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Research paper

Derivations of Second Type of Algebra of Second Class Filiform Leibniz Algebras of Dimension Derivation $(n+2)$

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Abstract

In this article, the derivations of second type of algebra from the second class filiform Leibniz algebras of dimension derivation $(n+2)$ is discussed. From the description of the derivations.It is found that the basis of the space $Der(L_n(b))$ of the algebra.

Keywords: *Filiform Leibniz algebra, Leibniz algebra, gradation, natural gradation, derivation.*

1. Introduction

Many mathematicians have actively studied the theory of Lie algebra for years. J.-L. Loday was the first Mathematician who introduces Leibniz algebra In (1993). Leibniz algebra is a generation of Lie algebra, and it is very common that any associative algebra motivates Lie algebra and Leibniz of algebras $[x,y] = xy - yx$. Actually, it was J.-L. Loday who suggested this new notion of algebra, which was one of the second class filiform Leibniz algebras. This paper discusses the low dimension of algebras. It aims to get the basis of the space $Der(L_n(b))$. By discussing the table of algebra from dimension 5 to 16,this paper will also try to discover the basis of the derivation of this algebra and the link between the algebra and its derivations.

2. Preliminary Results

Some definitions and essential results, needed in this paper are prrsented.

Definition 2.1. *(see[6]) An algebra L over a field K is called a Leibniz algebra if its bilinear operation $[.,.]$ satisfies the following Leibniz identity:*

$$[x,[y,z]] = [[x,y],z] - [[x,z],y],\ for\ any\ x,y,z \in L.$$

in other word, all algebras are assumed to be over the fields of complex numbers $\mathbb{C}$. Now let L be a Leibniz algebra. We put: $L^1 = L, L^{k+1} = [L^k, L], k \geqslant 1$.

Definition 2.2. *(see[8]) A Leibniz algebra L is said to be nilpotent if there exists $s \in N$ such that $L^1 \supset L^2 \supset \cdots \supset L^s = 0$.*

Definition 2.3. *(see[1]) A Leibniz algebra L is said to be filiform if $dimL^i = n - i$, where $n = dimL$ and $2 \leqslant i \leqslant n$.*

Definition 2.4. *(see[7])*
A k-linear transformation d of an algebra L is called a derivation of L if

$$d([x,y]) = [d(x),y] + [x,d(y)] \quad \text{for all } x,y \in L.$$

The set of all derivations of an algebra L is denoted by $Der(L)$. We also, denote by Leibn the set of all (n+1)-dimensional filiform Leibniz algebras. We now look at the following theorem from (see[3])which splits the set of fixed dimension filiform Liebniz algebras into three disjoint subsets.However we just take the result of this theorem regarding only $SLeib_{n+1}$.

Theorem 2.1. *Any $(n+1)$-dimensional complex filiform Leibniz algebra L admits a basis $e_0, e_1, \cdots, e_n$ called adapted, such that the table of multiplication of L has the following forms, where non defined products are zero:*

$$SLeib_{n+1} = \begin{cases} [e_0,e_0] = e_2, \\ [e_i,e_0] = e_{i+1}, \quad i \in [\![2;n-1]\!], \\ [e_0,e_1] = \sum\limits_{i=3}^{n} \beta_i\, e_i \\ [e_1,e_1] = \gamma e_{n-1} \\ [e_j,e_1] = \sum\limits_{i=3}^{n} \beta_i\, e_{j+i-1}, j \in [\![2;n-2]\!]. \end{cases}$$

for $\beta_3, \beta_4, \cdots, \beta_n, \beta \in \boldsymbol{C}$.

Lemma 2.1. *[7] Let $d \in Der(L_n)$. In this case $d = \sum\limits_{i=0}^{n-1} d_i$ where $d_k \in End(L_n)$ and $d_k(L_i) \subseteq L_{i+k}$ for $i \in [\![1;n]\!]$.*

Proof. Consider the natural filtration S_i of the algebra L_n. It can be easely seen from [(see[7], Corollary 1)] that this filtration coincides with the natural filtration of the algebra L_n. There for, $S_k = \bigoplus_{i \geq k} L_n$ and $d(S_i) \subseteq S_i$ by [(see[7],Corollary 3)] So,

$$d \in F_0Z^1(L_n,L_n) \text{ implies } d_k \in End(L_n) \text{ and } d_k(L_i) \subseteq L_{i+k}.$$

This proves the lemma. □

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3. Algebra $L_n(b)$ of the Second Class Filiform Leibniz Algebras

We denote by $L_n(b)$ an algebra of second class filiform Leibniz algebras, and it is defined by

$$L_n(b) = \begin{cases} [e_0, e_0] = e_2, \\ [e_i, e_0] = e_{i+1}, \quad i \in [\![2, n-1]\!], \\ [e_1, e_1] = e_n \, . \end{cases}$$

Where ⟦n; m ⟧ denotes all integers between n and m. Tables (1), (2) and (3) (see Appendix) presents the equation (dim Der), number equation of derivation and dim Der of Algebra in Low dimension:
Remarks:
In this work, we add some points to give more information about our study.

- A basis of $Der(L_n(b))$ can be found and this will be the main purpose of the next pages.
- Based on, $\dim Der(L_n(b)) = n+2$, $n \geqslant 4$ the dimension of Der can be computed.
- According to Tables 1,2 and 3, we suggest that the number of equations follows the rule:

  Number of equation $= \dfrac{(n+1)(n+2)}{2} + n.$
- In the next we will present some Lemma and proposition that are the main task of our paper.

Definition 3.1. *Let $n \geqslant 4$, $X = \sum_{i=0}^{n} \lambda_i e_i$ we denote by*

$$t_1(X) = \sum_{i=0}^{n} \lambda_i t_1(e_i)$$

where,

$$\begin{cases} t_1(e_0) = e_0, \\ t_1(e_1) = \frac{n}{2} e_1, \\ t_1(e_i) = i e_i \quad , i \in [\![2; n-1]\!]. \end{cases}$$

and we define d_0 such that

$$d_0(e_i) = \begin{cases} \alpha_0 e_0, \quad i = 0, \\ \alpha_1 \frac{n}{2} e_1 \quad i = 1, \\ \alpha_i i e_i, \quad i \in [\![2, n]\!]. \end{cases} \tag{1}$$

Lemma 3.1. *We have $d_0 = \alpha_0 t_1$.*

Proof. Consider $d_0 \in Der(L_n(b))$ which is defined by where

α_0, α_1, *and* α_i $i \in [\![2, n]\!]$ *are parameters.*

Consider the family of derivations

$d_0([e_i, e_j]) = [d_0(e_i), e_j] + [e_i, d_0(e_j)], \quad i, j = 0, 1, 2, \cdots, n-1.$

We now look at the problem case by case. In each case, we repeatedly use algebra $L_n(b)$ and (1).

- First Case: if $(i, j) = (0, 0)$

then

$d_0([e_0, e_0]) = [d_0(e_0), e_0] + [e_0, d_0(e_0)]$

so,

$d_0(e_2) = [\alpha_0 e_0, e_0] + [e_0, \alpha_0 e_0]$

which implies

$2\alpha_2 e_2 = \alpha_0 e_2 + \alpha_0 e_2$

thus

$$\alpha_2 = \alpha_0. \tag{2}$$

- Second Case: if $(i, j) \in ([\![2; n]\!], 0)$

then

$$d_0([\sum_{i=2}^{n} e_i, e_0]) = [d_0(\sum_{i=2}^{n} e_i), e_0] + [\sum_{i=2}^{n} e_i, d_0(e_0)]$$

so that,

$$d_0(\sum_{i=2}^{n-1} e_{i+1}) = [\sum_{i=2}^{n} \alpha_i e_i, e_0] + [\sum_{i=2}^{n} e_i, \alpha_0 e_0]$$

which implies

$$\sum_{i=2}^{n-1} \alpha_{i+1}(i+1) e_{i+1} = \sum_{i=2}^{n-1} i\alpha_i e_{i+1} + \sum_{i=2}^{n-1} \alpha_0 e_{i+1}$$

$$\sum_{i=2}^{n-1} \alpha_{i+1}(i+1) e_{i+1} = \sum_{i=2}^{n-1} (i\alpha_i + \alpha_0) e_{i+1}$$

and so,

$$(i+1)\alpha_{i+1} = i\alpha_i + \alpha_0, \quad i \in [\![2; n-1]\!]. \tag{3}$$

If i=2 in 3 then $3\alpha_3 = 2\alpha_2 + \alpha_0$ By (2) we obtain

$$\alpha_3 = \alpha_2 \tag{4}$$

If i=3 in 3 then $4\alpha_4 = 3\alpha_3 + \alpha_0$ By (2 and 4) we obtain

$$\alpha_4 = \alpha_3 \tag{5}$$

Similarly,
If i=n-1 in 3 then

$$(n)\alpha_n = (n-1)\alpha_{n-1} + \alpha_0$$

but $\alpha_0 = \alpha_{n-1}$
Thus,

$$\alpha_n = \alpha_{n-1} \tag{6}$$

From (2),(4),(5)and (6) we obtain

$$\alpha_0 = \alpha_2 = \alpha_3 = \alpha_4 = \cdots = \alpha_n. \tag{7}$$

Thus

$$\begin{aligned} d_0(\sum_{i=0}^{n} \lambda_i e_i) &= d_0(\lambda_0 e_0) + d_0(\lambda_1 e_1) + d_0(\lambda_2 e_2) + \sum_{i=3}^{n-1} d_0(\lambda_i e_i) \\ &= \lambda_0(\alpha_0 e_0) + \lambda_1(\alpha_1 \tfrac{n}{2} e_1) + 2\lambda_2(\alpha_2 e_2) + \sum_{i=3}^{n-1} i\lambda_i(\alpha_i e_i) \\ by(1) & \\ &= \lambda_0(\alpha_0 e_0) + \lambda_1(\alpha_1 \tfrac{n}{2} e_1) + \sum_{i=2}^{n-1} i\lambda_i(\alpha_i e_i) \\ &= \alpha_0[\lambda_0 e_0 + \lambda_1 \tfrac{n}{2} e_1 + \sum_{i=2}^{n-1} i\lambda_i e_i] \quad by(7) \\ &= \alpha_0 t_1 \, . \end{aligned}$$

From this we have

$$t_1(e_0) = e_0, \quad t_1(e_1) = \frac{n}{2} e_1 \text{ and } t_1(e_i) = i e_i, \quad i \in [\![2; n-1]\!]. \tag{8}$$

from this we have our result □

Lemma 3.2. *Consider $d_k \in Der(L_n(b))$, $k \in [\![1, n-2]\!], n \geqslant 3$ such that d_k is defined by*

$$d_k(e_i) = \begin{cases} \lambda_0 e_k, \quad i = 0, \quad k \in [\![1, n]\!], \\ \lambda_i e_{k+i-1}, \quad i \in [\![2, n-k]\!]. \end{cases} \tag{9}$$

where λ_0 and λ_i, $i \in [\![1, n-k]\!]$, are scalars.
Then $d_k(e_0) = e_k$, $d_k(e_i) = e_{i+k-1}$, $i \in [\![2, n-k]\!]$, and $k \in [\![1, n-2]\!]$.

Proof. Consider the family of derivations

$d_k([e_i,e_j]) = [d_k(e_i),e_j]+[e_i,d_k(e_j)]$.

Similarly, we calculate case by case.In which we repeatedly $L_n(a)$ and (9).

Case 1 : if $i = 0, j = 0$ then

$$\sum_{k=1}^{n-2} d_k([e_0,e_0]) = [\sum_{k=1}^{n-2} d_k(e_0),e_0]+[e_0,\sum_{k=1}^{n-2} d_k(e_0)].$$

Then

$$\sum_{k=1}^{n-2} d_k(e_2) = [\sum_{k=1}^{n-2} \lambda_0\, e_k,e_0]+[e_0,\sum_{k=1}^{n-2} \lambda_0\, e_k]$$

which implies

$$\sum_{k=1}^{n-2} \lambda_2\, e_{k+1} = \sum_{k=1}^{n-2} \lambda_0\, e_{k+1}$$

to obtain

$$\lambda_2 = \lambda_0. \tag{10}$$

Case 2 : if$(i, j) = ([\![1,n-1]\!],0)$ and $k \in [\![1,n-2]\!]$, then

$$\sum_{k=1}^{n-2}\sum_{i=1}^{n-1} d_k([e_i,e_0]) = [\sum_{k=1}^{n-2}\sum_{i=1}^{n-1} d_k(e_i),e_0]+[\sum_{i=1}^{n-1} e_i,\sum_{k=2}^{n-1} d_k(e_0)]$$

$$\sum_{k=1}^{n-2}\sum_{i=2}^{n-2} d_k(e_{i+1}) = [\sum_{k=1}^{n-2}\sum_{i=1}^{n-1} \lambda_i\, e_{k+i-1},e_0]+[\sum_{i=1}^{n-1} e_i,\sum_{k=1}^{n-2} d_k(e_0)]$$

$$\sum_{k=1}^{n-2}\sum_{i=1}^{n-2} \lambda_{i+1}(e_{i+k}) = [\sum_{k=1}^{n-2}\sum_{i=1}^{n-1} \lambda_i\, e_{k+i-1},e_0]+[\sum_{i=1}^{n-1} e_i,\sum_{k=1}^{n-2} \lambda_0(e_k)]$$

$$\sum_{k=1}^{n-2}\sum_{i=1}^{n-2} \lambda_{i+1}e_{i+k} = \sum_{k=1}^{n-2}\sum_{i=1}^{n-2} \lambda_i\, e_{k+i}$$

Thus,

$$\lambda_{i+1} = \lambda_i. \tag{11}$$

From (11),
if $i = 2$ then $\lambda_3 = \lambda_2$
also,
if $i = 3$ then $\lambda_4 = \lambda_3$
similarly,
if $i = n-1$ then $\lambda_n = \lambda_{n-1}$.

From (10) and (11) this implies

$$\lambda_0 = \lambda_2 = \lambda_3 = ... = \lambda_{n-1} = \lambda_n \tag{12}$$

and hence,

$$\begin{aligned} d_k(\sum_{i=0}^{n} \alpha_i\, e_i) &= d_k(\alpha_0\, e_0)+d_k(\alpha_1\, e_1)+\sum_{i=2}^{n} d_k(\alpha_i\, e_i) \\ &= \alpha_0(\lambda_0\, e_k)+\sum_{i=2}^{n} \lambda_i(\alpha_i\, e_{k+i-1}) \quad by(9) \\ &= \alpha_0(\lambda_0\, e_k+\sum_{i=2}^{n-k} \lambda_i\, e_{k+i}) \quad by(7)\ . \end{aligned}$$

Thus,

$$d_k(e_0) = e_k \text{ and } d_k(e_i) = e_{k+i}, i \in [\![2,n-k]\!]. \tag{13}$$

Thus we have our result. ☐

Lemma 3.3.

$$\textit{Let } X = \sum_{i=0}^{n} \lambda_i e_i$$

Then, $t_2(X) = \sum_{i=0}^{n} \lambda_i t_2(e_i)$*, where* $\delta_0^i e_n$ *and* δ_0^i *is the Kronecker symbol.*

Proof. Consider $d_{n-1} \in Der(L_n(a))$ where d_{n-1} is defined by

$$d_{n-1}(e_i) = \begin{cases} \pi_0\, e_n, & i = 0 \\ 0; & i \neq 0 \end{cases} \tag{14}$$

for scalar π_0.
Consider the family of derivations

$d_{n-1}([e_i,e_j]) = [d_{n-1}(e_i),e_j]+[e_i,d_{n-1}(e_j)]$.

Case 1 : if $i = 0, j = 0$ then

$d_{n-1}([e_0,e_0]) = [d_{n-1}(e_0),e_0]+[e_0,d_{n-1}(e_0)]$
by $L_n(b)$ and (13), thus

$d_{n-1}(e_2) = [\pi_0\, e_n,e_0]+[e_0,\pi_0\, e_n]$.
If $\pi_0 \neq 0$ then

$0 = 0+0$.

Hence

$$\begin{aligned} d_{n-1}(\sum_{i=0}^{n} \lambda_i\, e_i) &= d_{n-1}(\lambda_0\, e_0)+d_{n-1}(\lambda_1\, e_1)+\sum_{i=2}^{n} d_{n-1}(\lambda_i\, e_i) \\ &= \lambda_0(\pi_0\, e_n) \quad by(14) \\ &= \pi_0(\lambda_0\, e_n) \\ &= \pi_0\, t_2 \end{aligned}$$

and we obtain

$$t_2(e_0) = e_n. \tag{15}$$

And here we obtain the result. ☐

Lemma 3.4. *The mappings t_1, t_2 and d_k for $1 \leq k \leq (n-2)$ are linearly independent.*

Proof. Consider that

$$\alpha_1 t_1(e_i) + \alpha_2 t_2(e_i) + \sum_{k=1}^{n-2} \beta_k d_k(e_i) = 0 \tag{16}$$

where $e_i \in L_n(a), i = 0,1,2,3,...,n-1$.
We will show that $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = ... = \beta_k = 0$ for $1 \leq k \leq n-2$.

$$\sum_{i=0}^{n-1} [\alpha_1 t_1(e_i) + \alpha_2 t_2(e_i) + \sum_{k=1}^{n-2} \beta_k d_k(e_i)]$$

$$= \sum_{i=0}^{n-1} [\alpha_1 t_1(e_i) + \alpha_2 t_2(e_i) + \beta_1 d_1(e_i) + \beta_2 d_2(e_i) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_i) + \beta_{n-2} d_{n-2}(e_i)]$$

$$= \alpha_1 t_1(e_0) + \alpha_2 t_2(e_0) + \beta_1 d_1(e_0) + \beta_2 d_2(e_0) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_0) + \beta_{n-2} d_{n-2}(e_0)$$

$$+ [\alpha_1 t_1(e_1) + \alpha_2 t_2(e_1) + \beta_1 d_1(e_1) + \beta_2 d_2(e_1) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_1) + \beta_{n-2} d_{n-2}(e_1)]$$

$$+ [\alpha_1 t_1(e_2) + \alpha_2 t_2(e_2) + \beta_1 d_1(e_2) + \beta_2 d_2(e_2) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_2) + \beta_{n-2} d_{n-2}(e_2)]$$

$$+ [\alpha_1 t_1(e_3) + \alpha_2 t_2(e_3) + \beta_1 d_1(e_3) + \beta_2 d_2(e_3) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_3) + \beta_{n-2} d_{n-2}(e_3)]$$

$$+ [\alpha_1 t_1(e_4) + \alpha_2 t_2(e_4) + \beta_1 d_1(e_4) + \beta_2 d_2(e_4) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_4) + \beta_{n-2} d_{n-2}(e_4)]$$

$$+ ...$$

$$+ ...$$

$$+ [\alpha_1 t_1(e_{n-2}) + \alpha_2 t_2(e_{n-2}) + \beta_1 d_1(e_{n-2}) + \beta_2 d_2(e_{n-2}) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_{n-2}) + \beta_{n-2} d_{n-2}(e_{n-2})]$$

$$+ [\alpha_1 t_1(e_{n-1}) + \alpha_2 t_2(e_{n-1}) + \beta_1 d_1(e_{n-1}) + \beta_2 d_2(e_{n-1}) + ...$$

$$+ \beta_{n-3} d_{n-3}(e_{n-1}) + \beta_{n-2} d_{n-2}(e_{n-1})]$$

$$= 0.$$

This implies

$$(\alpha_1(e_0 + (2n-5)e_1) + (\alpha_1(n-1) + 2\beta_1)e_2 + (n\alpha_1 + 2\beta_2 + \beta_1)e_3$$

$$+ ((n+1)\alpha_1 + 2\beta_3 + \beta_2 + \beta_1)e_4$$

$$+ ((n+2)\alpha_1 + 2\beta_4 + \beta_3 + \beta_2 + \beta_1)e_5 + ...$$

$$+ (2(n-2)\alpha_1 + 2\beta_{n-2} + \beta_{n-1} + ... + \beta_3 + \beta_2 + \beta_1)e_{n-1}$$

$$+ ((n-1)(\alpha_1 + 3) + \alpha_2 + 2\beta_{n-2} + \beta_{n-1} + ... + \beta_3 + \beta_2 + \beta_1)e_n$$

$$= 0$$

Here we have these following results:

1) $\alpha_1 e_1 = 0$ which implies $\alpha_1 = 0$.

2) $(\alpha_1(n-1) + 2\beta_1)e_2 = 0$ which implies $\alpha_1(n-1) + 2\beta_1 = 0$

but since $\alpha_1 = 0$then $\beta_1 = 0$.

3) $(n\alpha_1 + 2\beta_2 + \beta_1)e_3 = 0$ which implies $n\alpha_1 + 2\beta_2 + \beta_1 = 0$

but since $\alpha_1 = \beta_1 = 0$ then $\beta_2 = 0$.

4) $((n-1)\alpha_1 + 2\beta_3 + \beta_2 + \beta_1)e_4 = 0$

which implies $(n-1)\alpha_1 + 2\beta_3 + \beta_2 + \beta_1 = 0$

but since $\alpha_1 = \beta_1 = \beta_2 = 0$ then $\beta_3 = 0$.

5) $((n-2)\alpha_1 + 2\beta_4 + \beta_2 + \beta_3 + \beta_1)e_5 = 0$

which implies $((n-2)\alpha_1 + 2\beta_4 + \beta_2 + \beta_3 + \beta_1) = 0$

but since $\alpha_1 = \beta_1 = \beta_2 = \beta_3 = 0$ then $\beta_4 = 0$.

Similarly,

6) $(2(n-2)\alpha_1 + 2\beta_{n-2} + \beta_{n-1} + \beta_n + ... + \beta_1)e_{n-1} = 0$

which implies $(2(n-2)\alpha_1 + 2\beta_{n-2} + \beta_{n-1} + \beta_n + ... + \beta_1) = 0$

but since $\alpha_1 = 2\beta_{n-2} = \beta_{n-1} = ... = \beta_1 = 0$ then $\beta_{n-2} = 0$.

7) $((n-2)(\alpha_1 + 3) + \alpha_2 + 2\beta_n + \beta_{n-1} + \beta_n + ... + \beta_1)e_n = 0$ which implies

$((n-2)(\alpha_1 + 3) + \alpha_2 + 2\beta_{n-1} + \beta_{n-1} + \beta_n + ... + \beta_1) = 0$

but since $\alpha_1 = \beta_{n-1} = \beta_{n-2} = \beta_{n-3} = ... = \beta_2 = \beta_1 = 0$,

then

$\alpha_2 = 0$.
From above we will obtain

$$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = ... = \beta_{n-1} = 0. \tag{17}$$

Those prove the mappings are linearly independent. □

Lemma 3.5. *The linear mappings $d_0, d_k, d_{n-1} \in Der(L_n(b)), 1 \leq k \leq n-2$ defined by (1), (9) and (14) are linearly composition.*

Proof. Let

$$x = \eta_0 e_0 + \eta_1 e_1 + \eta_2 e_2 + \eta_3 e_3 + \eta_4 e_4 + ... + \eta_{n-1} e_{n-1}.$$

First we observe that by (1), (9) and (14),

$$d_k(x) = \begin{cases} 2\eta_0 (\alpha_1 e_1) + \eta_1 (\alpha_0 (n-1))e_1 + \sum_2^n \alpha_i (\eta_i i e_i) \\ + \sum_{k=1}^{n} \eta_0 \lambda_0 e_k + \sum_{k=2}^{n-1} \sum_{i=2}^{n-1} \lambda_i \eta_i e_{k+i-1} + \eta_0 \pi_0 e_n. \end{cases} \quad (18)$$

Hence by using (7) and (12),

$d(x) = \alpha_0 [\eta_0 (e_1) + \eta_1 (n-1) e_1 + \sum_2^n \eta_i i e_i]$

$+ \lambda_0 [\sum_{k=1}^{n} \eta_0 e_k + \sum_{k=2}^{n-1} \sum_{i=2}^{n-1} \eta_i e_{k+i-1}] + \pi_0 \eta_0 e_n.$

Thus

$$d = \alpha_0 t_1 + \pi_0 t_2 + \lambda_0 d_k \;\; for \;\; 1 \le k \le n-2.$$

This prove linear composition of the mappings. □

Lemma 3.6. *The mappings t_1, t_2, d_k for $1 \le k \le n-2$ are derivations.*

Proof. Consider that

$$x = \alpha_0 e_0 + \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 + ... + \alpha_{n-1} e_{n-1}$$

and

$$y = \beta_0 e_0 + \beta_1 e_1 + \beta_2 e_2 + \beta_3 e_3 + ... + \beta_{n-1} e_{n-1}$$

Then

$$x.y = \beta_0(\alpha_0 .e_2 + \alpha_2 e_3 + \alpha_3 e_4 + ... + (\alpha_{n-1} + \alpha_1)e_n$$

and so,

$$t_1(x.y) = \beta_0(2\alpha_0 e_2 + 3\alpha_2 e_3 + 4\alpha_3 e_4 + ... + n(\alpha_{n-1} + \alpha_1) e_n \quad (19)$$

Thus,

$$t_1(x) = \alpha_0 e_0 + \alpha_1 (n-1) e_1 + 2\alpha_2 e_2 + 3\alpha_3 e_3 + ... + \alpha_{n-1}(n-1) e_{n-1}$$

Hence

$$t_1(x).y = \beta_0 [\alpha_0 e_2 + 2\alpha_2 e_3 + 3\alpha_3 e_4 + ... + (n-1)(\alpha_{n-1} + \alpha_1) e_n] \quad (20)$$

and thus,

$$t_1(y) = \beta_0 e_0 + \beta_1 (n-1) e_1 + 2\beta_2 e_2 + 3\beta_3 e_3 + ... + \beta_{n-1}(n-1) e_{n-1}$$

Therefore,

$$x.t_1(y) = \beta_0 [\alpha_0 e_2 + \alpha_2 e_3 + ... + (\alpha_{n-1} + \alpha_1) e_n] \quad (21)$$

By adding (20) to (21) we will obtain (19). This implies t_1 is a derivation.
We now show that t_2 is also a derivation .

$$t_2(x) = \alpha_0 e_n$$
$$t_2(y) = \beta_0 e_n$$

From easy calculation we have $t_2(x).y = 0$, $x.t_2(y) = 0$ and $t_2(x.y) = 0$

and thus t_2 is a derivation.
Now, since

$$d_k(x) = \sum_{k=1}^{n-1} [\alpha_0 d_k(e_0) + \alpha_1 d_k(e_1) + \alpha_2 d_k(e_2) + ... + \alpha_{n-1} d_k(e_{n-1})]$$

thus,

$$\sum_{k=1}^{n-1} d_k(x) = \sum_{k=1}^{n-1} [\alpha_0 e_k + \sum_{i=2}^{n-k} \alpha_i e_{k+i-1}]$$

then,

$$[d_k(x).y] = \beta_0 [\sum_{k=1}^{n-1} (e_{k+1} + \sum_{k=1}^{n-1} (\alpha_2 e_{k+2} + \alpha_3 e_{k+3} + ... + \alpha_{n-k} e_{n-k}))]. \quad (22)$$

In addition,

$$\sum_{k=1}^{n-1} d_k(y) = \sum_{k=1}^{n-1} [\alpha_0 e_k + \sum_{i=2}^{n-k} \alpha_i e_{k+i-1}]$$

and we have

$$[x, \sum_{k=1}^{n-1} d_k(y)] = 0 \quad (23)$$

Also,

$$\sum_{k=1}^{n-1} d_k(x.y) = \sum_{k=1}^{n-1} \beta_0 (\alpha_0 e_{k+1} + \alpha_2 e_{k+2} + \alpha_3 e_{k+3} + ... + \alpha_{n-k} e_{n-1}. \quad (24)$$

By adding (22) to (23) we will get (24),thus d_k is a derivation.
This completes the proof of the proposition. □

The following is our main result. we note that t_1, t_2, and d_k for $1 \le k \le n-2$ are defined in Lemma 3.1 to Lemma 3.3.

Proposition 3.1. *Let $L_n(b)$ be $e_0 e_0 = e_2, e_i e_0 = e_{i+1}$, $2 \le i \le n-1$ and $e_1 e_0 = e_n$. Then t_1, t_2 and d_k for $1 \le k \le n-2$ form a basis of the space Der($L_n(b)$) .*

Proof. The proof follows from Lemma 3.4 to Lemma 3.6 □

4. Conclusion

1. This algebra $L_n(b)$, is nilpotent, but it is not characteristically nilpotent.
2. This algebra$L_n(b)$ work with basis derivations from five dimension and above.
3. We can find number derivations of this algebra on any dimension by this rule:

   $$dimDer(L_n(b)) = n + 2$$

   .
4. We can determine number of equations from the result of derivations by this rule number equations $= \frac{(n+1)(n+2)}{2} + n$

Appendix

Table 1: Dimension derivation of Algebra from 5 to 9.

dimension	equation(dim Der)	dim Der	No.of equations
5	$d_1(e_0) = e_0,\ d_1(e_1) = 2e_1,$ $d_1(e_i) = ie_i, 2 \leqslant i \leqslant 4$ $d_2(e_i) = 0,\ 0 \leqslant i \leqslant 4,$ $d_3(e_0) = e_2,\ d_3(e_1) = 0,\ d_3(e_i) = e_{i+1},\ 2 \leqslant i \leqslant 3$ $d_4(e_0) = e_3,\ d_4(e_1) = 0,\ d_4(e_2) = e_4,$ $d_5(e_0) = e_4, d_6(e_1) = e_4.$	6	19
6	$d_1(e_0) = e_0,\ d_1(e_1) = \frac{5}{2}e_1,$ $d_1(e_i) = ie_i, 2 \leqslant i \leqslant 5$ $d_2(e_i) = 0,\ 0 \leqslant i \leqslant 5,$ $d_3(e_0) = e_2,\ d_3(e_1) = 0,\ d_3(e_i) = e_{i+1}, 2 \leqslant i \leqslant 4$ $d_4(e_0) = e_3,\ d_4(e_1) = 0,\ d_4(e_i) = e_{i+2}, 2 \leqslant i \leqslant 3$ $d_5(e_0) = e_4, d_5(e_1) = 0, d_5(e_2) = e_5,$ $d_6(e_0) = e_5, d_7(e_1) = e_5.$	7	26
7	$d_1(e_0) = e_0,\ d_1(e_1) = 3e_1,$ $d_1(e_i) = ie_i, 2 \leqslant i \leqslant 6$ $d_2(e_i) = 0,\ 0 \leqslant i \leqslant 6,$ $d_3(e_0) = e_2,\ d_3(e_1) = 0,\ d_3(e_i) = e_{i+1}, 2 \leqslant i \leqslant 5$ $d_4(e_0) = e_3,\ d_4(e_1) = 0,\ d_4(e_i) = e_{i+2}, 2 \leqslant i \leqslant 4$ $d_5(e_0) = e_4, d_5(e_1) = 0, d_5(e_i) = e_{i+3}, 2 \leqslant i \leqslant 3$ $d_6(e_0) = e_5, d_6(e_1) = 0, d_6(e_2) = e_6$ $d_7(e_1) = e_6\ \ d_7(e_1) = e_6.$	8	34
8	$d_1(e_0) = e_0,\ d_1(e_1) = \frac{7}{2}e_1,$ $d_1(e_i) = ie_i, 2 \leqslant i \leqslant 7$ $d_2(e_i) = 0,\ 0 \leqslant i \leqslant 7,$ $d_3(e_0) = e_2,\ d_3(e_1) = 0,\ d_3(e_i) = e_{i+1}, 2 \leqslant i \leqslant 6$ $d_4(e_0) = e_3,\ d_4(e_1) = 0,\ d_4(e_i) = e_{i+2}, 2 \leqslant i \leqslant 5$ $d_5(e_0) = e_4, d_5(e_1) = 0, d_5(e_i) = e_{i+3},\ 2 \leqslant i \leqslant 4$ $d_6(e_0) = e_5, d_6(e_1) = 0, d_6(e_2) = e_{i+4},\ 2 \leqslant i \leqslant 3$ $d_7(e_0) = e_6\ \ d_7(e_0) = 0\ \ d_7(e_2) = e_7$ $d_8(e_0) = e_7\ \ d_9(e_1) = e_7.$	9	43
9	$d_1(e_0) = e_0,\ d_1(e_1) = 4e_1,$ $d_1(e_i) = ie_i, 2 \leqslant i \leqslant 8$ $d_2(e_i) = 0,\ 0 \leqslant i \leqslant 8,$ $d_3(e_0) = e_2,\ d_3(e_1) = 0,\ d_3(e_i) = e_{i+1}, 2 \leqslant i \leqslant 7$ $d_4(e_0) = e_3,\ d_4(e_1) = 0,\ d_4(e_i) = e_{i+2}, 2 \leqslant i \leqslant 6$ $d_5(e_0) = e_4, d_5(e_1) = 0, d_5(e_i) = e_{i+3},\ 2 \leqslant i \leqslant 5$ $d_6(e_0) = e_5, d_6(e_1) = 0, d_6(e_2) = e_{i+4},\ 2 \leqslant i \leqslant 4$ $d_7(e_0) = e_6,\ d_7(e_1) = 0\ \ d_7(e_i) = e_{i+5},\ 2 \leqslant i \leqslant 3$ $d_8(e_0) = e_7,\ d_8(e_1) = 0,\ d_8(e_2) = e_8,$ $d_9(e_0) = e_8, d_{10}(e_0) = e_8.$	10	53

Table 2: Dimension derivation of Algebra from 10 to 13.

Dimension	Equation(dim Der)	dim Der	No.of equations
10	$d_1(e_0)=e_0,\ d_1(e_1)=4e_1,$ $d_1(e_i)=ie_i, 2\leqslant i\leqslant 9$ $d_2(e_i)=0,\ 0\leqslant i\leqslant 9,$ $d_3(e_0)=e_2,\ d_3(e_1)=0,\ d_3(e_i)=e_{i+1}, 2\leqslant i\leqslant 8$ $d_4(e_0)=e_3,\ d_4(e_1)=0,\ d_4(e_i)=e_{i+2}, 2\leqslant i\leqslant 7$ $d_5(e_0)=e_4, d_5(e_1)=0, d_5(e_i)=e_{i+3},\ 2\leqslant i\leqslant 6$ $d_6(e_0)=e_5, d_6(e_1)=0, d_6(e_2)=e_{i+4},\ 2\leqslant i\leqslant 5$ $d_7(e_0)=e_6,\ d_7(e_1)=0\ \ d_7(e_i)=e_{i+5},\ 2\leqslant i\leqslant 4$ $d_8(e_0)=e_7,\ d_8(e_1)=0,\ d_8(e_2)=e_{i+6},\ 2\leqslant i\leqslant 3$ $d_9(e_0)=e_8,\ d_9(e_1)=0,\ d_9(e_2)=e_9,$ $d_{10}(e_0)=e_9\ \ d_{11}(e_1)=e_9.$	10	64
11	$d_1(e_0)=e_0,\ d_1(e_1)=5e_1,$ $d_1(e_i)=ie_i, 2\leqslant i\leqslant 10$ $d_2(e_i)=0,\ 0\leqslant i\leqslant 10,\ d_3(e_0)=e_2,$ $d_3(e_1)=0,\ d_3(e_i)=e_{i+1}, 2\leqslant i\leqslant 9$ $d_4(e_0)=e_3,\ d_4(e_1)=0,\ d_4(e_i)=e_{i+2}, 2\leqslant i\leqslant 8$ $d_5(e_0)=e_4,\ d_5(e_1)=0,\ d_5(e_i)=e_{i+3},\ 2\leqslant i\leqslant 7$ $d_6(e_0)=e_5,\ d_6(e_1)=0,\ d_6(e_2)=e_{i+4},\ 2\leqslant i\leqslant 6$ $d_7(e_0)=e_6,\ d_7(e_1)=0\ \ d_7(e_i)=e_{i+5},\ 2\leqslant i\leqslant 5$ $d_8(e_0)=e_7,\ d_8(e_1)=0,\ d_8(e_2)=e_{i+6},\ 2\leqslant i\leqslant 4$ $d_9(e_0)=e_8,\ d_9(e_1)=0,\ d_9(e_2)=e_{i+7},\ 2\leqslant i\leqslant 3$ $d_{10}(e_0)=e_9\ \ d_{10}(e_1)=0\ \ d_{10}(e_2)=e_{10}$ $d_{11}(e_0)=e_{10},\ d_{12}(e_1)=e_{10}.$	12	76
12	$d_1(e_0)=e_0,\ d_1(e_1)=\frac{11}{2}e_1,$ $d_1(e_i)=ie_i, 2\leqslant i\leqslant 11$ $d_2(e_i)=0,\ 0\leqslant i\leqslant 11,\ d_3(e_0)=e_2,$ $d_3(e_1)=0,\ d_3(e_i)=e_{i+1}, 2\leqslant i\leqslant 10$ $d_4(e_0)=e_3,\ d_4(e_1)=0,\ d_4(e_i)=e_{i+2},\ 2\leqslant i\leqslant 9$ $d_5(e_0)=e_4,\ d_5(e_1)=0,\ d_5(e_i)=e_{i+3},\ 2\leqslant i\leqslant 8$ $d_6(e_0)=e_5,\ d_6(e_1)=0,\ d_6(e_2)=e_{i+4},\ 2\leqslant i\leqslant 7$ $d_7(e_0)=e_6,\ d_7(e_1)=0\ \ d_7(e_i)=e_{i+5},\ 2\leqslant i\leqslant 6$ $d_8(e_0)=e_7,\ d_8(e_1)=0,\ d_8(e_2)=e_{i+6},\ 2\leqslant i\leqslant 5$ $d_9(e_0)=e_8,\ d_9(e_1)=0,\ d_9(e_2)=e_{i+7},\ 2\leqslant i\leqslant 4$ $d_{10}(e_0)=e_9,\ d_{10}(e_1)=0,\ d_{10}(e_2)=e_{i+8},\ 2\leqslant i\leqslant 3$ $d_{11}(e_0)=e_{10},\ d_{11}(e_1)=0,\ d_{11}(e_2)=e_{11},$ $d_{12}(e_0)=e_{11},\ d_{13}(e_1)=e_{11}.$	13	89
13	$d_1(e_0)=e_0, d_1(e_1)=6e_1,$ $d_1(e_i)=ie_i, 2\leqslant i\leqslant 12$ $d_2(e_i)=0, 0\leqslant i\leqslant 12, d_3(e_0)=e_2,$ $d_3(e_1)=0, d_3(e_i)=e_{i+1}, 2\leqslant i\leqslant 11$ $d_4(e_0)=e_3, d_4(e_1)=0, d_4(e_i)=e_{i+2}, 2\leqslant i\leqslant 10$ $d_5(e_0)=e_4, d_5(e_1)=0, d_5(e_i)=e_{i+3}, 2\leqslant i\leqslant 9$ $d_6(e_0)=e_5, d_6(e_1)=0, d_6(e_2)=e_{i+4}, 2\leqslant i\leqslant 8$ $d_7(e_0)=e_6, d_7(e_1)=0\ d_7(e_i)=e_{i+5}, 2\leqslant i\leqslant 7$ $d_8(e_0)=e_7, d_8(e_1)=0, d_8(e_2)=e_{i+6}, 2\leqslant i\leqslant 6$ $d_9(e_0)=e_8, d_9(e_1)=0, d_9(e_2)=e_{i+7}, 2\leqslant i\leqslant 5$ $d_{10}(e_0)=e_9, d_{10}(e_1)=0, d_{10}(e_2)=e_{i+8}, 2\leqslant i\leqslant 4$ $d_{11}(e_0)=e_{10}, d_{11}(e_1)=0, d_{11}(e_i)=e_{i+9}, 2\leqslant i\leqslant 3$ $d_{12}(e_0)=e_{11}, d_{12}(e_1)=0, d_{12}(e_2)=e_{12},$ $d_{13}(e_0)=e_{12}, d_{13}(e_1)=e_{12}.$	14	103

Table 3: Dimension derivation of Algebra 16.

Dimension	Equation(dim Der)	dim Der	No.of equations
14	$d_1(e_0)=e_0,\ d_1(e_1)=\frac{13}{2}e_1,$ $d_1(e_i)=ie_i, 2\leqslant i\leqslant 13$ $d_2(e_i)=0,\ 0\leqslant i\leqslant 13,\ d_3(e_0)=e_2,$ $d_3(e_1)=0,\ d_3(e_i)=e_{i+1}, 2\leqslant i\leqslant 12$ $d_4(e_0)=e_3,\ d_4(e_1)=0,\ d_4(e_i)=e_{i+2},\ 2\leqslant i\leqslant 11$ $d_5(e_0)=e_4,\ d_5(e_1)=0,\ d_5(e_i)=e_{i+3},\ 2\leqslant i\leqslant 10$ $d_6(e_0)=e_5,\ d_6(e_1)=0,\ d_6(e_2)=e_{i+4},\ 2\leqslant i\leqslant 9$ $d_7(e_0)=e_6,\ d_7(e_1)=0\ d_7(e_i)=e_{i+5},\ 2\leqslant i\leqslant 8$ $d_8(e_0)=e_7,\ d_8(e_1)=0,\ d_8(e_2)=e_{i+6},\ 2\leqslant i\leqslant 7$ $d_9(e_0)=e_8,\ d_9(e_1)=0,\ d_9(e_2)=e_{i+7},\ 2\leqslant i\leqslant 6$ $d_{10}(e_0)=e_9,\ d_{10}(e_1)=0,\ d_{10}(e_2)=e_{i+8},\ 2\leqslant i\leqslant 5$ $d_{11}(e_0)=e_{10},\ d_{11}(e_1)=0,\ d_{11}(e_i)=e_{i+9},\ 2\leqslant i\leqslant 4$ $d_{12}(e_0)=e_{11},\ d_{12}(e_1)=0,\ d_{12}(e_2)=e_{i+10},\ 2\leqslant i\leqslant 3$ $d_{13}(e_0)=e_{12},\ d_{13}(e_1)=0,\ d_{13}(e_i)=e_{13},$ $d_{14}(e_0)=e_{13},\ d_{14}(e_1)=e_{13}.$	15	118
15	$d_1(e_0)=e_0,\ \ d_1(e_1)=7e_1,$ $d_1(e_i)=ie_i, 2\leqslant i\leqslant 14$ $d_2(e_i)=0,\ 0\leqslant i\leqslant 14,\ d_3(e_0)=e_2,$ $d_3(e_1)=0,\ d_3(e_i)=e_{i+1}, 2\leqslant i\leqslant 13$ $d_4(e_0)=e_3,\ d_4(e_1)=0,\ d_4(e_i)=e_{i+2},\ 2\leqslant i\leqslant 12$ $d_5(e_0)=e_4,\ d_5(e_1)=0,\ d_5(e_i)=e_{i+3},\ 2\leqslant i\leqslant 11$ $d_6(e_0)=e_5,\ d_6(e_1)=0,\ d_6(e_2)=e_{i+4},\ 2\leqslant i\leqslant 10$ $d_7(e_0)=e_6,\ d_7(e_1)=0\ d_7(e_i)=e_{i+5},\ 2\leqslant i\leqslant 9$ $d_8(e_0)=e_7,\ d_8(e_1)=0,\ d_8(e_2)=e_{i+6},\ 2\leqslant i\leqslant 8$ $d_9(e_0)=e_8,\ d_9(e_1)=0,\ d_9(e_2)=e_{i+7},\ 2\leqslant i\leqslant 7$ $d_{10}(e_0)=e_9,\ d_{10}(e_1)=0,\ d_{10}(e_2)=e_{i+8},\ 2\leqslant i\leqslant 6$ $d_{11}(e_0)=e_{10},\ d_{11}(e_1)=0,\ d_{11}(e_i)=e_{i+9},\ 2\leqslant i\leqslant 5$ $d_{12}(e_0)=e_{11},\ d_{12}(e_1)=0,\ d_{12}(e_2)=e_{i+10},\ 2\leqslant i\leqslant 4$ $d_{13}(e_0)=e_{12},\ d_{13}(e_1)=0,\ d_{13}(e_i)=e_{i+11},\ 2\leqslant i\leqslant 3$ $d_{14}(e_0)=e_{13},\ d_{14}(e_1)=0,\ d_{14}(e_2)=e_{14},$ $d_{15}(e_0)=e_{14},\ d_{16}(e_1)=e_{14}.$	16	134
16	$d_1(e_0)=e_0,\ d_1(e_1)=\frac{15}{2}e_1,$ $d_1(e_i)=ie_i, 2\leqslant i\leqslant 15$ $d_2(e_i)=0,\ 0\leqslant i\leqslant 15,\ d_3(e_0)=e_2,$ $d_3(e_1)=0,\ d_3(e_i)=e_{i+1}, 2\leqslant i\leqslant 14$ $d_4(e_0)=e_3,\ d_4(e_1)=0,\ d_4(e_i)=e_{i+2},\ 2\leqslant i\leqslant 13$ $d_5(e_0)=e_4,\ d_5(e_1)=0,\ d_5(e_i)=e_{i+3},\ 2\leqslant i\leqslant 12$ $d_6(e_0)=e_5,\ d_6(e_1)=0,\ d_6(e_2)=e_{i+4},\ 2\leqslant i\leqslant 11$ $d_7(e_0)=e_6,\ d_7(e_1)=0\ d_7(e_i)=e_{i+5},\ 2\leqslant i\leqslant 10$ $d_8(e_0)=e_7,\ d_8(e_1)=0,\ d_8(e_2)=e_{i+6},\ 2\leqslant i\leqslant 9$ $d_9(e_0)=e_8,\ d_9(e_1)=0,\ d_9(e_2)=e_{i+7},\ 2\leqslant i\leqslant 8$ $d_{10}(e_0)=e_9,\ d_{10}(e_1)=0,\ d_{10}(e_2)=e_{i+8},\ 2\leqslant i\leqslant 7$ $d_{11}(e_0)=e_{10},\ d_{11}(e_1)=0,\ d_{11}(e_i)=e_{i+9},\ 2\leqslant i\leqslant 6$ $d_{12}(e_0)=e_{11},\ d_{12}(e_1)=0,\ d_{12}(e_2)=e_{i+10},\ 2\leqslant i\leqslant 5$ $d_{13}(e_0)=e_{12},\ d_{13}(e_1)=0,\ d_{13}(e_i)=e_{i+11},\ 2\leqslant i\leqslant 4$ $d_{14}(e_0)=e_{13},\ d_{14}(e_1)=0,\ d_{14}(e_2)=e_{i+12}, 2\leqslant i\leqslant 3$ $d_{15}(e_0)=e_{14},\ d_{15}(e_1)=0,\ d_{15}(e_2)=e_{14},$ $d_{16}(e_0)=e_{15},\ d_{16}(e_1)=e_{15}.$	17	151

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