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Research paper

Stochastic inventory control models for single item replenishment with variable demand under constrained and unconstrained conditions

Nse Udoh [1] *, Wisdom Ekpenyong [2]

[1] *Senior Academics, Department of Statistics, University of Uyo, Nigeria*
[2] *Research Scholars, Department of Statistics, University of Uyo, Nigeria*
**Corresponding author E-mail: nsesudoh@uniuyo.edu.ng*

Abstract

This work examined stochastic inventory control models for single-items with and without constraints in a building material company, with equal importance to similar distribution companies with large inventories. The study is an improvement on inventory control for single-items with probabilistic demand. The chi-square goodness-of-fit test was used to identify the best-fit probability distribution of the number of demands of each considered items. The different costs considered in this work were purchase cost with added tax-cost, ordering cost, holding cost and shortage cost while the items covered were: 2 inches zinc nail (in cartons) which number of demand follows a Weibull distribution with estimated parameters, $\alpha = 6$ and $\beta = 36$; 10mm rod (in lengths) which number of demand equally follows a Weibull distribution with estimated parameters, $\alpha = 2$ and $\beta = 607$; 5 inches nail (in cartons) which number of demand follows a normal distribution with estimated parameters, $\mu = 10.696$ and $\sigma = 0.3228$ and urban zinc (in bundles) which number of demand follows a Rayleigh distribution with estimated parameter, $\sigma = 74.827$. These estimated parameters were used for computing the mean of each item as the basis for obtaining the Economic Order Quantity for the respective items. A modified Hadley Whitin algorithm and the trimming methods were respectively used to obtain the optimum order quantity and reorder point of items for the replenishment models without constraint and with constraints. Results obtained from models with constraint yields lower total variable cost and were therefore recommended for single-item replenishment.

Keywords: *Unconstrained Models; Constrained Models; Variable Demand; Probability Distribution; Single Replenishment; Tax-Cost.*

1. Introduction

Inventory control refers to the process of managing stock levels to satisfy customers demand while also trying to minimize costs. Some key components include ordering policies, safety stock, lead time and demand. Stochastic inventory models are quite different from deterministic models because they consider demand as a random variable and not as constant. These models use probability distributions in describing the number of demand and aim at optimizing inventory policies under uncertainty. Mathematically, concepts relating to unconstrained variable demand assumes that demand can widely vary but not subjected to specific constraints while in constrained variable demand, its variability is usually limited by certain constraints like warehouse capacity, number of items constraint, budget restriction and many more. Researches in the past years, have considered Economic Order Quantity as a determining factor for the quantity of items a company or retailer should order to help minimize the total inventory cost.

The origin of modern inventory management principles can be traced back to [10] when the first inventory model was developed. This model was expanded by [23] who derived the formula for Economic Order Quantity (EOQ). He proposed that EOQ assumes a consistent demand rate and timely supply replenishment, but real-world scenario often feature unpredictable market conditions and varying demand rates. Furthermore, [7] studied some probabilistic models of the case, where both demand and lead procurement lead time are identically and independently distributed random variables.

Early literature on inventory control and its development include; [8], [4], [5], [11] and [15]. Specifically, [3] examined an integrated inventory control and inspection policies with deterministic demand while [20] proposed deterministic inventory lot-size models with time-varying demand and cost under generalized holding cost. Additionally, [17] introduced the concept of optimal inventory policy for items having linear demand variable deterioration rate with trade credit while the concept of periodic review probabilistic inventory system with zero lead time under some constraint and varying holding cost was eventually developed by [6]. Furthermore, constrained probabilistic economic order quantity model under varying order cost and zero lead time via geometric programming was developed by [12]. The use of KKT conditions was used by [1] to examine a multi-item probabilistic inventory models with perishable products and warehouse constraints while a multi-item multi-period inventory control model that considered expiration factor, all unit discount policy and warehouse constraints was later developed by [16]. The characteristics considered in this study were probabilistic demand, perishable products, and warehouse constraints for multi-item inventory models. The results of the developed inventory model provided two optimal ordering times, namely ordering time-based on warehouse capacity and joint order time. In addition, [9] investigated a probabilistic multi-item single –source inventory model with varying shortage costs while a pricing inventory model for non-instantaneous deteriorating items with partial backordering and price dependent stochastic demand under two – level trade credit policy was examined by [14].

Furthermore, [21] analyzed probability distributions of variable demand rates in a multi-item inventory problem. The demands of selected products were distributed according to different probability distributions namely; normal, Weibull and uniform distributions and the optimal order quantity, probabilities of shortage and no shortage for these selected items were also obtained. Furthermore, a multi-objective particle swarm optimization algorithm was used to optimize total inventory cost and inventory layout management in a multi-item inventory control model by [19]. In a later development, [22] developed an inventory model for determining optimum order quantities for four different selected items with different probability demand functions subject to constraints: warehouse space, limited capital, specified level of inventory and number of order constraints. The location parameter estimate of the respective probability distribution provides a dynamic description of the behavior and value of the demand rate over a period of time.

Hence, this work seeks a modification of the total variable cost by incorporating tax as part of purchase cost for single items replenishment models with variable demand. The effect of environmental condition (constrained and unconstrained) on the optimal order quantity Q_i^* and reorder point R_i^* of the model shall also be investigated using data from building materials company.

2. Methodology

2.1. Notations

The following notations will be used in this work:
n = total number of items being controlled simultaneously
f_i = floor area (storage space) required per unit of item i $(i = 1,2, \dots, n)$
W = warehouse space limit to store all items in the inventory
D = average demand within one planning period
θ = purchase cost per unit of item
t = tax-cost on each item
Q = order quantity
R = reordered point
K = ordering cost per order for single replenishment
M = shortage cost per unit
T_1 = trimming constant
Y = holding cost fraction per unit, per planning period
TVC = Total Variable Cost

2.2. Single item replenishment model without constraint

In line with [13], the following costs will be considered:
1) Purchasing cost
2) Ordering cost
3) Holding cost
4) Shortage cost

Where, Purchasing cost = purchase cost per uit × average demand (θD) (1)

$$\text{Ordering cost} = \frac{\text{ordering cost}}{\text{order}} \times \text{ordering frequency in a year}$$

$$= k \times \frac{D}{Q} \quad (2)$$

Holding cost = (holding cost per unit) X (the average of stored items)

$$= \theta Y \times \frac{Q+E[R-V]}{2} + \frac{E[R-V]}{2}$$

$$= \{\theta Y\} \times \frac{Q}{2} + R - E[V] \quad (3)$$

Where V represents the average demand of each item, during lead time.
Shortage cost = (shortage cost per unit)X (expected shortage)X (shortage frequency in a year)

$$= M \times E[R-V] \times \frac{D}{Q} = \frac{MD}{Q}(\int_R^\infty V - R)f(V)dv \quad (4)$$

Combining Equations 1 – 4, the total annual inventory cost for the model becomes:

$$\text{TVC} = \{ \theta D + \frac{KD}{Q} + \theta Y\left[\frac{Q}{2} + R - E(V)\right] + \frac{MD}{Q}\int_R^\infty (V-R)f(V)dv\} \quad (5)$$

Equation (5) captures the summation of the different cost considered in [13] but without the inclusion of tax-cost either as an incorporated cost or as one of the several costs. This work seeks to consider the inclusion of tax as part of the purchase cost (tax at purchase). Therefore, (5) is modified by incorporating tax-cost, t, into the purchasing cost of each item. Thus;

$$\mathrm{TVC} = \{(\theta D + t) + \frac{KD}{Q} + \theta Y\left[\frac{Q}{2} + R - E(V)\right] + \frac{MD}{Q}\int_R^{\infty}(V - R)f(V)dv\} \quad (6)$$

Finding the total variable inventory cost entails satisfying the minimization conditions with respect to the order quantity, Q and the reorder point, R. Hence;

$$\frac{\partial}{\partial Q}\mathrm{TVC} = \frac{\partial}{\partial Q}\left\{(\theta D + t) + \frac{KD}{Q} + \theta Y\left[\frac{Q}{2} + R - E(V)\right] + \frac{MD}{Q}\int_R^{\infty}(V - R)f(V)dv\right\} = 0$$

$$= -\frac{KD}{Q^2} + \frac{\theta Y}{2} - \frac{MD}{Q^2}\int_R^{\infty}(V - R)f(V)dv = 0$$

$$= \frac{MD}{Q^2}\int_R^{\infty}(V - R)f(V)dv + \frac{KD}{Q^2} = \frac{\theta Y}{2}$$

$$Q^2 = \frac{2D[K + M\int_R^{\infty}(V-R)f(V)dv]}{\theta Y}$$

$$Q^* = \sqrt{\frac{2D[K + M\int_R^{\infty}(V-R)f(V)dv]}{\theta Y}} \quad (7)$$

Equation (7) represents the optimal ordering quantity for single item replenishment model without constraint. Also,

$$\frac{\partial}{\partial R}TVC = \frac{\partial}{\partial R}\{(\theta D + t) + \frac{KD}{Q} + \theta Y\left[\frac{Q}{2} + R - E(V)\right] + \frac{MD}{Q}\int_R^{\infty}(V - R)f(V)dv\} = 0$$

$$= \theta Y + \frac{MD}{Q}\left[\int_R^{\infty}\frac{\partial}{\partial R}(V - R)f(V)dv\right] = 0$$

$$\theta Y + \frac{MD}{Q}[-\int_R^{\infty}f(V)dv] = 0$$

$$\theta Y - \frac{MD}{Q}\int_R^{\infty}f(V)dv = 0$$

$$\frac{MD}{Q}\int_R^{\infty}f(V)dv = \theta Y$$

$$Hence;\ \frac{\partial}{\partial R}TVC = MD\int_R^{\infty}f(V)dv = \theta YQ$$

$$\int_R^{\infty}f(V)dv = \frac{\theta YQ}{MD} = R \quad (8)$$

Equation (8) represents the optimal reorder point involving single item replenishment model without constraint.

2.3. Single item replenishment model with a warehouse space availability constraint

In this section, total warehouse space constraint, according to [18] is introduced.
Let TVC be the objective function;

$$\mathrm{Min}(TVC) = \sum_{i=1}^{n}\left[\begin{array}{c}(\theta_i D_i + t) + \frac{K_i D_i}{Q_i} + \left\{\theta_i Y_i\left(\frac{Q_i}{2} + R_i - E(V)\right)\right\} + \\ \frac{M_i D_i}{Q_i}\int_{R_i}^{\infty}(V_i - R_i)f(V_i)dvi\end{array}\right] \quad (9)$$

Subject to constraint

$$\sum_{I=1}^{n} f_i Q_i \leq W$$

$$Q_i \geq 0\ \forall\ i$$

To determine the optimal order quantities for different items so as to achieve minimum value of TVC, let λ be the non–negative Lagrange multiplier. The Lagrange function, L, is given as;

$$L(Q_i, \lambda) = TVC + \lambda(\sum_{I=1}^{n} f_i Q_i - W)$$

The necessary conditions for L to be minimum are:

1) $\frac{\partial}{\partial_\lambda}(Q_i, \lambda) = \lambda(\sum_{I=1}^{n} f_i Q_i - W) = 0$

$$\therefore \sum_{i=1}^{n} f_i Q_i = W \quad (10)$$

2) $\frac{\partial L(Q_i,\lambda)}{\partial Q_i} = \frac{\partial}{\partial Q_i}\left[(\theta_i D_i + t) + \frac{K_i D_i}{Q_i} + \left\{\theta_i Y_i\left(\frac{Q_i}{2} + R_i - E(V)\right)\right\}\right] +$

$$\left[\frac{M_i D_i}{Q_i}\int_{R_i}^{\infty}(V_i - R_i)f(V_i)dvi\right] + \lambda\left(\sum_{i=1}^{n} f_i Q_i - w\right) = 0 \quad (11)$$

$$= -\frac{K_i D_i}{Q_i^2} + \frac{\theta_i Y_i}{2} - \frac{M_i D_i}{Q_i^2}\int_{R_i}^{\infty}(V_i - R_i)f(V_i)dv_i + \lambda f_i = 0$$

$$\frac{M_i D_i}{Q_i^2}\int_{R_i}^{\infty}(V_i - R_i)f(V_i)dv_i + \frac{K_i D_i}{Q_i^2} = \frac{\theta_i Y_i}{2} + \lambda f_i$$

$$\frac{M_i\, D_i \int_{R_i}^{\infty}(V_i - R_i)f(V_i)dv_i + K_i D_i}{Q_i^2} = \frac{Q_i Y_i + 2\lambda f_i}{2}$$

$$Q_i^2 = \frac{2D_i\left(K_i + M_i\int_{R_i}^{\infty}(V_i - R_i)f(v_i)dv_i\right)}{(\theta_i Y_i + 2\lambda f_i)}$$

$$Q_i^* = \sqrt{\frac{2D_i\left(K_i + M_i\int_{R_i}^{\infty}(V_i - R_i)f(v_i)dv_i\right)}{(\theta_i Y_i + 2\lambda f_i)}} \quad (12)$$

Equation (12) represents the optimal ordering quantity for single item replenishment model with warehouse capacity constraint. Similarly, subjecting reorder point of items to warehouse availability constraint, we have; $\sum_{i=1}^{n} f_i R_i \leq W$,
where R_i is the reorder point of each item. Therefore, the min(TVC) with Reorder point R_i is given as:

$$min(TVC) = \sum_{i=1}^{n}\left[(\theta_i D_i + t) + \frac{K_i D_i}{Q_i} + \left\{\theta_i Y_i\left(\frac{Q_i}{2} + R_i - E(V)\right)\right\} + \frac{M_i D_i}{Q_i}\int_{R_i}^{\infty}(V_i - R_i)f(V_i)dvi\right]$$

Subject to: (13)

$\sum_{I=1}^{n} f_i R_i \leq W$

$Q_i \geq 0 \;\forall\; i$

To determine the optimal reorder point for different items so as to achieve minimum value of TVC, the Lagrange function was obtained as follows;

$$L(Q_i,\lambda) = TVC + \lambda\left(\sum_{I=1}^{n} f_i R_i - W\right)$$

The necessary conditions for L to be minimum are:

1) $\frac{\partial}{\partial_\lambda}(Q_i,\lambda) = \lambda\left(\sum_{I=1}^{n} f_i R_i - W\right) = 0$

$\therefore \sum_{i=1}^{n} f_i R_i = W$

2) $\frac{\partial L(Q_i,\lambda)}{\partial R_i} = \frac{\partial}{\partial R_i}\left[(\theta_i D_i + t) + \frac{K_i D_i}{Q_i} + \left\{\theta_i Y_i\left(\frac{Q_i}{2} + R_i - E(V)\right)\right\}\right] + \left[\frac{M_i D_i}{Q_i}\int_{R_i}^{\infty}(V_i - R_i)f(V_i)dvi\right] + \lambda\left(\sum_{i=1}^{n} f_i Q_i - w\right) = 0$

$$= Q_i Y_i - \frac{M_i D_i}{Q_i}\int_{R_i}^{\infty} f(V_i) + \lambda f_i = 0$$

$$\frac{M_i D_i}{Q_i}\int_{R_i}^{\infty} f(V_i) = \theta_i Y_i + \lambda f_i \quad (14)$$

By multiplying both sides of (14) by $\frac{Q_i}{M_i D_i}$, we obtain;

$$\int_{R_i}^{\infty} f(V_i) = \frac{Q_i(\theta_i Y_i + \lambda f_i)}{M_i D_i} = R_i^* \quad (15)$$

Equation (15) represents the optimal reorder point for single item replenishment model with warehouse capacity constraint.

2.4. Probability distribution of demand of items

Demand of items vary from time to time due to need and purchasing power. Thus, stochastic inventory models incorporate the variation in demand and uncertain lead time. In this case, demand of an item is not known to be deterministic but are considered variables. Hence, demand is a variable and there is need to obtain the probability distributions for the demand of the items. The following probability distributions for the demand of the selected items were considered after necessary preliminary analysis:

1) When Demand of item is assumed to follow normal distribution.
2) When demand of item is assumed to follow Weibull distribution.
3) When demand of item is assumed to follow Rayleigh distribution.

2.4.1. Probability inventory model when demand follows a Weibull distribution

The probability density function of a two parameter Weibull random variable is;

$$f(V,\alpha,\beta) = \begin{cases} \left(\frac{\alpha}{\beta}\right)\left(\frac{V}{\beta}\right)^{(\alpha-1)} e^{-\left[\frac{V}{\beta}\right]^{\alpha}}, V \geq 0 \\ 0, V < 0 \end{cases} \quad (16)$$

The mean of a Weibull distribution is given by;

$$E[V] = \beta\Gamma\left(1+\frac{1}{\alpha}\right) \quad (17)$$

2.4.2 Probability inventory model when the demand follows a normal distribution

The probability density function of a normal distribution is given by;

$$f(V,\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(V-\mu)^2}{2\sigma^2}}\ ;\ V \geq 0;\ \sigma \geq 0 \quad (18)$$

The mean of a normal distribution is given by;

$$E(V) = \mu \quad (19)$$

2.4.3 Probability inventory model when demand follows a Rayleigh distribution

The probability density function of Rayleigh distribution with a scale parameter, $\sigma \geq 0$ is given by;

$$f(V,\sigma) = \frac{V}{\sigma^2}\, e^{-\frac{V^2}{(2\sigma^2)}} \quad (20)$$

The mean of a Rayleigh distribution is given by;

$$E[V] = \sigma\sqrt{\frac{\pi}{2}} \quad (21)$$

2.5. A modified Hadley-whitin algorithm for calculating the optimal order quantity and reorder point without constraint

The Hadley–Whitin algorithm [8] was modified in this work by incorporating the terminating condition and the user specified level of accuracy. It is given as follows:

1) Set $\varepsilon = 0.05$ or any other specified level of accuracy
2) Compute $Q = \sqrt{\frac{2DK}{(\theta+t)Y}}$
3) Compute R_i using $\int_R^{\infty} f(V)dv = \frac{\theta YQ}{MD} = R$
4) Compute $Q_i\ using\ Q^* = \sqrt{\frac{2D[K+M\int_R^{\infty}(V-R)f(V)dv]}{\theta Y}}$
5) Compute $Q_{i_{+1}}$ from step 4
6) Compute R_{i+1} from step 3
7) Is $|Q_i - Q_{i+1}| \leq \varepsilon$? If yes, stop and go to step 8. Otherwise return to step 5
8) Is $|R_i - R_{i+1}| \leq \varepsilon$? If yes, stop and go to step 9. Otherwise return to step 6
9) Terminate the algorithm if steps 7 and 8 are satisfied.

2.6. Trimming method for finding the optimal ordering quantity of single replenished item with constraints

This is an improved method of arriving at the optimum order quantity when compared to the trial-and-error method of minimizing the cost equations considering some limitations (constraints).

According to [18], the Lagrange method of solving multi–item deterministic inventory models involve the systematic trial and error method used to find λ^* (the optimum value of λ) resulting in simultaneous values of q_i^* satisfying the given constraint equations. However, results shown by [2] in comparing the minimized cost of an inventory using trial and error method on one side and trimming method on the other end reveals the trimming method to be a better option that leads to optimum order quantity without having to go through so many iterations which is peculiar to the trial-and-error method. Therefore, the trimming method eases computation and speed over the trial-and-error method.

The trimming algorithm for the determination of optimal ordering quantity for single item replenishment involves the following steps:

1) Determine the unconstrained value of Q_i using Equation 7
2) Compute the value of $\sum_{i=1}^{n} f_i Q_i$
3) If the value $\sum_{i=1}^{n} f_i Q_i \leq W$; stop and set the obtained Q_i to Q_i^*. Otherwise go to step 4
4) Introduce a trimming constant, $T_1 = \frac{W}{\sum_{i=1}^{4} f_i Q_i}$
5) $Set\ Q_i^* = T_1 Q_i\ \forall\ i = 1,2 \ldots n$

6) If $\sum_{i=1}^{n} f_i Q_i \leq W$, stop. Otherwise go to step 4.

Similarly, the above algorithm can be used to obtain the optimal reorder point, by replacing Q_i with R_i at each step of the algorithm.

3. Application to the building materials inventory

Inventory on four selected building materials in Table 1 were obtained from Ubotex limited, Uyo in Nigeria for practical implementation and adaptation of the models and algorithms.

Table 1: Cost Values for Selected Items

	Zinc nail	10mm rod	5 inches nail	Urban zinc
Annual demand	409	6443	2960	1125
Purchased cost/unit	23800	2950	24500	95000
Tax paid at Purchase	1190	148	1225	4750
Shortage per unit	1500	186	1554	5985
Ordering cost per unit	9996	1239	10290	39900
Holding cost fraction	0.04	0.02	0.05	0.01
Floor space (Sq. Ft/Unit)	0.611	1.5	0.95	0.8

4. Results

4.1. Probability distribution for the quantity demand of items

The quantity demand of each item followed different probability distributions. The most suitable probability distribution for the demand rate of each item was determined using the chi-square goodness-of-fit test with the aid of Easyfit software. The quantity demand data of zinc nail was found to follow Weibull distribution in chi-square goodness-of-fit test with a rank of 2, having scale parameter, $\alpha = 6$ and shape parameter, $\beta = 36$. This is because the probability distribution with rank one did not represent a demand curve and cannot be used to model same. The demand quantity of 10mm rod was found to equally follow a Weibull distribution in a chi-square goodness-of-fit test with a rank of 1, having a scale parameter, $\alpha = 2$ and shape parameter, $\beta = 607$.

Furthermore, the demand of 5 inches nail followed a normal distribution in a chi-square goodness-of-fit test with rank of 1, having mean value, $\mu = 10.696$ and variance, $\sigma = 607$. Finally, the quantity demand of urban zinc was found to have followed a Rayleigh distribution in a chi-square goodness-of-fit test with rank of 4, having mean value, $\mu = 93.78$ and scale parameter, $\sigma = 74.827$. The Rayleigh distribution with rank of 4 was chosen because other distributions that preceded it didn't represent a demand curve.

Easyfit (5.6) software was used to perform the goodness-of-fit test for these items as well as the parameter estimates as shown in Table 2.

Table 2: Summary of Probability Distribution for the Demand of Each Product, Rank of Chi-Square Goodness-of-Fit Test and Parameters Estimates

Item	Probability distribution	Chi-square ranks	Parameter estimates
2 inches nail	Weibull	2	$\alpha = 6$; $\beta = 36$
10mm rod	Weibull	1	$\alpha = 2$; $\beta = 607$
5 inches nail	Normal	1	$\mu = 10.696$; $\sigma = 607$
Urban zinc	Rayleigh	4	$\sigma = 74.827$

4.2. Computation of optimal economic order quantity (EOQ) and cost components for single replenished items without constraints

The modified Hadley-Whitin algorithm of section 2.5 was used to obtain the optimal order quantity and reorder point of single replenished items without constraint in Table 3. Also, the computation of the different cost components involving these items was done using equations (1), (2), (3) and (4) which represent the purchase, ordering, holding and shortage costs respectively.

Table 3: Results of Reorder Point, EOQ and Cost Components for Single Item Replenishment without Constraint

	Item1 (cartons)	Item 2 (length)	Item3 (cartons)	Item4 (bundles)
Q	96	1676	218	300
R	44	930	18	164
Total purchase cost		N218,546,074		
Total ordering cost		N337, 208		
Total holding cost		N5, 433,953		
Total shortage cost		N9, 480		
Total variable cost		N224,326,715		

4.3. EOQ, Reorder point and cost components for single replenished items with warehouse space availability constraint

The algorithm of section 2.6 was used to determine the optimal order quantity of each item with a warehouse space constraint. Furthermore, equations (1), (2), (3), (4) were equally used to determine the purchase cost, ordering cost, holding cost and shortage cost respectively in Table 4.

Table 4: Results of Eoq, Reorder Point and Cost Components for Single Item Replenishment with Warehouse Space Constraint

	Item1 (cartons)	Item 2 (length)	Item3 (cartons)	Item4 (bundles)
Q	62	1089	141	197
R	44	930	18	164
Total purchase cost		N218,546,074		
Total ordering cost		N517, 654		

Total holding cost	N4,089,263
Total shortage cost	N14,640
Total variable cost	N223,167,631

5. Discussion

5.1. Comparing the economic order quantity (EOQ) and cost components of single replenished items without constraint with those of single replenished items with constraint

The total inventory cost involving single item replenishment with constraint in Table 4 is less than that of the model without an imposed constraint in Table 3. Also, the order quantities and reorder points of single item replenishment with constraint in Table 4 were equally less than the values obtained without constraint in Table 3. These outcomes are logical on the premise that constraints have a way of controlling costs by restricting inventory levels, ordering quantities and reorder points. This ensures that inventory-related expenses such as purchase costs, ordering costs, holding costs and shortage costs, are minimized. The obtained results are in line with [13] who concluded that single replenishment model with constraints do minimize total variable cost than the model without constraint.
Also, the results in Table 4 reveals that the imposition of the warehouse capacity constraint also ensured that a minimum level of inventory is maintained, thus reducing the risk of stock outs and lost sales thereby enhancing customer satisfaction and service levels - a result that agrees with [12].

5.2. Inclusion of tax-cost

The inclusion of tax-cost in the models increases the purchase cost of each item, because tax adds to the total acquisition cost of inventory and this cost data is important for statistical analysis, allowing businesses to evaluate mean and other location and dispersion measures of inventory costs for comprehensive business decision. It also helps the company in predicting the future by adjusting pricing strategies and anticipating changes in demand based on price sensitivity. However, it didn’t really affect the ordering quantity of these items. This is because the partial derivatives of the total variable cost function with respect to the variables of interest (Q and R) left the tax-cost (t) as a constant equal to zero in each case.

6. Conclusion

This work has revealed that imposing constraints related to storage capacity can help in ensuring minimum inventory level of a company in line with financial and physical limitations; hence, preventing overstocking or understocking. Also, the inclusion of tax-cost increases total inventory cost, a layout for holistic business analysis and decision making. These will help in creating a realistic and practical inventory model which agrees with real-world limitations and goals, which in turn leads to better decision-making and profitability.

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