International Journal of Advanced Statistics and Probability, 2 (1) (2014) 15-20
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www.sciencepubco.com/index.php/IJASP
doi: 10.14419/ijasp.v2i1.1475
Short Communications

Asymptotic sampling distribution of inverse coefficient of variation and its applications: revisited

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Abstract

Sharma and Krishna [16] derived mathematically an appealing asymptotic confidence interval for the population Signal-to-Noise Ratio (SNR). In this paper, an evaluation of the performance of this interval using Monte Carlo simulations using randomly generated data from normal, log-normal, $\chi 2$, Gamma, and Weibull distributions three of which are discussed in Sharma and Krishna [16]. Simulations revealed that its performance, as measured by coverage probability, is totally dependent on the amount of noise introduced. A proposal for using ranked set sampling (RSS) instead of simple random sampling (SRS) improved its performance. It is recommended against using this confidence interval for data from a log-normal distribution. Moreover, this interval performs poorly in all other distributions unless the SNR is around one.

Keywords: *Signal-to-noise ratio, coefficient of variation, sampling distribution, confidence interval, ranked set sample, simple random sample.*

1. Introduction

The population coefficient of variation (CV) is a unit-free measure of variability relative to the mean. This measure can be used to make comparisons across several populations that have different units of measurement. The population CV is defined as a ratio of the population standard deviation (σ) to the population mean (μ), namely CV = σ/μ. In real life cases the population parameters σ and μ are estimated by the sample estimators s and $\bar{x}$, respectively. The CV is widely used in health sciences in descriptive and inferential manners see Kelley [6] and Gulhar et al. [4] for a discussion. Some applications include measuring the variation in the mean synaptic response of the central nervous system and measuring the variability in socioeconomic status and prevalence of smoking among tobacco control environments, see Faber and Korn [2] and Bernatm et al., [1]. Because the CV is a unit free measure, the variation of two or more different measurement methods can be compared to each other. In public health setting, the CV is used in assessing the overall health of an individual where the CV may be useful in comparing the variability in blood pressure measurement (mmHg) and cholesterol measurement (mg/dL). If variance σ^2 were used, rather than the CV, then these two measures would not be comparable as their units of measurement would differ.

In analog and digital communications, the reciprocal of the CV is called Signal-to-Noise Ratio (SNR) which is a measure of signal strength relative to background noise, while in quality control; the SNR represents the magnitude of the mean of a process compared to its variation. The SNR measures how much signal has been corrupted by noise see McGibney and Smith [10] for a discussion. The SNR is commonly used in image processing, where the SNR of an image is usually calculated as the ratio of the mean pixel values over a given neighborhood. For a population with mean μ and standard deviation σ, the SNR is defined as the ratio of the population mean to the population standard deviation where the parameters μ and σ are estimated by $\bar{x}$ and s, respectively. Hence a sample estimate of the population SNR is given by$\widehat{SNR} = \bar{x}/s$.

It is of great interest to find confidence interval estimate for SNR. Confidence interval estimation allows the researcher to have an idea about the precision of the point estimate rather than only a p value for rejection or no rejection of a specified null hypothesis. Confidence intervals for the SNR are limited in the literature. Sharma and Krishna [16] derived mathematically an appealing asymptotic confidence interval for the population SNR. The authors indicated that such derivation solved a problem concerning inference about population CV and SNR in addition to making inferences (point & interval estimation, and hypothesis testing) about the shape parameter of three popular life-testing distributions: Gamma, Weibull, and log-normal, when the scale parameter is unknown. George and Kibria [3] performed a simulation study that compares several confidence intervals estimate for estimating SNR by inverting confidence intervals for estimating CV.

A powerful alternative sampling technique to simple random sampling (SRS) is ranked set sampling (RSS) which is used to obtain estimates of the μ and σ^2 in situations where the variable of interest is too expensive or can't be easily measured, but can be easily ranked. McIntyre [9] was the first to suggest using RSS to estimate the population means instead of SRS and the idea was later developed by Takahasi and Wakimoto [18] using mathematical theory to support their claim. Takahasi and Wakimoto [18] proved that the sample mean obtained using RSS is unbiased and has smaller variance compared to that obtained using SRS using the same sample size, see Samawi and Muttlak [14] and Samawi [15] for discussion. Stokes [17] proposed an estimator for the variance of a ranked set sample data and showed that the estimator is asymptotically unbiased and asymptotically more efficient than the sample variance of a simple random sample of a the same number of observations. MacEachern et al. [7] proposed an alternative estimator for the variance which is unbiased and more efficient than Stokes's estimator even when the underlying distribution is not normal and the ranking of the elements is not perfect. MacEachern et al. [7] estimator of σ^2 performs well for small to moderate sample sizes and is asymptotically equivalent to Stokes [17] estimator. Stokes [17] used RSS to derive an asymptotically unbiased estimate of the variance regardless of presence of errors in ranking. MacEachern et al. [7] provided an estimator for the variance which is unbiased and more efficient than the one proposed by Stokes's [17] estimator, even for non-normal underlying distribution and where judgment rankings are not perfect.

Terpstra and Nelson [19] used unbalanced RSS to compare maximum likelihood estimator (MLE) and weighted average (WA) estimate for the population proportion. Later Terpstra and Wang [20] compared several confidence intervals for estimating the population proportion using RSS, which is by far the only work found about implementation and discussing corresponding properties of RSS in interval estimation for the population proportion. On the other hand, Samawi and Muttlak [14] compared RSS to SRS in estimating ratio and proved that the efficiency of the estimator has increased when using RSS relative to SRS. In this paper, rather than using the usual SRS technique, the more powerful RSS technique will be used to generate the samples. This makes the current paper very unique in using RSS to estimate SNR rather than SRS. This provided the motivation and ground to implement RSS in estimating a confidence interval for the SNR.

In this paper, an evaluation of the performance of Sharma and Krishna [16] confidence interval for estimating the SNR using simulated data from several distributions. Also RSS is proposed for interval estimation of the SNR instead of SRS with comparison based on coverage probability.

Table 1: Balanced RSS with *m* cycles and set size *k*

Cycle 1	$X_{[1]1}$	$X_{[1]2}$	$X_{[1]3}$	$\cdots$	$X_{[1]k}$
Cycle 2	$X_{[2]1}$	$X_{[2]2}$	$X_{[2]3}$	$\cdots$	$X_{[2]k}$
Cycle 3	$X_{[3]1}$	$X_{[3]2}$	$X_{[3]3}$	$\cdots$	$X_{[3]k}$
.	.	.	.	.	.
Cycle m	$X_{[m]1}$	$X_{[m]2}$	$X_{[m]3}$	$\cdots$	$X_{[m]k}$

This paper is organized as follows: in section 2 an unbiased estimates μ and σ using RSS are introduced. Section 3 provides a description of the asymptotic distribution of the SNR. Section 4 describes the simulation study and simulation results, while a conclusion is presented in section 5.

2. Unbiased estimates of μ and σ using RSS

Suppose that we are interested in obtaining a RSS of size *k* from a population. First, a SRS of size *k* observations are selected and rank ordered on an attribute of interest. The observation that is determined to be the smallest is the first element of the RSS and is denoted $X_{[1]}$ and the remaining *k*-1 units are discarded. A second SRS of size *k* is selected from the population and ranked the same way and the second smallest observation is selected and denoted $X_{[2]}$. In a similar fashion, $X_{[3]}$, $X_{[4]}\ldots X_{[k]}$ are selected hence $X_{[1]}$, $X_{[2]}\ldots X_{[k]}$ represent our first balanced RSS of size *k*. To obtain a balanced RSS of size $n=km$, the process is repeated *m* independent cycles yielding the balanced RSS of size *n* shown in Table 1.

The complete balanced RSS with set size *k* and *m* cycles is given by $\{X_{[r]i} : r = 1,2,\ldots m;\ i = 1,2,\ldots, k\}$. The term $X_{[r]i}$ is called the *r*th judgment order statistic from the *i*th cycle. It is the observation that is judged to be the *r*th order statistic from the *k* sets in the *i*th cycle, see MacEachern et al. [7] for discussion. Assume that the underlying distribution has finite mean μ and variance σ^2. Stokes [17] proposed an estimator of σ^2 based on RSS given by

$$\hat{\sigma}^2 = \frac{1}{nm-1}\sum_{i=1}^{n}\sum_{r=1}^{m}(X_{[r]i}-\hat{\mu})^2 \text{ , where } \hat{\mu} = \frac{1}{nm}\sum_{i=1}^{n}\sum_{r=1}^{m}X_{[r]i} \tag{1}$$

Stokes [17, page 37] showed that this estimator is a biased estimator of σ^2, but it is asymptotically unbiased as either n or m approach ∞. Moreover, Stokes [17] indicated that the RSS estimator $\hat{\mu}$ has more precision over the sample mean; say $\bar{Y}$ obtained using SRS because of independence of the order statistics composing the ranked set sample. In fact, the author showed that n^2 var $(\bar{Y}) \geq n^2$ var $(\hat{\mu})$.
The balanced RSS is used in this paper. The estimator of the variance of a RSS proposed by MacEachern et al. [7] will be implemented in the simulations since it has been shown to perform very well for small as well as large ranked set samples, which is an improvement on Stokes [17] estimator. The evaluation of Sharma and Krishna [16] interval will be based on coverage probability which seems to be the major factor for comparison, see Mahmoudvand and Hassani [8], Terpstra and Wang [20], Panichkitkosolkul [11,12], and Kang and Schmeiser [5] for discussion.

3. Asymptotic distribution of SNR

Let x_1, x_2 …xn be an independently and identically distributed (iid) random sample of size n from a population with finite mean μ and finite variance σ^2. Let $\bar{x}$ be the sample mean and s being the sample standard deviation. Using the central limit theorem $\bar{x}$ converges in distribution to standard normal distribution and in probability to μ, while s converges in probability to σ. The following Lemma is from Sharma and Krishna [16].

Lemma 1: *Let* $[X_n, Y_n]$, $n = 1, 2\ldots$ *be a sequence of pairs of random variables. Then for a constant* $k\neq 0$,

If $X_n \xrightarrow{L} X, Y_n \xrightarrow{P} k$, *then* $\dfrac{X_n}{Y_n} \xrightarrow{L} \dfrac{X}{k}$

Applying Lemma 1 to $\bar{x}$ and s to obtain $\frac{\bar{x}}{s}$ converges in distribution to the standard normal with mean $\frac{\mu}{\sigma}$ and variance $\frac{1}{n}$, $\frac{\bar{x}}{s} \xrightarrow{P} \frac{\mu}{\sigma}$ in probability. Sharma and Krishna [16] indicated that the method of moments can be used to show that $\frac{\bar{x}}{s}$ is an estimator of $\frac{\mu}{\sigma}$ and that $\frac{\bar{x}}{s}$ is an unbiased and consistent estimator of $\frac{\mu}{\sigma}$. Moreover, $\sqrt{n}\left(\frac{\bar{x}}{s} - \frac{\mu}{\sigma}\right)$ is asymptotically distributed as standard normal. Sharma and Krishna [16] indicated that the asymptotic distribution of $\frac{\bar{x}}{s}$ is useful for testing and estimating population value of $\frac{\mu}{\sigma}$ and hence solved an important problem in survival analysis involving inferences on the shape parameter of Gamma, Weibull, and log-normal distributions. Sharma and Krishna [16, page 631] provided a (1-α) 100% confidence interval estimate for the population SNR= $\frac{\mu}{\sigma}$ which is given by

$$\frac{\bar{x}}{s} \pm Z_{\frac{\alpha}{2}} \frac{1}{\sqrt{n}} \tag{2}$$

Note that the width of this interval is $2\, Z_{\frac{\alpha}{2}} \frac{1}{\sqrt{n}}$ which is a function of the sample size n only. The width will decrease as the sample size n increases. Therefore, for a fixed n, the width will be constant for any value of $\bar{x}$ and s obtained using either SRS or RSS. For this reason, the width was not considered as a criterion for performance comparison. In the following section, a critical evaluation of the performance of this interval is performed using coverage probability as a performance criterion. Data were generated from five distributions, three of which are used by Sharma and Krishna [16], with a range of sample sizes and values of SNR using both SRS and RSS techniques.

4. Simulation study

The objective of this paper is to evaluate the performance of Sharma and Krishna [16] interval estimator of the SNR using SRS compared to RSS. This interval has an appealing mathematical derivation, but to evaluate its performance a simulation study has been conducted using R© statistical software. Five distributions are used in simulations, namely: normal, log-normal, χ^2, Gamma, Weibull distribution, three of which have been used in Sharma and Krishna [16]. The nominal confidence level is set to 95%.

4.1. Simulation technique

In these simulations, random samples of sizes n=15, 25, 50, 100 are generated with specific parameters from normal, log-normal, χ^2, Gamma, and Weibull distributions. For each combination of n and SNR, 5000 replications were generated. The corresponding simulated coverage probabilities are presented in Table 2.

4.1.1. Normal Distribution

Let x_1, x_2… x_n be an *iid* random sample from a normal distribution with finite mean μ and variance σ^2. For a random variable X such that $X \sim N(\mu, \sigma^2)$, the population SNR= μ/σ. Thus, for σ = 10, 5, 3, 1 and μ = 10 the corresponding population values of SNR are 1, 2, 3.33, and 10 are obtained.

4.1.2. Log-Normal Distribution

Let x_1, x_2… x_n be *iid* random sample from a log-normal distribution *ln* $N(\mu,\sigma^2)$ where the shape parameter $\sigma^2 > 0$ and the scale parameter μ a real number. The mean and variance of a log normal random variable are given by

$\mu^* = e^{\mu+\frac{1}{2}\sigma^2}$ And $\sigma^{*2} = (e^{\sigma^2} - 1)(e^{2\mu+\sigma^2})$.

Therefore, the population SNR is given by

$$SNR = (\sqrt{e^{\sigma^2} - 1})^{-1} \quad (3)$$

Clearly the value of SNR in equation (3) depends only on the shape parameter σ. For simulation purposes, set μ = 10 (arbitrary), and in order to get SNR= 10, 3.33, 2, 1, we must set σ = 0.09975, 0.2938, 0.4724, 0.8326, respectively.

4.1.3. χ^2 distribution

Let x_1, x_2… x_n be an *iid* random sample from a $\chi^2_{(v)}$ distribution with v degrees of freedom. Then the mean and variance of a χ^2 random variable are given by $\mu = v$ and $\sigma^2 = 2v$, respectively. Therefore, the population SNR for a χ^2 random variable is given by

$$SNR = \sqrt{\frac{v}{2}} \quad (4)$$

For simulation purposes, and in order to get SNR= 10, 3.33, 2, 1, we must set v = 200, 22, 8, 2, respectively.

4.1.4. Gamma Distribution

Let x_1, x_2… x_n be an *iid* random sample from a Gamma distribution with shape and scale parameters α and β, respectively. The mean and variance of a random variable from Gamma (α, β) distribution are $\mu = \alpha\beta$ and $\sigma^2 = \alpha\beta^2$. Therefore, the value of the population SNR is given by

$$SNR = \sqrt{\alpha} \quad (5)$$

Clearly, SNR in equation (5) is free of the scale parameter β. For simulations purposes, we set β = 2 (arbitrary) and in order to get SNR= 10, 3.33, 2, 1, we must set α = 100, 11.0889, 4, 1, respectively.

4.1.5. Weibull distribution

Let x_1, x_2… x_n be an *iid* random sample from a Weibull distribution with scale parameter λ > 0 and shape parameter k > 0. Then the mean and variance of a random variable which has a Weibull distribution are $\mu = \lambda\Gamma(1+\frac{1}{k})$ and $\sigma^2 = \lambda^2\Gamma(1+\frac{2}{k}) - \mu^2$. Thus, the value of SNR is given by

$$SNR = \frac{\lambda\Gamma(1+\frac{1}{k})}{\sqrt{\lambda^2\Gamma(1+\frac{2}{k}) - \mu^2}} = \frac{\Gamma(1+\frac{1}{k})}{\sqrt{\Gamma(1+\frac{2}{k}) - \Gamma^2(1+\frac{1}{k})}} \quad (9)$$

For simulations purposes, and in order to get SNR= 10, 3.33, 2, 1, we set λ = 4 (arbitrary) and k = 12.16, 3.71, 2.105, 1, respectively.

4.2 Simulation results

To evaluate the performance of Sharma and Krishna [16] confidence interval, 5000 random samples were generated for each combination of n and SNR. The random samples were generated from each of the following distributions: normal, log-normal, χ^2, Gamma, and Weibull distributions. Coverage probabilities were calculated using SRS and RSS techniques. Table 2 presents the simulation results which reveals that for normal data and sample sizes greater than or equal to 25, coverage probabilities using RSS were closer to the 0.95 nominal value only when SNR=1. For log-normal data the interval performed the worse for all sample sizes and SNR values, with the highest coverage attained of 0.793. on the other hand, for χ^2, Gamma, and Weibull distributions, the interval attained coverage probability close to the nominal value for all sample sizes, using both RSS and SRS, only when SNR=1. As the SNR value increases from 1 to 10, the coverage probability got worse. Based on these simulations, this interval estimate of the SNR should not be used in interval estimation of the SNR unless the value of the SNR is close to one, with a preference of RSS over SRS especially when the data is normally distributed. In real life settings, it could be difficult for SNR to equal one for this interval to perform well. This is a severe limitation of the performance of this interval, which could not been identified without extensive simulations, given the appealing mathematical formation of this interval.

Table 2: Estimated coverage probabilities using normal (I), log-normal (II), χ^2 (III), Gamma (IV), and Weibull (V) distributions for sample sizes 15,25,50,100 and SNR of 1, 2, 3.33, and 10

Distribution	I	II	III	IV	V
			n=15		
SNR=1					
RSS	0.927	0.793	0.960	0.960	0.953
SRS	0.861	0.777	0.940	0.944	0.939
SNR=2					
RSS	0.778	0.695	0.802	0.800	0.845
SRS	0.727	0.692	0.763	0.776	0.805
SNR=3.33					
RSS	0.580	0.551	0.596	0.590	0.598
SRS	0.539	0.531	0.559	0.551	0.553
SNR=10					
RSS	0.211	0.229	0.221	0.216	0.181
SRS	0.215	0.206	0.211	0.209	0.175
			n=25		
SNR=1					
RSS	0.947	0.755	0.955	0.953	0.960
SRS	0.878	0.766	0.951	0.947	0.945
SNR=2					
RSS	0.795	0.677	0.798	0.809	0.851
SRS	0.719	0.680	0.779	0.785	0.823
SNR=3.33					
RSS	0.598	0.541	0.601	0.585	0.626
SRS	0.553	0.513	0.561	0.570	0.548
SNR=10					
RSS	0.220	0.238	0.227	0.234	0.183
SRS	0.208	0.216	0.214	0.209	0.177
			n=50		
SNR=1					
RSS	0.968	0.741	0.966	0.966	0.968
SRS	0.882	0.741	0.951	0.954	0.950
SNR=2					
RSS	0.817	0.696	0.811	0.819	0.868
SRS	0.738	0.672	0.787	0.770	0.835
SNR=3.33					
RSS	0.613	0.527	0.621	0.609	0.651
SRS	0.550	0.518	0.575	0.561	0.578
SNR=10					
RSS	0.239	0.242	0.240	0.240	0.199
SRS	0.214	0.214	0.212	0.218	0.165
			n=100		
SNR=1					
RSS	0.966	0.715	0.965	0.964	0.966
SRS	0.881	0.718	0.949	0.949	0.950
SNR=2					
RSS	0.818	0.673	0.829	0.822	0.889
SRS	0.743	0.653	0.769	0.781	0.830
SNR=3.33					
RSS	0.616	0.542	0.630	0.613	0.661
SRS	0.552	0.517	0.573	0.572	0.581
SNR=10					
RSS	0.240	0.228	0.244	0.254	0.196
SRS	0.215	0.217	0.214	0.204	0.170

5. Concluding remarks

In this paper, a simulation study is conducted to evaluate the performance of Sharma and Krishna [16] confidence interval of SNR. Data were simulated from normal, log-normal, χ^2, Gamma, and Weibull distributions with sample sizes 15, 25, 50, and 100 with SNR values of 1, 2, 3.33, and 10. A ranked set sampling strategy is proposed instead of simple random sampling which improved the performance as measured by coverage probability. Even though this confidence interval for estimating the SNR has an appealing statistical reasoning, it was found that the performance of this interval is totally dependent on the value of the SNR, with best performance when SNR is one. This interval should not be used when the data follow a log-normal distribution due to low coverage probability. For data from χ^2, Gamma, and Weibull, the interval attained the nominal value coverage of 0.95 when SNR=1 using either RSS or SRS. For data from normal distribution, the interval attained nominal coverage value when SNR=1 and a sample size greater than or equal to 25 (not surprising since this interval is based on an asymptotic result), with RSS performing better than SRS in terms of coverage probability. The more noise introduced in the data, the worse the performance of this interval. This was noticed across all distributions discussed. The conclusions of this paper are restricted to the set of simulation conditions we have considered. For definite statement, one might need more simulation under different conditions. All simulations were performed using *R* [13] statistical software.

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