



A simple confidence interval for the difference between two normal population means with one variance unknown

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Abstract

In this paper, asymptotic coverage probabilities and expected lengths of confidence intervals for the difference between the means of two normal populations with one variance unknown are derived. Monte Carlo simulation results indicate that our proposed confidence interval, which is easy to use, performs as well as the existing confidence intervals.

Keywords: confidence interval, coverage probability, expected length, normal means.

1 Introduction

Recently, confidence intervals for the difference between two normal population means have been investigated. It is known that a confidence interval based on the t -distribution with pooled sample variances is preferable when it is assumed that two population variances are equal; otherwise, the Welch-Satterthwaite (WS, hereafter) confidence interval is preferable, see e.g. Satterthwaite [1-2], Welch [3], Miao and Chiou [4], Moser et al. [5], Schechtman and Sherman [6] and references therein. Miao and Chiou [4] mentioned that the WS, Satterthwaite [1, 2], confidence interval also performs well, based on its coverage probability, in the case of two equal population variances. Niwitpong and Niwitpong [7] also derived coverage probabilities and expected lengths of confidence intervals for the difference between two normal means with a known ratio of variances. They revealed that the confidence interval based on t -statistic of Schechtman and Sherman [6] has a shorter expected length than that of the WS confidence interval when the ratio of variances is large.

A confidence interval for the difference of two normal population means with one variance unknown is also of interest. Maity and Sherman [8] described that this situation arises, for instance, when one is interested in comparing a standard treatment with a new treatment. A known variance comes from the standard treatment while an unknown variance comes from the new treatment.

Maity and Sherman found that their proposed t -test has more power than an existing Satterthwaite's test. Peng and Tong [9] proved that the t -test statistic with the number of degrees of freedom ν_2 (see, section 2.2) of Maity and Sherman [8] is the biased estimator of the number of degrees of freedom. Hence, they proposed an unbiased degrees of freedom of the t -test statistic which is described in section 2.2. Peng and Tong [9] showed that the t -test statistic with the unbiased estimator of the number of degrees of freedom ν_3 performs better than that of the Maity and Sherman's method especially when the variance of the unknown variance is large. In practice, both test statistics, however, need time to compute the number of degrees of freedom.

In this paper, we therefore, propose a simple and easy method to construct the confidence interval with one variance unknown as in Niwitpong [10]. We also proved the coverage probability and the expected length of each confidence interval in comparison with the WS confidence interval. The paper is organized as follows. Section 2 presents confidence intervals for the difference between two normal population means with one variance unknown. Coverage probabilities and expected lengths of confidence intervals in section 2 are derived in section 3. Section 4 contains a discussion of the results and conclusions.

2 Confidence intervals for the difference of two normal population means

Let X_1, X_2, \dots, X_n and Y_1, Y_2, \dots, Y_m be random samples from two independent normal distributions with means μ_x, μ_y and standard deviations σ_x and σ_y , respectively. The sample means and variances for X and Y are also denoted as \bar{X}, \bar{Y}, S_x^2 and S_y^2 , respectively, where

$$\bar{X} = \sum_{i=1}^n X_i / n, \bar{Y} = \sum_{j=1}^m Y_j / m, S_x^2 = (n-1)^{-1} \sum_{i=1}^n (X_i - \bar{X})^2, S_y^2 = (m-1)^{-1} \sum_{j=1}^m (Y_j - \bar{Y})^2.$$

We are interested in $100(1-\alpha)$ % confidence interval for $\theta = \mu_x - \mu_y$ when one of the variance is unknown.

2.1 The Confidence interval for θ based on Welch-Satterthwaite’s method

It is known that when the two variances differ, i.e. $\sigma_x^2 \neq \sigma_y^2$, the confidence interval for θ is constructed using the test statistic T_1 , as the pivot quantity,

$$T_1 = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}}.$$

It is also well-known that T_1 is *approximately* distributed as a t -distribution with degrees of freedom equal to

$$v_1 = \frac{(A+B)}{\frac{A^2}{n-1} + \frac{B^2}{m-1}}, A = \frac{S_x^2}{n}, B = \frac{S_y^2}{m}.$$

The $100(1-\alpha)$ % confidence interval for θ is therefore

$$CI_{WS} = \left[(\bar{X} - \bar{Y}) - t_{1-\alpha/2, v_1} \sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}, (\bar{X} - \bar{Y}) + t_{1-\alpha/2, v_1} \sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}} \right]$$

where $t_{1-\alpha/2, v_1}$ is the $(1-\alpha/2)$ th percentile of T_1 distribution with v_1 degrees of freedom.

The confidence interval CI_{WS} is known as the WS confidence interval.

2.2 Existing confidence intervals for θ with one variance unknown

Following Maity and Sherman [8], suppose σ_y^2 is unknown, Maity and Sherman proposed the test statistic T_2 ,

$$T_2 = \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}}}$$

of the hypothesis $H_0 : \mu_x - \mu_y = \Delta_0$ against the alternative hypothesis $H_1 : \mu_x - \mu_y \neq (<, >) \Delta_0$. By means of Monte Carlo simulation, they found that the test statistic T_2 has better power than the test statistic T_1 when one variance is unknown. However, they did not study the confidence interval for θ using the pivotal test statistic T_2 .

As a result, it is of interest to construct the confidence interval for θ when one variance is unknown. Niwitpong [10] proposed the confidence interval for θ using the pivotal quantity T_2 with v_2 degrees of freedom where

$$v_2 = \frac{\left(\frac{\sigma_x^2}{n} + \frac{S_y^2}{m} \right)^2}{\frac{(S_y^2 / m)^2}{m-1}}.$$

Maity and Sherman [8] pointed out that the test statistic T_2 has an approximate t -distribution with v_2 degrees of freedom. Define $t_{1-\alpha/2, v_2}$ be the $(1-\alpha/2)$ th percentile of the t -distribution with

v_2 Degrees of freedom, such that

$$\Pr \left[(\bar{X} - \bar{Y}) - t_{1-\alpha/2, v_2} \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}} < \theta < (\bar{X} - \bar{Y}) + t_{1-\alpha/2, v_2} \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}} \right] = 1 - \alpha.$$

Therefore, $100(1-\alpha)\%$ confidence interval for θ is

$$CI_{MS} = \left[(\bar{X} - \bar{Y}) - t_{1-\alpha/2, v_2} \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}}, (\bar{X} - \bar{Y}) + t_{1-\alpha/2, v_2} \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}} \right].$$

Niwitpong [10] derived the coverage probability and the expected length of the confidence intervals CI_{WS} and CI_{MS} for θ , when one variance unknown. She found that coverage probabilities of both confidence intervals CI_{WS} and CI_{MS} are almost the same but CI_{MS} has a smaller expected length when the unknown variance is small.

Peng and Tong [9] showed that the number of degrees of freedom v_2 of the test statistic T_2 is a positive biased estimator of v where

$$v = \frac{\left(\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m} \right)^2}{\left(\frac{\sigma_y^2}{m} \right)^2} \cdot \frac{1}{m-1}.$$

Hence, they proposed an unbiased estimator of v which is

$$v_3 = (m-1) \left\{ \frac{m^3(m-3)(m-5)\sigma_x^2}{n^2(n-1)^2} \frac{1}{S_y^2} + \frac{2m(m-3)\sigma_x^2}{n(n-1)} \frac{1}{S_y^2} + 1 \right\}$$

for the test statistic T_2 .

By means of simulation, Peng and Tong [9] showed that the test statistic T_2 with degrees of freedom v_3 can control type I error better than that of the test statistic T_2 with degrees of freedom v_2 , especially when σ_y^2 is large.

Similarly to CI_{MS} , we construct a new confidence interval for $\theta = \mu_x - \mu_y$ by using the pivotal statistic T_2 with degrees of freedom v_3 , and this confidence interval is denoted as CI_{JT} , where

$$CI_{JT} = \left[(\bar{X} - \bar{Y}) - t_{1-\alpha/2, v_3} \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}}, (\bar{X} - \bar{Y}) + t_{1-\alpha/2, v_3} \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}} \right].$$

2.3. Proposed confidence interval for θ with one variance unknown

It is argued that, in practice, confidence intervals CI_{MS} and CI_{JT} are not easy to use. This is due to the calculation of the number of degrees of freedom v_2 and v_3 . We now propose an easy method, based on Zou et al. [11-12], to construct a new confidence interval for θ with one variance unknown. Our proposed confidence interval constructed using the test statistics Z (standard normal distribution) and T with degrees of freedom $m-1$.

Zou et al. [11-12] proposed the method called "The Method of Variance Estimates Recovery (MOVER)" to construct confidence intervals for lognormal and normal data. Their strategy is to recover variance estimates from confidence interval of each parameter and then approximate confidence intervals for functions of parameters by using the central limit theorem. A general approach to construct two-sided confidence interval for $\theta_1 + (-\theta_2)$ is $[L, U]$ where

$$L = (\hat{\theta}_1 - \hat{\theta}_2) - \sqrt{(\hat{\theta}_1 - l_1)^2 + (u_2 - \hat{\theta}_2)^2}$$

and

$$U = (\hat{\theta}_1 - \hat{\theta}_2) + \sqrt{(u_1 - \hat{\theta}_1)^2 + (\hat{\theta}_2 - l_2)^2},$$

$\hat{\theta}_1, \hat{\theta}_2$ are respectively estimators of $\theta_i, i=1,2$ and $\theta_i \in (l_i, u_i)$.

For $(l_i, u_i), (i=1,2)$, setting $\theta_1 = \mu_x$ and $\theta_2 = \mu_y$, it is known that confidence intervals for each $\theta_i, i=1,2$ are respectively

$$(l_1, u_1) = \left(\bar{X} - Z_{1-\alpha/2} \frac{\sigma_x}{\sqrt{n}}, \bar{X} + Z_{1-\alpha/2} \frac{\sigma_x}{\sqrt{n}} \right)$$

$$(l_2, u_2) = \left(\bar{Y} - t_{1-\alpha/2, m-1} \sqrt{\frac{S_y^2}{m}}, \bar{Y} + t_{1-\alpha/2, m-1} \sqrt{\frac{S_y^2}{m}} \right)$$

Putting $\hat{\theta}_1 = \bar{X}$, and $\hat{\theta}_2 = \bar{Y}$, $l_1 = \bar{X} - Z_{1-\alpha/2} \frac{\sigma_x}{\sqrt{n}}$, $u_1 = \bar{X} + Z_{1-\alpha/2} \frac{\sigma_x}{\sqrt{n}}$ and

$$l_2 = \bar{Y} - t_{1-\alpha/2} \frac{S_y}{\sqrt{m}}, \quad u_2 = \bar{Y} + t_{1-\alpha/2} \frac{S_y}{\sqrt{m}} \text{ in } [L, U], \text{ we have}$$

$$\begin{aligned} L' &= \bar{X} - \bar{Y} - \sqrt{\left(\bar{X} - \bar{X} + Z_{1-\alpha/2} \frac{\sigma_x}{\sqrt{n}}\right)^2 + \left(\bar{Y} + t_{1-\alpha/2, m-1} \frac{S_y}{\sqrt{m}} - \bar{Y}\right)^2} \\ &= \bar{X} - \bar{Y} - \sqrt{Z_{1-\alpha/2}^2 \frac{\sigma_x^2}{n} + t_{1-\alpha/2, m-1}^2 \frac{S_y^2}{m}} \end{aligned}$$

$$\begin{aligned} U' &= \bar{X} - \bar{Y} + \sqrt{\left(\bar{X} + Z_{1-\alpha/2} \frac{\sigma_x}{\sqrt{n}} - \bar{X}\right)^2 + \left(\bar{Y} - \bar{Y} + t_{1-\alpha/2, m-1} \frac{S_y}{\sqrt{m}}\right)^2} \\ &= \bar{X} - \bar{Y} + \sqrt{Z_{1-\alpha/2}^2 \frac{\sigma_x^2}{n} + t_{1-\alpha/2, m-1}^2 \frac{S_y^2}{m}}. \end{aligned}$$

A new confidence interval for $\theta = \mu_x - \mu_y$ with unknown σ_y^2 is therefore a confidence interval $CI_N = [L', U']$ where

$$CI_N = \left[\bar{X} - \bar{Y} - \sqrt{Z_{1-\alpha/2}^2 \frac{\sigma_x^2}{n} + t_{1-\alpha/2, m-1}^2 \frac{S_y^2}{m}}, \bar{X} - \bar{Y} + \sqrt{Z_{1-\alpha/2}^2 \frac{\sigma_x^2}{n} + t_{1-\alpha/2, m-1}^2 \frac{S_y^2}{m}} \right].$$

We evaluate these confidence intervals i.e. CI_{WS} , CI_{MS} , CI_{JT} and CI_N using their coverage probabilities and expected lengths which are derived in the next section. Generally, we prefer a confidence interval with minimum coverage probability equal to a pre-specified value $1 - \alpha$ and with a shorter expected length.

2. Main results: Coverage probabilities and expected lengths of confidence intervals for θ with one variance unknown

In this section, the coverage probabilities and the expected lengths of confidence intervals CI_{WS} , CI_{MS} , CI_{JT} and CI_N are derived.

Theorem 1 : (Niwitpong and Niwitpong [7]) The coverage probability and the expected length of CI_{WS} when one variance (σ_y^2) is unknown are respectively

$$\begin{aligned} & E [\Phi(w_1) - \Phi(-w_1)] \text{ and} \\ & \begin{cases} 2d \zeta \sqrt{s_1} F \left[\frac{-1}{2}, \frac{m-1}{2}, \frac{m+n-2}{2}, \frac{s_1-s_2}{s_1} \right] ; \text{ if } s_2 < 2s_1, \\ 2d \zeta \sqrt{s_2} F \left[\frac{-1}{2}, \frac{n-1}{2}, \frac{m+n-2}{2}, \frac{s_2-s_1}{s_2} \right] ; \text{ if } 2s_1 \leq s_2 \end{cases} \\ \text{where } w_1 &= t_{1-\frac{\alpha}{2}, v} \frac{\left(\frac{S_x^2}{n} + \frac{S_y^2}{m}\right)^{1/2}}{\left(\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}\right)^{1/2}}, \end{aligned}$$

$\Phi[\cdot]$ is the cumulative distribution function of $N(0, 1)$,

$$s_1 = \frac{\sigma_X^2}{n(n-1)}, s_2 = \frac{\sigma_Y^2}{m(m-1)}, \zeta = \frac{\sqrt{2}\Gamma\left(\frac{m+n-1}{2}\right)}{\Gamma\left(\frac{m+n-2}{2}\right)},$$

$F(a, b, c; k)$ is the hypergeometric function defined by $F(a, b, c; k) = 1 + \frac{ab}{c} \frac{k}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)} \frac{k^2}{2!} + \dots$

and $\Gamma[\cdot]$ is the gamma function.

Proof: See Niwitpong and Niwitpong [7].

Theorem 2: (Niwitpong [10]) The coverage probability and the expected length of CI_{MS} with one variance unknown (σ_Y^2) are respectively

$$E[\Phi(w_2) - \Phi(-w_2)] \quad \text{and} \quad 2t_{1-\alpha/2, v_2} \left(\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m} \right)^{1/2}$$

where

$$w_2 = t_{1-\alpha/2, v_2} \frac{\sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}}.$$

Proof : See Niwitpong [10]

Theorem 3: The coverage probability and the expected length of CI_{IT} with one variance unknown (σ_Y^2) are respectively

$$E[\Phi(w_3) - \Phi(-w_3)] \quad \text{and} \quad 2t_{1-\alpha/2, v_3} \left(\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m} \right)^{1/2}$$

where

$$w_3 = t_{1-\alpha/2, v_3} \frac{\sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}}.$$

Proof: It is straightforward to see that the coverage probability of CI_{IT} is

$$\begin{aligned} 1 - \alpha &= \Pr \left((\bar{X} - \bar{Y}) - d_1 \sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}} \leq \theta \leq (\bar{X} - \bar{Y}) + d_1 \sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}} \right), \quad d_1 = t_{1-\alpha/2, v_3} \\ &= \Pr \left(-d_1 \sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}} \leq \theta - (\bar{X} - \bar{Y}) \leq d_1 \sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}} \right) \\ &= \Pr \left(-d_1 \frac{\sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \leq Z \leq d_1 \frac{\sqrt{\frac{\sigma_X^2}{n} + \frac{S_Y^2}{m}}}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \right) \\ &= E \left[I_{\{-w_3 < Z < w_3\}}(\tau) \right], \quad I_{\{-w_3 < Z < w_3\}}(\tau) = \begin{cases} 1, & \tau \in \{-w_3 < Z < w_3\} \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

$$\begin{aligned}
 &= E \left[E \left[I_{\{-w_3 < Z < w_3\}}(\tau) \mid S_Y^2 \right] \right] \\
 &= E \left[\Phi(w_3) - \Phi(-w_3) \right].
 \end{aligned}$$

The length of $CI_{\mathcal{I}}$ is $2d_1 \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}}$ and the expected length of $CI_{\mathcal{I}}$ is

$$E \left(2d_1 \sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}} \right) = 2d_1 E \left(\sqrt{\frac{\sigma_x^2}{n} + \frac{S_y^2}{m}} \right) \leq 2d_1 \left(E \left(\frac{\sigma_x^2}{n} + \frac{S_y^2}{m} \right) \right)^{1/2},$$

using Jensen's inequality and (Niwitpong [10], Lemma 1), we have $E_r(f(r)) \leq f(E(r))$

$$\begin{aligned}
 &= 2d_1 \left(\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m(m-1)} E \left(\frac{(m-1)S_y^2}{\sigma_y^2} \right) \right)^{1/2} \\
 &= 2d_1 \left(\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m(m-1)} E(U_2) \right)^{1/2}, \text{ since } U_2 \sim \chi_{m-1}^2 \\
 &= 2d_1 \left(\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m(m-1)} (m-1) \right)^{1/2}, E(U_2) = m-1 \\
 &= 2d_1 \left(\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m} \right)^{1/2}.
 \end{aligned}$$

This ends the proof.

Theorem 4: The coverage probability and the expected length of CI_N with one variance unknown (σ_y^2) are respectively

$$E[\Phi(w_5) - \Phi(-w_5)] \quad \text{and} \quad 2 \left(\frac{Z_{1-\alpha/2}^2 \sigma_x^2}{n} + \frac{t_{1-\alpha/2, v_3}^2 \sigma_y^2}{m} \right)^{1/2}$$

where

$$w_5 = \frac{\sqrt{\frac{Z_{1-\alpha/2}^2 \sigma_x^2}{n} + \frac{t_{1-\alpha/2, v_3}^2 \sigma_y^2}{m}}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}}.$$

Proof: Let $C_1 = Z_{1-\alpha/2}^2, C_2 = t_{1-\alpha/2, v_3}^2$, the coverage probability of CI_N is

$$\begin{aligned}
 1 - \alpha &= \Pr(\theta \in CI_N) = P(\theta \in (L, U)) \\
 &= P \left(\bar{X} - \bar{Y} - \sqrt{C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m}} < \theta < \bar{X} - \bar{Y} + \sqrt{C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m}} \right) \\
 &= P \left(-\sqrt{C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m}} < \theta - (\bar{X} - \bar{Y}) < +\sqrt{C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m}} \right) \\
 &= P \left(-w_4 < \theta - (\bar{X} - \bar{Y}) < w_4 \right), w_4 = \sqrt{C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m}} \\
 &= P \left(w_4 > (\bar{X} - \bar{Y}) - \theta > -w_4 \right) \\
 &= P \left(-\frac{w_4}{\sqrt{A}} < \frac{(\bar{X} - \bar{Y}) - \theta}{\sqrt{A}} < \frac{w_4}{\sqrt{A}} \right), A = \frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}
 \end{aligned}$$

$$\begin{aligned}
&= P(-w_5 < Z < w_5), w_5 = \frac{w_4}{\sqrt{A}} \\
&= E \left[I_{\{-w_5 < Z < w_5\}}(\xi) \right], I_{\{-w_5 < Z < w_5\}}(\xi) = \begin{cases} 1, \xi \in \{-w_5 < Z < w_5\} \\ 0, \text{ otherwise} \end{cases} \\
&= E \left[E \left[I_{\{-w_5 < Z < w_5\}}(\xi) \mid S_Y^2 \right] \right] = E[\Phi(w_5) - \Phi(-w_5)]
\end{aligned}$$

where $Z \sim N(0, 1)$.

The Length of CI_C is

$$\begin{aligned}
E \left(2 \sqrt{C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m}} \right) &= 2E \left(C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m} \right)^{\frac{1}{2}} \\
&\leq 2 \left(E \left(C_1 \frac{\sigma_x^2}{n} + C_2 \frac{S_y^2}{m} \right) \right)^{\frac{1}{2}} \\
&= 2 \left(C_1 \frac{\sigma_x^2}{n} + C_2 E \left(\frac{(m-1)}{(m-1)} \frac{S_y^2}{m} \frac{\sigma_y^2}{\sigma_y^2} \right) \right)^{\frac{1}{2}} \\
&= 2 \left(C_1 \frac{\sigma_x^2}{n} + C_2 \frac{\sigma_y^2}{m(m-1)} E(\chi_{m-1}^2) \right)^{\frac{1}{2}} \\
&= 2 \left(C_1 \frac{\sigma_x^2}{n} + \frac{C_2 \sigma_y^2}{m(m-1)} (m-1) \right)^{\frac{1}{2}} \\
&= 2 \left(\frac{C_1 \sigma_x^2}{n} + \frac{C_2 \sigma_y^2}{m} \right)^{\frac{1}{2}}.
\end{aligned}$$

This ends the proof.

3. Discussion and conclusions

We proposed, in this paper, the new confidence interval for the difference between two normal population means with one variance unknown, CI_N . We derived coverage probability of the new confidence interval compared to existing confidence intervals: CI_{WS} , CI_{MS} and CI_{JT} . Theorems 1-4 show that all confidence intervals have coverage probability $1-\alpha$, for large samples and α is a level of significance for all test statistics in this paper. The expected length of the confidence interval CI_{JT} is as short as the confidence interval CI_{MS} and the expected length of our proposed confidence interval CI_N is at least shorter than that of the confidence interval CI_{JT} but our proposed confidence interval is simple to use more than those of existing confidence intervals which are needed to compute the number of degrees of freedoms v_2 and v_3 .

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