n),\;\; j,k=\overline{1,n-1},\;\; i=1,2. \tag{16}$$

The outside integral term in (15) should be written as

$$v_{i,p}^{N,n-2}(t_{0j}^n)=\sum_{k=1}^{n-1}v_{i,p}^{N,n-2}(t_{0k}^n)\delta_{j,k},\quad t_{0k}^n=\cos\left(\tfrac{j\pi}{n}\right),\;\; j=\overline{1,n-1}. \tag{17}$$

After discretization (15) and with the help of special quadrature formulas of interpolation type for all integrals with the corresponding collocation points [29], thus the mathematical model in (6) resulted in deriving a system for approximate solutions:

$$\begin{cases} B\frac{\beta_p^N-\alpha_p^N}{2}\sqrt{1-(t_{0j}^n)^2}\sum\limits_{k=1}^{n-1}v_{1,p}^{N,n-2}(t_{0k}^n)\delta_{k,j}-\sum\limits_{k=1,j\neq k}^{n-1}v_{1,p}^{N,n-2}(t_{0k}^n)\times \\ \frac{(1-(-1)^{j+k})(1-(t_{0k}^n)^2)}{(t_{0k}^n-t_{0j}^n)^2}\frac{1}{n}+\frac{n}{2}\sum\limits_{k=1}^{n-1}v_{i,p}^{N,n-2}(t_{0k}^n)\delta_{k,j}+\frac{\kappa^2}{2}\left(\frac{\beta_p^N-\alpha_p^N}{2}\right)^2\left(\frac{-1}{n}\right)\sum\limits_{k=1}^{n-1}v_{1,p}^{N,n-2}(t_{0k}^n)\left(1-(t_{0k}^n)^2\right)\times \\ \left[\ln 2+2\sum\limits_{r=1}^{n-1}\frac{T_r(t_{0k}^n)}{r}T_r(t_{0j}^n)+\frac{(-1)^k}{n}T_r(t_{0j}^n)\right]+\sum\limits_{q=1}^{2^N}\frac{1}{n}\sum\limits_{k=1}^{n-1}Q_{1,qp}^{N,n-2,n-2}(g_q^{(N)}(t_{0k}^n),g_p^{(N)}(t_{0j}^n))\times \\ v_{1,q}^{N,n-2}(t_{0k}^n)\left(1-(t_{0k}^n)^2\right)=f_{1,p}^{N,n-2}(g_p^{(N)}(t_{0j}^n)), \\ \frac{\beta_p^N-\alpha_p^N}{2}\sqrt{1-(t_{0k}^n)^2}\sum\limits_{k=1}^{n-1}v_{2,p}^{N,n-2}(t_{0k}^n)\delta_{k,j}-\left(\frac{\beta_p^N-\alpha_p^N}{2}\right)^2B\left(-\frac{1}{n}\sum\limits_{k=1}^{n-1}v_{2,p}^{N,n-2}(t_{0k}^n)\left(1-(t_{0k}^n)^2\right)\right)\times \\ \left[\ln 2+2\sum\limits_{r=1}^{n-1}\frac{T_r(t_{0k}^n)}{r}T_r(t_{0j}^n)+\frac{(-1)^k}{n}T_r(t_{0j}^n)\right]+\sum\limits_{q=1}^{2^N}\frac{1}{n}\sum\limits_{k=1}^{n-1}Q_{2,qp}^{N,n-2,n-2}(g_q^{(N)}(t_{0j}^n),g_p^{(N)}(t_{0k}^n))\times \\ v_{2,q}^{N,n-2}(t_{0k}^n)\left(1-(t_{0k}^n)^2\right)=f_{2,p}^{N,n-2}(g_p^{(N)}(t_{0j}^n)),\;\; p=\overline{1,2^N},\;\; j=\overline{1,n-1}. \end{cases} \tag{18}$$

where $Q_{i,qp}^{N,n-2,n-2}(g_q^{(N)}(t_{0j}^n),g_p^{(N)}(t_{0k}^n)),\;\; f_{i,p}^{N,n-2}(g_p^{(N)}(t_{0k}^n)),\;\; i=1,2$ are known smooth functions.

We can reduce the system (18) to compact form for $i=1,2,\;\; p=\overline{1,2^N},\;\; j=\overline{1,n-1}$

$$\sum_{q=1}^{2^N}\sum_{k=1}^{n-1}M_{i,qp}^{N,n-2,n-2}\left(g_q^{(N)}\left(t_{0k}^n\right),g_p^{(N)}\left(t_{0j}^n\right)\right)v_{i,q}^{N,n-2}\left(t_{0k}^n\right)=f_{i,p}^{N,n-2}\left(g_p^{(N)}\left(t_{0j}^n\right)\right), \tag{19}$$

where

$$M_{1,qp}^{N,n-2,n-2}\left(g_q^{(N)}\left(t_{0k}^n\right),g_p^{(N)}\left(t_{0j}^n\right)\right)\overset{p=q}{=}B\frac{\beta_p^N-\alpha_p^N}{2}\sqrt{1-(t_{0j}^n)^2}\delta_{k,j}-\frac{(1-(-1)^{j+k})(1-(t_{0k}^n)^2)}{(t_{0k}^n-t_{0j}^n)^2}\frac{1}{n}+\frac{n}{2}+$$

$$\frac{\kappa^2}{2}\left(\frac{\beta_p^N-\alpha_p^N}{2}\right)^2\left(\frac{-1}{n}\right)\left(1-(t_{0k}^n)^2\right)\left[\ln 2+2\sum_{r=1}^{n-1}\frac{T_r(t_{0k}^n)}{r}T_r(t_{0j}^n)+\frac{(-1)^k}{n}T_r(t_{0j}^n)\right]+$$

$$\frac{1}{n}Q_{1,qp}^{N,n-2,n-2}(g_q^{(N)}(t_{0k}^n),g_p^{(N)}(t_{0j}^n))\left(1-(t_{0k}^n)^2\right),$$

$$M_{2,qp}^{N,n-2,n-2}\left(g_q^{(N)}\left(t_{0k}^n\right),g_p^{(N)}\left(t_{0j}^n\right)\right)\overset{p=q}{=}\frac{\beta_p^N-\alpha_p^N}{2}\sqrt{1-(t_{0j}^n)^2}\delta_{k,j}-B\left(\frac{\beta_p^N-\alpha_p^N}{2}\right)^2\left(\frac{-1}{n}\right)\left(1-(t_{0k}^n)^2\right)\times \tag{20}$$

$$\left[\ln 2+2\sum_{r=1}^{n-1}\frac{T_r(t_{0k}^n)}{r}T_r(t_{0j}^n)+\frac{(-1)^k}{n}T_r(t_{0j}^n)\right]+\frac{1}{n}Q_{2,qp}^{N,n-2,n-2}(g_q^{(N)}(t_{0k}^n),g_p^{(N)}(t_{0j}^n))\left(1-(t_{0k}^n)^2\right),$$

$$M_{i,qp}^{N,n-2,n-2}\left(g_q^{(N)}\left(t_{0k}^n\right),g_p^{(N)}\left(t_{0j}^n\right)\right)\overset{p\neq q}{=}\frac{1}{n}Q_{i,qp}^{N,n-2,n-2}(g_q^{(N)}(t_{0k}^n),g_p^{(N)}(t_{0j}^n))\left(1-(t_{0k}^n)^2\right).$$

The final system of linear equations (19) does not contain any integrals, unlike projection methods Galerkin or collocation. Therefore, the simple implementation, after appropriate analytical transformations have the advantage of this approach.

After evaluate the values of unknown function in node points $v_{i,q}^{N,n-2}\left(t_{0k}^n\right)$, we have to return for calculating unknown coefficients (5) in discrete form using the same aproach as for system (19)

$$B_i^N\left(\lambda\right)=\frac{1}{2}\sum_{q=1}^{2^N}\left(\frac{\beta_q^N-\alpha_q^N}{2}\right)^2\sum_{k=1}^{n-1}v_{i,q}^{N,n-2}\left(t_{0k}^n\right)\left(1-\left(t_{0k}^n\right)^2\right)\exp^{-i\lambda g_q^{(N)}(t_{0k}^n)}. \tag{21}$$

And looking up to formula (5) and (2) we can evaluate the scattering and diffraction fields for the magnetic field component of the total field (1). As good knowing if found a component of the magnetic field thus it is allow to restore electric component and can evaluate the total field using Maxwell equations.

4. Numerical results

Numerical results of an applied mathematics problems are most important while we can choose the true and right input data. It will give us new knowledge for future scientific investigations. For this reason in current paper as input data we use previous results from papers [18], [19] which was calculated in related field but their mathematical model was based on IEs with logarithmic and singular singularities. In work [18] was shown main numerical results for superconducting strips from Niobium, Stannum, Plumbum and for impedance strips from constantan. Last material is efficient as for practical use in modern mobile devices because this is copper-nickel alloy, which is characterized by weak dependence of electrical resistance from temperature. That it is used in Fraktus company which produces fractal antenna. In paper [19] was performed scientific computing for PEC strips and investigated the surface charge density, relief distribution of the total field for different incident angels, RPs, scattered and total fields for different frequencies and orders of pre-Cantor gratings.

Obtained results from system (19) we use for calculating electromagnetic characteristics such as radiation pattern in far field, diffraction pattern in near field and main as total and scattered fields in near zone.

There is shown on Fig. 2 and Fig. 3 radiation pattern (RPs) and diffraction patterns depending on the incident angles and how to change the magnetic component of total field.

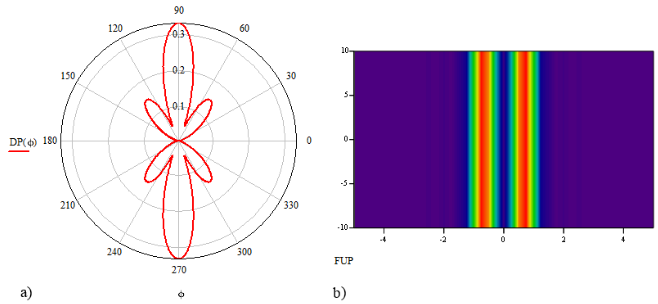

Figure 2: Scattered fields for molybdenum strips where N=3, l=0.02m, f=15 GHz, α=0: a) far-field radiation pattern, b) near-field diffraction pattern

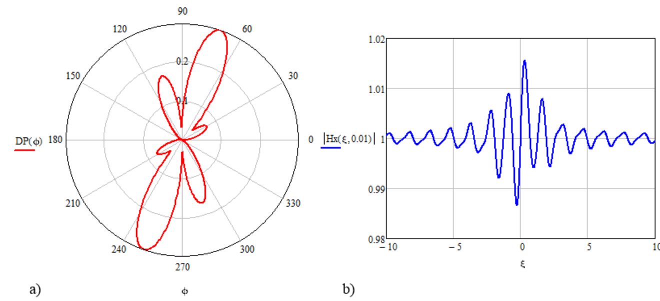

Figure 3: Magnetic fields for molybdenum strips where N=3, l=0.02m, f=15GHz, α=20: a) far-field radiation pattern of the scattered field, b) near-field of the total field

The electrical resistivity and conductivity of copper (Cu), aluminum (Al), magnesium (Mg), molybdenum (Mo), nickel (Ni) have fall values from Cu to Ni. This tendency we can see for total field for different values of impedance pre-Cantor strips as shown on Fig. 4. Yet stay in incident angle by 20 and change the frequency to 11.2 GHz, these computational results in diffraction pattern are present on Fig. 5.

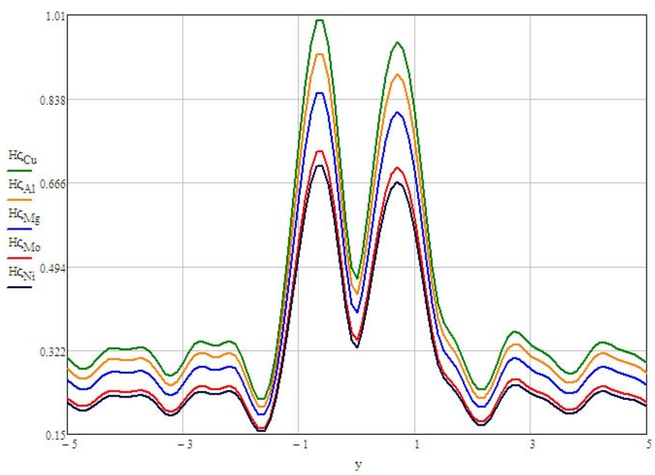

Figure 4: Normalized near zone of the total field depending on different impedance strips, where N = 3, α = 20, l = 0.02m, f = 15 GHz

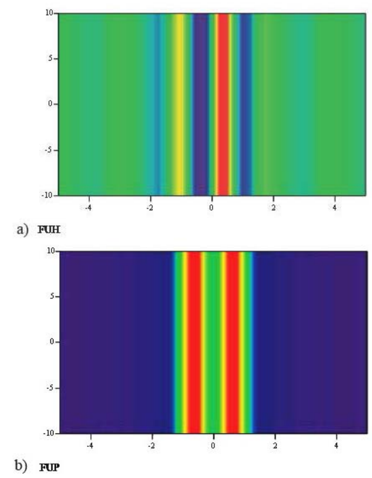

Figure 5: Near-field diffraction pattern for the a) total and b) scattered fields for molybdenum strips where N=3, l=0.02m, f=11.2 GHz, α=20

The magnetic field components was illustrated graphically in near and far zone on Fig. 6.

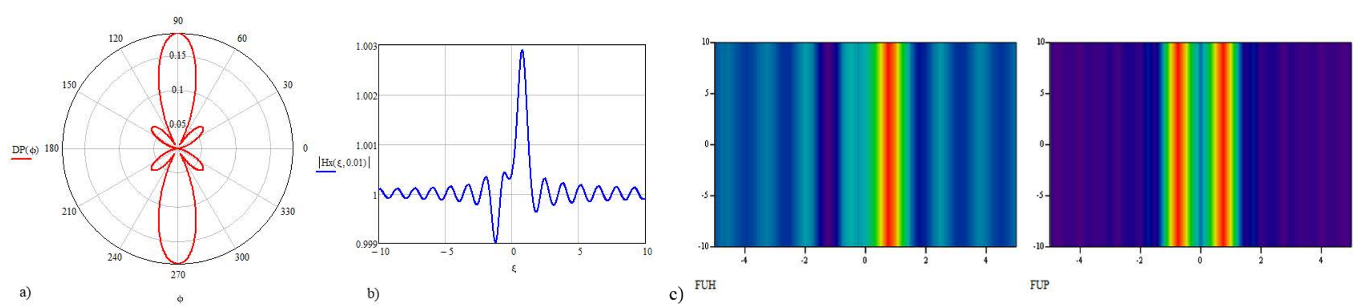

Figure 6: Magnetic fields for molybdenum strips where N=3, l=0.02 m, f=11.2 GHz, α=0: a) radiation pattern of the scattered field, b) near field of the total field, c) diffraction pattern for total and scattered fields

Electrodynamic analysis program of FEKO was used for comparison and reliability of obtained computational results while the recommending of input data (Fig 7).

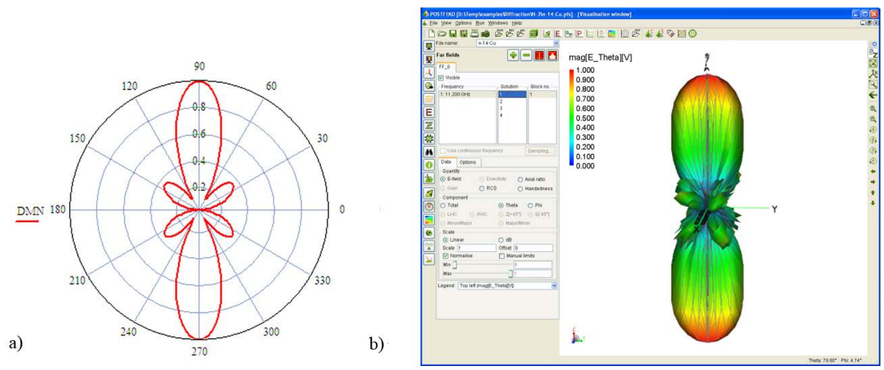

Figure 7: Compare RPs for cooper strips where f=11.2 GHz, N=3, l=0.01 m, α=0, calculated by the: a) created program; b) program FEKO

The comparing the calculated results was demonstrated for the scattering problems based on HSIEs and SIEs. Time obtaining of the numerical results for the total field values in near field where N=3 (Table 1) based on HSIE is 1 min., and based on SIE 5 min. Note, that physical statement of the problem is the same but the mathematical models are different. The results coincide with the average accuracy $2 \cdot 10^{-8}$.

Table 1: Compare the component of the total field based on mathematical models on SIEs and HSIEs where N=3, l=0.02 m, f=11.2 GHz, molybdenum strips

ξ	**SIE** $H_\varsigma(\xi, 0.1)$	**HSIE** $H_\varsigma(\xi, 0.1)$	Δ
-3	$0.8931587157 - 0.4479097861i$	$0.8931586946 - 0.4479098279i$	$2.11e-8+4.18e-8i$
-2.4	$0.8899082031 - 0.4575927905i$	$0.8899081817 - 0.4575928324i$	$2.14e-8+4.19e-8i$
$-1,8$	$0,8954471217 - 0,4445156149i$	$0,8954471008 - 0,4445156568i$	$2.09e-8+4.19e-8i$
-1.2	$0.879840424 - 0.4688726682i$	$0.8798404017 - 0.4688727095i$	$2.23e-8+4.13e-8i$
-0.6	$0.880254173 - 0.4946200733i$	$0.8802541509 - 0.4946201148i$	$2.21e-8+4.15e-8i$
0	$0.8916430486 - 0.4654700071i$	$0.8916430274 - 0.4654700489i$	$2.12e-8+4.18e-8i$
0.6	$0.9005422001 - 0.4953638328i$	$0.9005421799 - 0.4953638744i$	$2.02e-8+4.16e-8i$
1.2	$0.9024421723 - 0.4698703838i$	$0.9024421521 - 0.4698704252i$	$2.02e-8+4.14e-8i$
1.8	$0.8890329728 - 0.444327671i$	$0.8890329512 - 0.4443277128i$	$2.16e-8+4.18e-8i$
2.4	$0.8936472402 - 0.4576809538i$	$0.8936472192 - 0.4576809957i$	$2.1e-8+4.19e-8i$
3	$0.8909997859 - 0.4478734168i$	$0.8909997646 - 0.4478734586i$	$2.13e-8+4.18e-8i$

5. Conclusions

Some difficulties need to resolve in order to perform a discretization of the HSIEs and the Fredholm IEs of the second kind. And within this investigation was shown how can resolve their. A discrete mathematical model for the pre-fractal plane-parallel diffraction structure has been investigated and derived. The results, discribed in this paper, allowed to do the computational analysis of such IEs for investigating the total, scattered and the diffracted fields on the pre-Cantor plane-parallel structure. We can conclude that

- ✔ mathematical model based on HSIEs of the 2-nd kind was used for impedance pre-Cantor boundary gratings which are equivalent to corresponding boundary-value problem for the Helmholtz equation in TM cases using the method of parametric representations of corresponding integral operators;
- ✔ computational model have been created on this base of boundary IEs using discrete singularities method with specific quadrature formulas for scattering problem of a plane wave on boundary pre-Cantor impedance gratings;
- ✔ algorithm and software have been created for performing the numerical experiments based on the developed models from pre-Cantor structures with regard to calculate important electrodynamics characteristics;

- ✓ scientific computing have been performed by developed software for considered diffraction problem on pre-Cantor plane-parallel structure for calculate EM fields, RPs, diffraction patterns of scattered fields in near zone depending on the different frequency EM waves, the incident angle, the grating pre-Cantor order, the different values of impedance metals;
- ✓ results of computational experiments have been investigated and analyzed for pre-Cantor structures with regard to compare the solutions based on HSIEs and SIEs, between created software and software of electrodynamic analysis FEKO.

The electromagnetic scattering on complex and difficult structures has become very interesting and important for in-depth investigations and their subsequent analysis. Considering this importance, the relevant calculations and numerical experiments for 3D such structures are planned to be carried out using the same approach with some modification of quadrature method for hypersingular IEs.

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