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Research paper

A new extended distribution and its application to using survival analysis of cancer patients

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Abstract

Medical research is one of the most important aspects of statistical analysis and application. In this paper, present a new three-parameter continuous distribution referred to as power exponentiated Shanker distributions with application to real-life datasets. The proposed distribution possesses a density function, a distribution function with three parameters, a survival function, and a hazard function. They studied the nature of the distribution with the help of its statistical properties, including moments, moment-generating functions, and entropy. The probability density function of order statistics for this distribution is also obtained. The classical distribution is the estimation of parameters by using the technique of maximum likelihood estimation. The application of the model selection technique criteria AIC, BIC, AICC, and goodness of fit on two real data sets is finally presented and compared to the fit attained by some other well-known distributions.

Keywords: *Entropy; Moments; Maximum Likelihood Estimation; Order Statistics; Power Exponentiated.*

1. Introduction

Medical research is mostly interested in studying the survival of cancer patients, as applied to statistical research. The statistical distributions have been extensively utilized for analyzing time-to-event data, also referred to as survival or reliability data, in different areas of applicability, including medical science. In recent years, an impressive set of new statistical distributions has been explored by statisticians. The necessity of developing an extended class of classical distribution is analysis, biomedicine, reliability, insurance, and finance. Recently, many researchers have been working on this area and have proposed new methods to develop improved probability distributions with utility. A new family of distribution, namely the exponentiated exponential distribution was introduced by Gupta et al. [4]. Gupta and Kundu [3] the family have two parameters (scale and shape) of the Weibull or gamma family; properties of the distribution were studied. Mud Holkar and Srivastava [7] introduced by exponentiated Weibull family of the distribution. Pal et al. [9] discussed by exponentiated Weibull distribution has been compared with the two parameter Weibull and gamma distribution with respect to failure rate. Samir et. al [1] the exponentiated power Lindley distribution, the distribution is containing as special-sub models some widely well-known distribution. Chrisogonus keleeh onyekwere [8] the author discussed by exponentiated Rama distribution properties and application. Rajitha and Vaishnavi. [11]. Power Exponentiated Weibull Distribution: Application in Survival Rate of Cancer Patients. New techniques for extending lifetime distribution have piqued the curiosity of researchers. Kanak Modi [6] introduced a new method that adds two shape parameters $\beta, v > 0$ to an arbitrary base line distribution called the power exponentiated family of continuous distributions. With a cumulative distribution (cdf) defined as

$$F(x) = \frac{v^{(G(x))^{\beta}} - 1}{v - 1} \quad (1)$$

And the probability distribution function (pdf) defined as

$$f(x) = \frac{\beta v^{(G(x))^{\beta}} \ln v (G(x))^{\beta-1} g(x)}{v - 1}$$

Where X is a continuous random variable whose baseline (cdf) is $G(x, \theta)$ a vector of parameter (θ).
Rama Shanker [10] introduced the shanker distribution with properties and its application. The proposed distribution can be obtained by assuming $G(x)$ the shanker distribution with shape parameters $\theta > 0$. Thuse, the cdf and probability density function (pdf) of the shanker distribution are obtained.

$$G(x) = 1 - \frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}$$

$g(x) = \frac{\theta^2}{\theta^2+1}(\theta + x)e^{-\theta x}$

The proposed distribution describes the survival time by analyzing some cancer patients. Additionally, another objective is to estimate the unknown model parameters using a maximum likelihood estimator. In this paper, we adopt the idea of a proposing a new three-parameter distribution. The proposed distribution, such as the power exponentiated shanker distribution, is also shown over the other well-known classical distributions.
The present paper is organized, as a in section 2, derive the pdf and cdf of the proposed distribution. In section 3, discusses in reliability analysis of (PES) distribution. In section, 4 some of the statistical properties of the proposed distribution are discussed. In section 5, hormonic mean are derived. In section 6, derive the mean deviation. Also derive mean deviation from median, In section 7. In section 8, explore the limiting distribution of order statistics. In section 9, likelihood ratio test has been derived. In section 10, Bonferroni and Lorene curves are obtained. Entropies are derived in section 11. The maximum likelihood estimators of the model parameter are derived in section 12. Finally, different application of the (PES) distribution on two real data sets are presented in section 13. All computations throughout this paper were performed using the statistical programing language R.

1.1. Power exponentiated family

The power exponentiated family of probability distributions to life time model. The cdf $F(x)$ and pdf $f(x)$ of power exponentiated family is given by

$$F(x) = \frac{v^{(G(x))^{\beta}} - 1}{v-1} \tag{1}$$

And

$$f(x) = \frac{\beta v^{(G(x))^{\beta}} \ln v((Gx))^{\beta-1} g(x)}{v-1}, x > 0, v > 0, \beta > 0 \tag{2}$$

1.2. Shanker distribution

The shanker distribution is well-known distribution and its importance in study
A continuous random variable X has Shanker distribution, if its cdf $G(x)$ and pdf $g(x)$ is given by

$$G(x) = 1 - \frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1} \tag{3}$$

$$g(x) = \frac{\theta^2}{\theta^2+1}(\theta + x)e^{-\theta x} \tag{4}$$

2. Power exponentiated shanker distribution

The power exponentiated family using cdf and pdf defined in (3) and (4) respectively, we proposed a new power exponentiated shanker distribution. Thus, the power exponentiated shanker distribution with β and v as shape parameters and θ as scale parameter, we have obtained
The pdf of Power Exponentiated Shanker Distribution (PESD) is given by

$$F(x) = \frac{v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}} - 1}{v-1} \tag{5}$$

$$f(x) = \frac{\beta v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}} \ln v\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta-1} \frac{\theta^2}{\theta^2+1}(\theta+x)e^{-\theta x}}{v-1}, x > 0,\ \beta, v > 0,\ \text{and } \theta > 0$$

$$f(x) = \frac{\beta\, \theta^2 \ln v}{(v-1)(\theta^2+1)} \sum_{i=0}^{\infty} \frac{(\ln v)^i}{i!} \left(1 - \frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1} (\theta + x)e^{-\theta x} \tag{6}$$

Then, using the following power series expansion is defined as

$$a^n = \sum_{i=0}^{\infty} \frac{(x \ln a)^i}{i!} \tag{7}$$

Where, a and x are any real numbers.

Then, the binomial series expansion, is defined as

$$(1 - z)^a = \sum_{j=0}^{\infty} (-1)^j \binom{a}{j} z^j \tag{8}$$

If z is a positive real non-integer.
$(a + b)^z = \sum_{k=0}^{\infty} \binom{z}{k} a^k b^{-k}$

For n, a positive integer.

$$(1+x)^k = \sum_{l=0}^{\infty} \binom{k}{l} x^l$$

3. Reliability analysis

In this section, we will discuss the reliability function, hazard function, reverse hazard function, cumulative hazard function, Odds rate, Mills ratio and, Mean Residual function for the proposed (PES) distribution.

3.1. Survival function

The survival function of Power Exponentiated Shanker (PES) distribution is obtained as

$$S(x) = 1 - F(x)$$

$$S(x) = 1 - \frac{v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}} - 1}{v-1}$$

$$S(x) = \left(\frac{v - v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}}}{v-1}\right)$$

3.2. Hazard rate function

The hazard rate function of power Exponentiated Shanker distribution is given by $h(x) = \frac{f(x)}{1-F(x)}$ is an important measure for characterizing life phenomenon.

$$h(x) = \left(\frac{\frac{\beta\theta^2}{(\theta^2+1)} v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}} \ln v \left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta-1} (\theta+x)e^{-\theta x}}{v - v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}}}\right)$$

3.3. Revers hazard rate

The Revers hazard rate of (PES) distribution is obtained as

$$h_r(x) = \frac{f(x)}{F(x)}$$

$$h_r(x) = \left(\frac{\beta\, \theta^2 \ln v \sum_{i=0}^{\infty} \frac{(\ln v)^i}{i!} \left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1} (\theta+x)e^{-\theta x}}{(\theta^2+1)\, v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}} - 1}\right)$$

3.4. Cumulative hazard function

The Cumulative hazard function of (PES) distribution is obtained as

$$H(x) = -\ln(1 - F(x))$$

$$H(x) = -\ln\left(\frac{v - v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}}}{v-1}\right)$$

3.5. Odds rate function

The Odds rate function of (PES) distribution is obtained as

$$O(x) = \frac{F(x)}{1-F(x)}$$

$$O(x) = \left(\frac{v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}} - 1}{v - v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}}}\right)$$

3.6. Mean residual function

The mean residual function of (PES) distribution is obtained as

$$M(x)=\frac{1}{S(x)}\int_x^{\infty} t\, f(t)dt-x$$

$$M(x)=\frac{1}{S(x)}\int_x^{\infty} t\,\frac{\beta\,\theta^2 \ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta t\, e^{-\theta t}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+t)e^{-\theta t}dt-x$$

Then, the following binomial series expansion equation (8), and simplify the expression is

$$M(x)=\frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\int_x^{\infty} t^{k+1}(\theta+t)e^{-\theta t(j+1)}dt$$

Let, assuming $u=\ e^{-\theta x(j+t+1)}$ then the mean residual function is given by
Then, solving the integral also obtained as,
Upper incomplete gamma function is defined as

$$\Gamma(s,x)=\int_x^{\infty} t^{s-1}e^{-t}dt$$

$$M(x)=\frac{\beta\,\theta^{2l}\ln v}{v-v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta}}(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\left(\frac{\theta^2(j+1)\Gamma(k+2,\theta x(j+1))+\Gamma(k+3,\theta x(j+1))}{\theta(j+1)^{k+3}}\right)-x$$

4. Statistical properties

In this section, we derived the structural properties, the moment generating function, Characteristic function and r^th moment for the (PES) distribution of the random variable is also derived. Including, the mean and variance investigated.

4.1. Moments

If a random variable X has the pdf of power exponentiated shanker distribution, then the corresponding r^{th} is given

$$E(X^r)=\mu_r'=\int_0^{\infty}x^r\ f(x)\,dx$$

$$\mu_r'=\int_0^{\infty}x^r\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}dx \tag{9}$$

Using the following binomial series expansion equation (8), and simplify the expression.

$$\mu_r'=\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{k}{l}\int_0^{\infty}x^{r+k}(\theta+x)\,e^{-\theta x(j+1)}dx$$

To, solving the integral also obtained as

$$\mu_r'=\frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{k}{l}\frac{\theta^2(j+1)\Gamma(r+k+1)+\Gamma(r+k+2)}{\theta^r(j+1)^{r+k+2}} \tag{10}$$

Where $\Gamma(.)$ is the gamma function. Subsequently, the mean and variance of (PES) distribution is obtained as substituting $r=1,2$ in equation (10)

$$\mu_1'=\frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{k}{l}\frac{\theta^2(j+1)\Gamma(k+2)+\Gamma(k+3)}{\theta(j+1)^{k+3}}$$

$$\mu_2'=\frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{k}{l}\frac{\theta^2(j+1)\Gamma(k+3)+\Gamma(k+4)}{\theta^2(j+1)^{k+4}}$$

4.2. Moment generating function and characteristic function

If a random variable X has the power exponentiated shanker distribution pdf is given then the corresponding r^{th} moments is obtained as

$$M_X(t)=E(e^{tx})=\int_0^{\infty}e^{tx}\ f(x)\,dx$$

$$M_X(t)=\int_0^{\infty}e^{tx}\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}dx$$

Then, using the following binomial series expansion equation (8), and simplify the expression

$$M_X(t) = \frac{\beta\,\theta^{k+2+2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\int_0^{\infty}x^k(\theta+x)\,e^{-\theta x(j+t+1)}\,dx$$

Let, assuming $u = e^{-\theta x(j+t+1)}$ then the corresponding r^{th} moments are given by
Solving the integral also obtained as

$$M_X(t) = \frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\frac{\theta^2(j+t+1)\Gamma(k+1)+\Gamma(k+2)}{\theta(j+t+1)^{k+2}}$$

Similarly, the characteristic function of (PES) distribution can be obtained as

$$\phi_X(t) = M_X(e^{itx})$$

$$\phi_X(t) = \frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\frac{\theta^2(j+it+1)\Gamma(k+1)+\Gamma(k+2)}{\theta(j+it+1)^{k+2}}$$

5. Hormonic mean

The Hormonic mean of the (PES) distribution is defined as

$$H.M = \int_0^{\infty}\frac{1}{x}f(x)dx$$

$$H.M = \int_0^{\infty}\frac{1}{x}\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}\,dx$$

Then, using substation method $u = e^{-\theta x(j+1)}$ then the hormonic mean is given by

$$H.M = \frac{\beta\,\theta^{k+2l+2}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\int_0^{\infty}x^{k-1}(\theta+x)e^{-\theta x(j+1)}\,dx$$

To solving the integral also obtained as

$$H.M = \frac{\beta\,\theta^{2l+2}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\frac{\theta^2(j+1)\Gamma(k)+\Gamma(k+1)}{\theta(j+1)^{k+1}}$$

6. Mean deviation

Let X be a random variable from (PES) distribution with mean μ. Then the deviation from mean is defined as

$$D(\mu) = E(|X-\mu|)$$

$$D(\mu) = \int_0^{\infty}|X-\mu|\ f(x)dx$$

$$D(\mu) = \int_0^{\mu}(\mu-x)f(x)dx + \int_{\mu}^{\infty}(x-\mu)f(x)dx$$

$$D(\mu) = \mu\int_0^{\mu}f(x)dx - \int_0^{\mu}x\,f(x)dx + \int_{\mu}^{\infty}x\,f(x)dx - \int_{\mu}^{\infty}\mu\,f(x)dx$$

$$D(\mu) = \mu F(\mu) - \int_0^{\mu}x\,f(x)dx - \mu[1-F(\mu)] + \int_{\mu}^{\infty}x\,f(x)dx$$

$$D(\mu) = 2\mu F(\mu) - 2\int_0^{\mu}x\,f(x)dx$$

Then,

$$\int_0^{\mu}x\,f(x)dx = \int_0^{\mu}x\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}\,dx$$

Then, using the following binomial series expansion in equation (8), and simplify the expression

$$= \frac{\beta\,\theta^{k+2l+2}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\int_0^{\mu}x^{k+1}(\theta+x)e^{-\theta x(j+1)}\,dx$$

Let, assuming $u = e^{-\theta x(j+1)}$ then the mean deviation is given by

$$= \frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\left(\frac{\theta^2(j+1)\Gamma(k+2,\theta\mu(j+1))+\Gamma(k+3,\theta\mu(j+1))}{\theta(j+1)^{k+3}}\right)$$

$$D(\mu) = 2\mu \frac{v^{\left(1-\frac{(\theta^2+1)+\theta\mu\, e^{-\theta\mu}}{\theta^2+1}\right)^{\beta}} - 1}{v-1} - 2\frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{G(x)^{\beta}}\sum_{j=0}^{\beta i+\beta-1}\sum_{k=0}^{j}\sum_{l=0}^{k}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\left(\frac{\theta^2(j+1)\Gamma(k+2,\theta\mu(j+1))+\Gamma(k+3,\theta\mu(j+1))}{\theta(j+1)^{k+3}}\right)$$

7. Mean deviation from median

Let X be a random variable from (PES) distribution with median M. Then the mean deviation from median is defined as

$$D(M) = E(|X-M|)$$

$$D(M) = \int_0^{\infty}|X-M|\ f(x)dx$$

$$D(M) = \int_0^{M}(M-x)f(x)dx + \int_M^{\infty}(x-M)f(x)dx$$

$$D(M) = MF(M) - \int_0^{M}x\,f(x)dx - M[1-F(M)] + \int_M^{\infty}x\,f(x)dx$$

$$D(M) = \mu - 2\int_0^{M}x\,f(x)dx$$

Then,

$$\int_0^{M}x\,f(x)dx = \int_0^{M}x\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}\,dx$$

Let, assuming $u = e^{-\theta x(j+1)}$ then the Midian deviation is given by

$$D(M) = \mu - 2\frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\left(\frac{\theta^2(j+1)\Gamma(k+2,\theta M(j+1))+\Gamma(k+3,\theta M(j+1))}{\theta(j+1)^{k+3}}\right)$$

8. Order statistics

The derived pdf of the i^{th} order statistics of the power exponentiated shanker distribution. Let $X_1, X_2, \dots, X_n$ be a simple random sample from power exponentiated shanker distribution with cdf and pdf given by (5) and (6), respectively. Let $X_{(1:n)} \le X_{(2:n)} \le \dots \le X_{(n:n)}$ denote the order statistics obtained from this sample. We now given the pdf of $X_{r:n}$, say $f_{r;n}(x)$ of $X_{r:n}$, $i = 1,2,\dots,n$. The pdf of the r^{th} order statistics $X_{r:n}$, $r = 1, 2, \dots, n$ is given by

$$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(F(x)\right)^{r-1}\left(1-F(x)\right)^{n-r}f(x), x > 0 \tag{11}$$

Where $F(.)$ and $f(.)$ are given by (5) and (6) respectively,

$$C_{r:n} = \frac{n!}{(r-1)!(n-r)!}$$

$$f_{r:n} = C_{r:n}\left(F(x)\right)^{r-1}\left(1-F(x)\right)^{n-r}f(x)$$

Using the following binomial series expansion in equation (8), and simplify the expression.

$$f_{r:n} = C_{r:n}\sum_{s=0}^{\infty}\binom{n-r}{s}(-1)^s\left(F(x)\right)^{(r+s-1)}f(x)$$

$$f_{r:n} = C_{r:n}\sum_{s=0}^{\infty}\binom{n-r}{s}(-1)^s\left(\frac{v^{\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta}}-1}{v-1}\right)^{(r+s-1)}\frac{\beta\theta^2}{(v-1)(\theta^2+1)}v^{\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta}}\ln v\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta-1}(\theta+x)e^{-\theta x}$$

$$f_{r:n} = C_{r:n}\frac{\beta\theta^2\ \ln v}{(v-1)(\theta^2+1)}\sum_{s=0}^{\infty}\sum_{q=0}^{\infty}\binom{n-r}{s}\binom{\beta(r+s-1)}{q}(-1)^{r+2s+q-1}\frac{1}{(1-v)^{(r+s-1)}}\ v^{\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta(q+1)}}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta-1}(\theta+x)e^{-\theta x}$$

First order statistics

$$f_{1:n} = C_{1:n}\frac{\beta\theta^2\ \ln v}{(v-1)(\theta^2+1)}\sum_{s=0}^{\infty}\sum_{q=0}^{\infty}\binom{n-1}{s}\binom{\beta(s)}{q}(-1)^{2s+q}\frac{1}{(1-v)^{(s)}}\ v^{\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta(q+1)}}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta-1}(\theta+x)e^{-\theta x}$$

n^{th} order statistics

$$f_{n:n} = C_{n:n} \frac{\beta\theta^2 \ln v}{(v-1)(\theta^2+1)} \sum_{q=0}^{\infty} \binom{\beta(n+s-1)}{q} (-1)^{n+2s+q-1} \frac{1}{(1-v)^{(n+s-1)}} v^{\left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta(q+1)}} \left(1-\frac{(\theta^2+1)+\theta x\, e^{-\theta x}}{\theta^2+1}\right)^{\beta-1} (\theta+x)e^{-\theta x}$$

9. Likelihood ratio test

In this section, we derive the likelihood ratio test from the (PES) distribution.
Let $x_1, x_2, x_3, \dots, x_n$ be a random sample from the (PES) distribution.
To testing the hypothesis, we have the null and alternative hypothesis.
$H_0 : f(x) = f(x, \theta)$ against $H_1 : f(x) = g(x)$
In test whether the random sample of size n comes from the Shanker distribution or Power Exponentiated Shanker distribution, the following test statistics is used.

$$\Delta = \frac{L_1}{L_2} = \prod_{i=1}^{n} \frac{f(x_i,\theta)}{g(x_i)}$$

$$\Delta = \prod_{i=1}^{n} \left(\frac{\frac{\beta\theta^2}{(v-1)(\theta^2+1)} v^{\left(1-\frac{(\theta^2+1)+\theta x_i\, e^{-\theta x_i}}{\theta^2+1}\right)^{\beta}} \ln v \left(1-\frac{(\theta^2+1)+\theta x_i\, e^{-\theta x_i}}{\theta^2+1}\right)^{\beta-1} (\theta+x_i)e^{-\theta x_i}}{\frac{\theta^2}{\theta^2+1}(\theta+x_i)e^{-\theta x_i}} \right)$$

$$\Delta = \prod_{i=1}^{n} \left(\frac{\beta \ln v}{(v-1)} v^{\left(1-\frac{(\theta^2+1)+\theta x_i\, e^{-\theta x_i}}{\theta^2+1}\right)^{\beta}} \left(1-\frac{(\theta^2+1)+\theta x_i\, e^{-\theta x_i}}{\theta^2+1}\right)^{\beta-1} \right)$$

Then, using the following binomial series expansion in equation (8), and simplify the expression

$$\Delta = \left(\frac{\beta \ln v}{(v-1)}\right)^{n} \sum_{i=0}^{\infty} \frac{(\ln v)^i}{i!} \prod_{i=1}^{n} \left(1-\frac{(\theta^2+1)+\theta x_i\, e^{-\theta x_i}}{\theta^2+1}\right)^{\beta i+\beta-1}$$

$$\Delta = \frac{L_1}{L_2} \left(\frac{\beta\theta^{k+2l} \ln v}{(v-1)(\theta^2+1)^j}\right)^{n} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\beta i+\beta-1}{j} \binom{j}{k} \binom{j-k}{l} \frac{(\ln v)^i (-1)^j}{i!} \prod_{i=1}^{n} x_i^k$$

Then, rejected the null hypothesis if

$$\Delta = \left(\frac{\beta\theta^{k+2l} \ln v}{(v-1)(\theta^2+1)^j}\right)^{n} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\beta i+\beta-1}{j} \binom{j}{k} \binom{j-k}{l} \frac{(\ln v)^i (-1)^j}{i!} \prod_{i=1}^{n} x_i^k > k$$

Equivalently, also reject the null hypothesis

$$\Delta^* = \prod_{i=1}^{n} x_i^{j-k} > k \left(\frac{\beta\theta^{k+2l} \ln v}{(v-1)(\theta^2+1)^j}\right)^{n} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\beta i+\beta-1}{j} \binom{j}{k} \binom{j-k}{l} \frac{(\ln v)^i (-1)^j}{i!}$$

$\Delta^* = \prod_{i=1}^{n} x_i^k > k^*$ where k^*

$$= k \left(\frac{\beta\theta^{k+2l} \ln v}{(v-1)(\theta^2+1)^j}\right)^{n} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \binom{\beta i+\beta-1}{j} \binom{j}{k} \binom{j-k}{l} \frac{(\ln v)^i (-1)^j}{i!}$$

For large sample size n, $2 \log \Delta$ is distribution as chi-square variates with one degree of freedom. Thus, we rejected the null hypothesis, when the probability value is given by $p(\Delta^* > \alpha^*)$, where $\alpha^* = \prod_{i=1}^{n} x_i^k$ is less than level of significance and $\prod_{i=1}^{n} x_i^k$ is the observed value of the statistics Δ^*.

10. Bonferroni and Lorenz curves

In this section, we have derived the Bonferroni and Lorenz curves from the (PES) distribution.
The Bonferroni and Lorenz curve is a powerful tool in the analysis of distributions and has applications in many fields, such as economies, insurance, income, reliability, and medicine. The Bonferroni and Lorenz cures for a X be the random variable of a unit and $f(x)$ be the probability density function of x. $f(x)dx$ will be represented by the probability that a unit selected at random is defined as

$$B(p) = \frac{1}{p\mu} \int_0^q x\, f(x) dx \quad L(p) = p\, B(p)$$

And

Where, $q = F^{-1}(p);\ q \epsilon [0,1]$ and $\mu = E(X)$

$$B(p)=\frac{1}{p\mu}\int_0^q x\,\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}\,dx$$

Then, using the following binomial series expansion in equation (8), and simplify the expression is

$$B(p)=\frac{\beta\,\theta^{k+2l+2}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\int_0^q x^{k+1}(\theta+x)e^{-\theta x(j+1)}\,dx$$

Then, assuming $u=\ e^{-\theta x(j+1)}$ then the Bonferroni and Lorenz curves is given by
Let's make a substitution to simplify the integral is

$$B(p)=\frac{\beta\,\theta^{2l}\ln v}{(v-1)(\theta^2+1)^{j+1}}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j(\ln v)^i}{i!}\binom{\beta i+\beta-1}{j}\binom{j}{k}\binom{j-k}{l}\left(\frac{\theta^2(j+1)\Gamma(k+2,\theta q(j+1))+\Gamma(k+3,\theta q(j+1))}{\theta(j+1)^{k+3}}\right)$$

After simplification we get

$$B(p)=\left(\frac{\theta^2(j+1)\Gamma(k+2)+\Gamma(k+3)}{p\left(\theta^2(j+1)\Gamma(k+2,\theta q(j+1))+\Gamma(k+3,\theta q(j+1))\right)}\right)$$

$$L(p)=\left(\frac{\theta^2(j+1)\Gamma(k+2)+\Gamma(k+3)}{\theta^2(j+1)\Gamma(k+2,\theta q(j+1))+\Gamma(k+3,\theta q(j+1))}\right)$$

11. Entropies

In this section, we derived the Rényi entropy, and Tsallis entropy from the (PES) distribution.
It is well known that entropy and information can be considered measures of uncertainty, or the randomness of a probability distribution. It is applied in many fields, such as engineering, finance, information theory, and biomedicine. The entropy functionals for probability distribution were derived on the basis of a variational definition of uncertainty measure.

11.1. Rényi entropy

Entropy is obtained as a random variable X is a measure of the variation of the uncertainty. It is used in many fields, such as engineering, statistical mechanics, finance, information theory, biomedicine, and economics. The entropy measure is the Rényi of order which is defined as

$$R_\lambda=\frac{1}{1-\lambda}\log\int_0^{\infty}[f(x)]^{\lambda}\,dx\,;\lambda>0,\lambda\neq 1$$

$$R_\lambda=\frac{1}{1-\lambda}\log\int_0^{\infty}\left(\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}\right)^{\lambda}dx$$

$$R_\lambda=\frac{1}{1-\lambda}\log\int_0^{\infty}\left(\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\right)^{\lambda}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\lambda(\beta i+\beta-1)}(\theta+x)^{\lambda}e^{-\theta\lambda x}\,dx$$

Then, using the following binomial series expansion in equation (8), and simplify the expression is

$$R_\lambda=\frac{1}{1-\lambda}\log\left(\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)^{j+1}}\right)^{\lambda}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\frac{(-1)^j\lambda^i(\ln v)^i}{i!}\binom{\lambda(\beta i+\beta-1)}{j}\binom{j}{k}\binom{j-k}{l}\binom{\lambda}{m}\int_0^{\infty}x^{k+m}\,e^{-\theta(\lambda+j)x}\,dx$$

To, solving the integral also obtained as

$$R_\lambda=\frac{1}{1-\lambda}\log\left(\frac{\beta\,\theta^{2l+m+2}\ln v}{(v-1)(\theta^2+1)^{j+1}}\right)^{\lambda}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\frac{(-1)^j\lambda^i(\ln v)^i}{i!}\binom{\lambda(\beta i+\beta-1)}{j}\binom{j}{k}\binom{j-k}{l}\binom{\lambda}{m}\frac{\Gamma(k+m+1)}{\theta^{m+1}(\lambda+j)^{k+m+1}}$$

11.2 Tsallis entropy

The Boltzmann-Gibbs (B-G) statistical properties initiated by Tsallis have received a great deal of attention. This generalization of (B-G) statistics was first proposed by introducing the mathematical expression of Tsallis entropy (Tsallis, (1988) for continuous random variables, which is defined as

$$T_\lambda=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}[f(x)]^{\lambda}\,dx\right);\lambda>0,\lambda\neq 1$$

$$T_\lambda=\frac{1}{\lambda-1}\left(1-\int_0^{\infty}\left(\frac{\beta\,\theta^2\ln v}{(v-1)(\theta^2+1)}\sum_{i=0}^{\infty}\frac{(\ln v)^i}{i!}\left(1-\frac{(\theta^2+1)+\theta x\,e^{-\theta x}}{\theta^2+1}\right)^{\beta i+\beta-1}(\theta+x)e^{-\theta x}\right)^{\lambda}dx\right)$$

Then, using the following binomial series expansion in equation (8), and simplify the expression is

$$T_\lambda = \frac{1}{\lambda-1}\left(1-\left(\frac{\beta\,\theta^2 \ln v}{(v-1)(\theta^2+1)^{j+1}}\right)^{\lambda} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^j \lambda^i (\ln v)^i}{i!}\binom{\lambda(\beta i+\beta-1)}{j}\binom{j}{k}\binom{j-k}{l}\binom{\lambda}{m} \times \int_0^\infty x^{k+m}\, e^{-\theta(\lambda+j)x}\,dx\right)$$

Solving the integral also obtained as

$$T_\lambda = \frac{1}{\lambda-1}\left(1-\left(\frac{\beta\,\theta^2 \ln v}{(v-1)(\theta^2+1)^{j+1}}\right)^{\lambda} \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^j \lambda^i (\ln v)^i}{i!}\binom{\lambda(\beta i+\beta-1)}{j}\binom{j}{k}\binom{j-k}{l}\binom{\lambda}{m} \times \frac{\Gamma(k+m+1)}{\theta^{m+1}(\lambda+j)^{k+m+1}}\right)$$

12. Estimations of parameter

In this section, the maximum likelihood estimates of the power exponentiated shanker distribution (PESD) parameter is given.

12.1. Maximum likelihood estimation (MLE)

Consider $x_1, x_2, x_3, \dots, x_n$ be a random sample of size n from the power exponentiated shanker distribution with parameter β, θ and v the likelihood function, which is defined as

$$L(x;\beta,\theta,v) = \prod_{i=1}^{n} f(x_i;\beta,\theta,v)$$

$$L(x;\beta,\theta,v) = \prod_{i=1}^{n}\left(\frac{\beta\theta^2}{(v-1)(\theta^2+1)} v^{\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right)^{\beta}} \ln v \left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right)^{\beta-1} (\theta+x_i)\, e^{-\theta x_i}\right)$$

Then its log-likelihood function is given by

$$\ell = \log L = n\ln\beta + n\ln\theta^2 - n\ln(\theta^2+1) + n\ln(\ln v) - n\ln(v-1) - \theta\sum_{i=1}^{n} x_i + (\beta-1)\sum_{i=1}^{n}\ln\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right) + \ln v \sum_{i=1}^{n}\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right)^{\beta} + \sum_{i=1}^{n}(\theta+x_i)$$

Differentiating with respect to β, θ and v

$$\frac{\partial \log L}{\partial v} = \frac{n}{v\ln v} - \frac{n}{v-1} + \sum_{i=1}^{n}\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right)^{\beta} = 0 \quad (12)$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} + \ln v \sum_{i=0}^{n}\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right)^{\beta} \ln\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right) + \sum_{i=1}^{n}\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right) = 0 \quad (13)$$

$$\frac{\partial \log L}{\partial \theta} = \frac{n2}{\theta} - \frac{n2\theta}{(\theta^2+1)} - \sum_{i=1}^{n} x_i - (\beta-1)\sum_{i=1}^{n}\frac{x_i^2(1-\theta^2)}{(\theta^2+1)\theta x_i} - \beta\ln v\sum_{i=1}^{n}\left(1-\frac{(\theta^2+1)+\theta x_i e^{-\theta x_i}}{\theta^2+1}\right)^{\beta-1}\frac{x_i^2(1-\theta^2)}{(\theta^2+1)\theta x_i} + 1 = 0 \quad (14)$$

The equation (12), (13) and (14) gives the maximum likelihood estimation of the parameters for the (PES) distribution. Although, the equation cannot be solved analytically, thus we solved numerically using R programming with data set.

13. Applications

Dat set. 1: This data consists of the life time (in years) of 40-blood cancer (leukemia) patients from one of ministry of health hospitals in Sdudhi Arabia reported in (25). This actual data is:

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036	2.162
2.211	2.370	2.532	2.693	2.805	2.910	2.912	3.192	3.263	3.348
3.427	3.499	3.534	3.767	3.751	3.858	3.986	4.049	4.244	4.323
4.381	4.392	4.397	4.647	4.753	4.929	4.973	5.074	5.381	

Data Set. 2: The Data Under Consideration are the Life Times of 20 Leukemia Patients Who Were Treated by A Certain Drug (20). the Data Are:

1.013	1.034	1.109	1.226	1.509	1.533	1.563	1.716	1.929	1.965	2.061	2.344	2.546
2.626	2.778	2.951	3.413	4.118	5.136							

To compare the goodness of fit of the fitted distribution, the following criteria: Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), Akaike Information Criteria Corrected (AICC) and $-2\log L$.
AIC, BIC, AICC and $-2\log L$ can be evaluated by using the formula as follows.

$$AIC = 2k - 2\log L,\; BIC = k\log n - 2\log L \text{ and } AICC = AIC + \frac{2k(k+1)}{(n-k-1)}$$

Where, $k =$ number of parameters, n sample size and $-2\log L$ is the maximized value of loglikelihood function.

Table 1: MLE's AIC, BIC, AICC, And -2Log L of the Fitted Distribution for the Given Data Set 1

Distribution	ML Estimates	-2log L	AIC	BIC	AICC
Power Exponentiated shanker Distribution	$\hat{\beta} = 1.7502840\ (0.8702718)$ $\hat{\theta} = 0.9903102\ (0.1214810)$ $\hat{v} = 21.6949251\ (35.7883473)$	139.4219	141.2556	142.9192	141.9414
Power Exponentiated Exponential Distribution	$\hat{\beta} = 2.2612389\ (1.0884226)$ $\hat{\theta} = 0.7897817\ (0.1106604)$ $\hat{v} = 25.7645262\ (39.0785412)$	140.925	146.925	151.9157	147.6108
Exponentiated Exponential Distribution	$\hat{\alpha} = 3.43564629\ (0.86309940)$ $\hat{\theta} = 0.60903417\ (0.09188663)$	146.6542	150.6542	153.9813	151.3382
Shanker	$\hat{\theta} = 0.54972161\ (0.05806214)$	144.7945	155.9545	157.6181	156.0597

Table 2: MLE's AIC, BIC, AICC, and -2logl of the Fitted Distribution for the Given Data Set 2

Distribution	ML Estimates	-2log L	AIC	BIC	AICC
Power Exponentiated shanker Distribution	$\hat{\beta} = 5.8552727(2.8921143)$ $\hat{\theta} = 1.2974573\ (0.5114641)$ $\hat{v} = 0.3380986\ (0.8529180)$	50.58192	56.58192	59.4152	58.18192
Power Exponentiated Exponential Distribution	$\hat{\beta} = 7.9230929\ (3.8378895)$ $\hat{\theta} = 1.0917748\ (0.4808820)$ $\hat{v} = 0.4081817\ (1.03372427)$	51.38647	57.38647	60.21978	58.98647
Exponentiated Exponential Distribution	$\hat{\alpha} = 8.0836867\ (3.9394382)$ $\hat{\theta} = 1.2260264\ (0.2644192)$	54.52526	60.52526	63.41414	62.12526
Shanker	$\hat{\theta} = 0.7124395\ (0.10777871)$	63.08856	65.08856	66.033	65.3107

From table 1 and 2, it can be clearly observed and seen from the results that the Power Exponentiated Shanker distribution have the lesser AIC, BIC, AICC, -2log L, and values as compared to the Power Exponentiated Exponential Distribution, Exponentiated Exponential Distribution, and Shanker distributions, which indicates that the Power Exponentiated Shanker distribution better fits than the Power Exponentiated Exponential Distribution, Exponentiated Exponential Distribution, and Shanker distributions. Hence, it can be concluded that the (PES) distribution leads to a better fit over the other distributions.

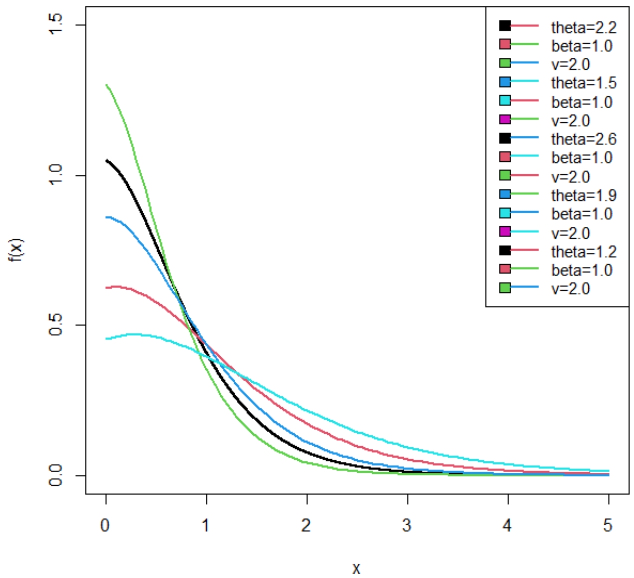

Fig. 1: PDF Plot of Power Exponentiated Shanker Distribution.

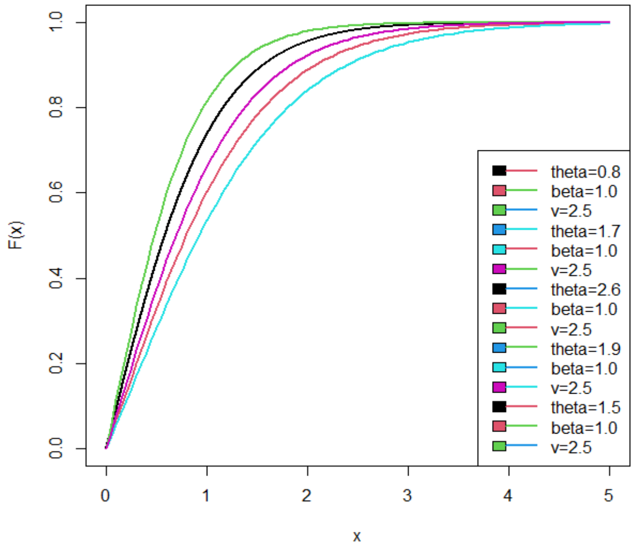

Fig. 2: CDF Plot of Power Exponentiated Shanker Distribution.

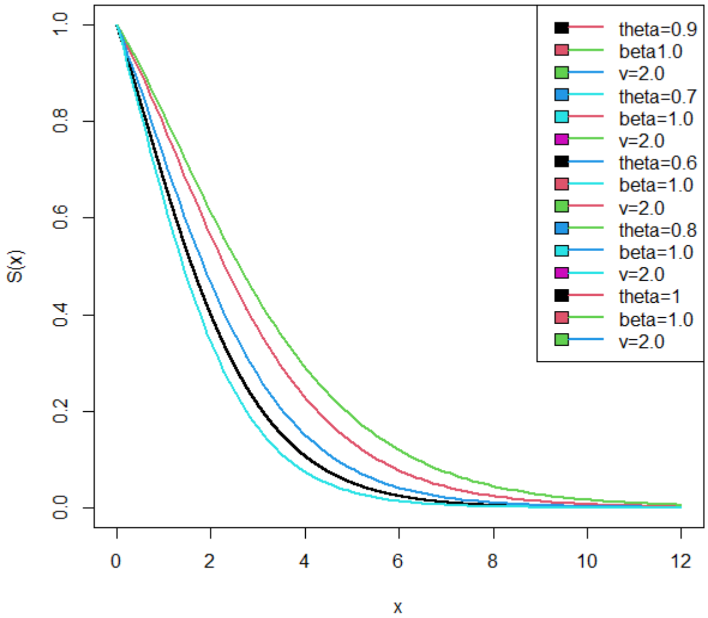

Fig. 3: Survival Plot of Power Exponentiated Shanker Distribution.

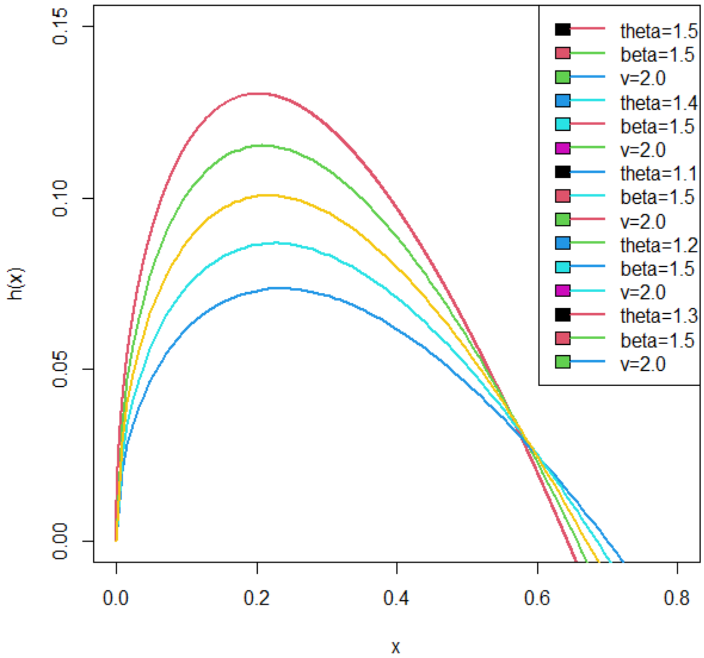

Fig. 4: Hazard Plot of Power Exponentiated Shanker Distribution.

14. Conclusion

The researchers' greatest concern has been selecting an appropriate model for fitting survival data. The Shanker distribution is one of the most well-liked distributions for lifespan data. In this paper, the shanker distribution is extended to provide a new distribution is extended to provide a new distribution called the power Exponentiated Shanker (PES) distribution to the model life time data. There are various specific cases that are addressed in the paper. The proposed distribution's many properties have been studied, including survival function and hazard function, moments, entropy's, the Bonferroni and Lorenz curves and order statistics. The Inference of parameters for a (PES) distribution was obtained using the method of the maximum likelihood estimator. When the parameter has been estimated using the method of maximum likelihood, good performance has been observed. Medical research for cancer patients is incredibly important and frequently uses statistical distributions. Thus, by applying this distribution to some actual data sets that determine the survival of various cancer patients, its value is demonstrated. The results indicate the superior performance of the (PES) distribution compared to the other competitive distributions by means of different goodness of fit-criteria. Overall, the proposed power exponentiated shanker distribution provides a better fit than other existing distributions.

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