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Research paper

Analysis of semi-parametric single-index models by using MAVE-method based on some kernel functions

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Abstract

In this paper, we used many forms of kernel functions with minimum average variance estimation (MAVE) method [Xia2002] we called the proposed methods (MAVE-Biweight), (MAVE-Epanechnikov) and .(MAVE-Gaussian) for estimation the parameters and the link function of the single – index model (SIM) comparing with other methods of estimation . to evaluate the performing of the various methods s simulation and a real data have been used, conclusions showed that the (MAVE- Gaussian) method in this paper gave better results compared with other methods depending on the mean squared error (MSE) and mean Absolute error (MAE) criterion for comparison.

Keywords: *Single-Index Model; MAVE Method; Kernel Function; Bandwidth Parameter; Curse of Dimensionality.*

1. Introduction

SIMs are widely used in sciences of applied quantitative. SIMs looking for a single linear combination of Xs variables that can catch information on a relation between Y and X, to avoid the curse of dimension specifically the model (SIM) can be written as :

$$Y = g(X^T \beta) + \in \quad (1)$$

$$E[Y|X = x_i] = E[Y|X^T\beta] = P[Y|X^T\beta] = g(X^T\beta)$$

Where $E[\in|X] = E[\in|X^T\beta] = 0$ almost surely, $g(X^T\beta)$ is unknown .link function and β is vector of parameters such that $\|\beta\| = 1$ or $\beta^T\beta = 1$,an,and the first component is positive for identification model . Estimation of (SIM) is very important in theory and in practice, In the last years many papers [Tong , Li and Zhu(2002) ; Yu and .Ruppert (2002) ; Yin and cook ,(2005) ; Delecrax , Hristache and patilea (2006)] have investigated the estimation of the parametric index β with focusing on root – n estimability and efficiency issues . There are three type of methods have been proposed to estimate β in the literature ,the first one that include the average derivative estimation method [Hardel and Stoker , (1989)] , the structure adaptive method [Hristach et al, (2002)] and the outer product of gradients (OPG) method [Xia et al ,(2002)], the second type contains methods that estimate g and β at same time whereas the third type used regressing X onY instead of regressing Yon X and were originally suggested dealing with sufficient dimension reduction (SDR) .([`1] , [4] , [7] , [16]) . In This paper used the second type from methods it is MAVE method with some kernel functions (Biweight, Epanechnikov and Gaussian) to obtain three methods (MAVE – Biweight, MAVE– Epanechnikov and MAVE –Gaussian) for analysis the semi-parametric single – index models.

The objective of this paper is identify the best method based on the Two comparison criterion mean squared error (MSE) and mean absolute error (MAE).
The reminder of this paper is arranged as follows. In section 2, a brief review with advantages of semi-parametric single index models and a brief explain for kernel functions with bandwidth parameter. MAVE method reviewed in section 3. A simulation studies are conducted under different setting and the real data and applications of methods are recorded at section 4 and at section 5 the conclusions are summarized.

2. The semi-parametric single –index model (SSIM)

Most estimation problems contain both unknown. Finite – dimensional parameter (β) and an unknown link function g ($X^T\beta$). These kinds of models are. Called "semi-parametric", the linearity assumption $X^T\beta$ is still valid but no additional assumption is made related to the error term in other a specific link function is not assumed in the model.
In statistic, the technique of regression analysis includes modeling and analysis many variables. it focus on the relationship between a dependent variable and one or more independent variables. More specifically regression analysis applied for estimating the conditional expectation of the dependent variable given the independent variables.
In multiple linear regression models, the conditional mean relationship between the response and each of the predictions is assumed linear. The most flexible models do not make any assumption about the form of the p-variate function. The problem is to fit a p-dimensional surface to the observed data $\{(x_i^T, y_i)$, i=1, 2… n}. An obvious approach is trying for generalizing the univariate smoothing techniques. There is a sveral problems appear called "curse of dimensionality". A popular way to beat the dimensionality problem is to first project all covariates on to a linear space spanned. by the covariates and then to fit a non-parametric curve.

To their linear combinations .this is lead to the single – index model (SIM):

$y_i = g(x_i^T \beta) + \epsilon_i$, i = 1,2,…,n

Where $g(x_i^T \beta)$ is a smooth unknown function , x_i is a p×1 vector of covariates , $\beta = (\beta_1, \beta_2, \dots, \beta_p)^T$ is a p×1 vector of parameters and ϵ_i .white noise identical .independent normal distribution with unknown variance σ^2 and mean zero.([6] ,[10],[13],[14]).

2.1. The advantages of semi-parametric single – index model (SSIM)

1) The semi-parametric single – index model can avoid the problem of .error distribution misspecification.
2) The (SSIM) is more general than the .binary choice model.
3) The response variable (Y) can be discrete or continuous in semi-parametric single – index model.
4) The (SSIM) is an alternative approach .designed to mitigate effects arising from the curse of dimensionality.
5) A single – index model dose not .assume that g (xiT β) is known. And hence it is more flexible and. less restrictive than parametric models for conditional mean functions, such as linear models and binary probit models, use of a semi-parametric single – index model reduces the risk obtaining misleading results.
6) A single – index model .avoids the curse of dimensionality because the index XT β aggregates the dimension of X . At the same time β can be estimated with same rate of .convergence $n^{-\frac{1}{2}}$ that is achieved in a parametric model.
7) in non- parametric estimation does not .permit extrapolation . it does not .provide predictions of E [Y | X = x] at points x That are not provide in the support of X . this is a serious .draw back in policy analysis and forecasting. A semi-parametric single –index model .by contrast permits extrapolation within limits it yields predictions of E [Y | X = x] at values of x that .are not in the support of X but are in the support of XT β . ([4],[10],[11],[13])

2.2. Kernel function selection

In nonparametric functional estimation , the kernel function analysis with different weight to each data point .the weights are depending on the bandwidth and the estimator that are used the function k(.) is generally a symmetric probability density functions satisfied the conditions:

1) $\int_{-\infty}^{\infty} k(u)du = 1$ is pdf, A kernel function k(u) :R→R is any function which satisfies This condition .
2) kernel function is symmetric, k(u) = k (-u)
3) $\int_{-\infty}^{\infty} uk(u)du = 0$
4) $\int_{-\infty}^{\infty} u^2 k(u)du = \mu_2(k) \neq 0$, The moments of a kernel.
5) $k(u) \geq 0$, Anon-negative kernel for all u when k(u) is a kernel function, commonly used kernel functions include the Gaussian, the Tricube, Uniform, Biweight, Cosine, Triweight and Epanechnikove kernel function. Gaussian kernel , Biweight kernel and Epanechnikove kernel function are chosen .([2],[3],[11])

Common second-order kernels are listed in the following table.

Table 1: Common Second-Order Kernels

kernel	Equation
Uniform	$K(u) = \frac{1}{2}\, 1(\lvert u\rvert \leq 1)$
Epanechnikov	$K(u) = \frac{3}{4}(1-u^2)\, 1(\lvert u\rvert \leq 1)$
Biweight	$K(u) = \frac{15}{16}(1-u^2)^2\, 1(\lvert u\rvert \leq 1)$
Triweight	$K(u) = \frac{35}{32}(1-u^2)^3\, 1(\lvert u\rvert \leq 1)$
Gaussian	$K(u) = \frac{1}{\sqrt{2\pi}} \exp(\frac{-u^2}{2})\, 1(\lvert u\rvert \leq \infty)$
Tricube	$K(u) = \frac{70}{81}(1-\lvert u\rvert^3)^3\, 1(\lvert u\rvert \leq 1)$
Cosine	$K(u) = \frac{\pi}{4}\cos(\frac{\pi}{2}u)\, 1(\lvert u\rvert \leq 1)$

2.3. Bandwidth parameter selection

The bandwidth parameter. also called the smooth parameter (h) , controls for the smoothing level of the estimation . (h) Plays very important turn at the performance for kernel estimators. different methods like the cross-validation, penalized functions, , bootstrap , etc.. have been developed to obtain the optimal bandwidths . The cross –validation method in general is preferred due to it is easier in computing and implementation .structure for any regression model the bandwidth value which minimizes the cross-validation (CV) function with a non-negative weight function w ($X^T \beta$) given as:

$$CV(h) = n^{-1} \sum_{i=1}^{n} \left[Y_i - \hat{g}_{-i}\left(X_i^T \hat{\beta}\right)\right]^2 \quad (2)$$

Is considered the optimal one. The (CV) function contains the leave-one-out kernel defined as follows.

$$\hat{g}_{-i}(X_i^T \hat{\beta}) = \frac{\sum_{i\neq j}^{n} Y_i K_h(X_i^T \hat{\beta} - X_j^T \hat{\beta})}{\sum_{i\neq j}^{n} K_h(X_i^T \hat{\beta} - X_j^T \hat{\beta})} \quad (3)$$

It used to estimate our mean function, a Nadaraya - watson kernel smoothing.

The leave-one-out estimator is obtained. by leaving out the observations i

(The concerned observations x_i and y_i) from. The data each time for satisfying. The unbiased estimate of the bandwidth parameter (h). The procedure is replicated (n) times. (For all observations). The final optimal bandwidth value. Required for the kernel estimation .is the mean of all these values computed. The bandwidth that minimizes the cross-validation function also minimizes the MSE is a performance criterion of an estimator. ([2], [9], [12]). 3.

3. Minimum average variance estimation (MAVE) method

For estimation of single – index model E [Y | X = x] = g(X^T β) proceed in two step:

First, .the coefficient vector β has to estimation methods to calculate the coefficients for discrete and continuous variables will be covered depthly later.

Secondly , for estimation the unknown link function g(X^T β) by non-parametrically regression the dependent variable Y on the fitting index $X^T \hat{\beta}$ where $\hat{\beta}$ is the coefficient vector. we estimated the kernel estimator. [11]

In (2002) Xia, Tong, Li and Zhu proposed a general estimation method termed minimum average variance estimation (MAVE) for semi-parametric models.

Let $\{x_i, y_i ;\ i = 1,2,\dots,n\}$ be a random samples from model (1) the basic concept of our estimation method is to linearly approximation the smooth link function g(X^T β) and estimation β by minimizing the overall approximation errors. The single – index model (1) is a special case of what they considered and we can estimate it as follows. Assuming function g (X^T β) and parameter β . Then the Taylor. expansion of g(x_i^T β) at g(x^T β) is:

$$g(x_i^T \beta) \approx a + b\,(x_i - x)^T \beta$$

Where a = g(x T β) , b = $g^{/}$(x T β) with fixed β the local estimator of conditional variance is:

$$\sigma_n^2(x\,|\beta) = \min\{n\hat{f}_\beta(x)\}^{-1} \sum_{i=1}^{n} \{ Y_i - \{a + b\,(Xi - x)^T\ \beta\}^2 . k_h \{(X_i - x)^T \beta\} \quad (4)$$

Where $\hat{f}_\beta(x) = n^{-1} \sum_{i=1}^{n} kh\{(x\,i - x)^T \beta\}$ where k is density function , h is the bandwidth and $k_h(u) = k_h(u/h)/h$.

$\sigma_n^2(x\,|\beta)$ can also be understood, the best approximation minimize the overall departure at all x=x_j , j=1,2,…,n , Thus , β estimator is to minimize:

$$Q_n(\beta) = \sum_{i=1}^{n} \sigma_n^2(x_j\,|\beta)$$

The basic algorithm. for estimation g(.) in the single – index model is based on observing that

$$\beta = \text{argmin } E\,[\,Y - g\,(X^T\beta\,)]^2 \quad (5)$$

Subject to $\beta^T\beta = 1$, by condition. on $X^T\beta$ we see that (5) equals

$E_u\,\sigma_\beta^2\,(x^T\beta)$ where $\sigma_\beta^2(x^T\beta) = E\,[(Y - g(X^T\beta)\,)^2\,|\,X^T\beta = u\,]$

$E_u\sigma_\beta^2\,(x^T\,\beta)$ where $\sigma_\beta^2\,(x^T\,\beta) = E\,[\,\left(Y - g(X^T\,\beta)\right)^2\,|\,X^T\beta = u]$

If follows that:

$E\,[Y - g\,(X^T\beta)\,]^2 = E_u\,\sigma_\beta^2\,(x^T\beta)$

Therefore, minimizing (5) is equivalent to:

$$\beta = \text{argmin } E_u\,\sigma_\beta^2\,(x^T\beta) \quad (6)$$

The conditional expectation in (6) is now approximation by the sample analogue for x_i , we have the following local linear approximation.

$$y_i - g\,(x_i^T\beta) \approx y_i - g(x^T\beta) - \acute{g}(x^T\beta).\,x_{ij}^T\,\beta$$

Where $x_{ij} = x_i - x_j$.

Following. The idea of local linear smoothing, we may estimate $\sigma_\beta^2\,(x^T\beta)$ by:

$$\sigma_\beta^2\,(x^T\beta) = \min_{a,b} \sum_{i=1}^{n}\{\,Y_i - a - b\,\,X_{ij}^T\,\beta\,\}^2\,.\,w_{i0} \quad (7)$$

Here, $w_{i0} \geq 0$, i = 1,2,...,n , are some weights. With $\sum_{i=1}^{n} w_{i0} = 1$,typically .centering at x . By (6) and (7) our estimation procedure is to minimize.

$$\frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{n}\,\{\,Y_i - a_j - b_j\,X_{ij}^T\beta\,\}^2\,\,.\,w_{ij}\;n \quad (8)$$

With .respect to (a_j, b_j) and β. if the kernel smoothing is used with kernel function k(u) and bandwidth (h) , Then the weight function :

$$w_{ij} = k_h\left(X_{ij}^T\right) \text{, where } w_{ij} = \frac{k_h(X_{ij}^T\beta)}{\sum_{i=1}^{n} k_h\left(X_{ij}^T\beta\right)}$$

We call the estimation procedure. The Minimum Average Variance Estimation (MAVE) method. ([5], [8], [15], [16]).

Our estimation procedure for β and g($x^T\,\beta$) is described. in details as follows:

Step (0): initialization step obtain an initial estimate of β, let $\hat{\beta}^{(0)}$ be an estimator of the OLS method or arbitrary.

Step (1): put $\hat{\beta}^{(0)} = \beta$ and calculate the solution of $(a_j\,, b_j)$.

$$\begin{pmatrix} \hat{a}_j \\ \hat{b}_j \end{pmatrix} = [\,\sum_{i=1}^{n} k_h\left(X_{ij}^T\beta\right)\begin{pmatrix} 1 \\ \frac{X_{ij}^T\beta}{h} \end{pmatrix}\begin{pmatrix} 1 \\ \frac{X_{ij}^T\beta}{h} \end{pmatrix}^T\,]^{-1}\sum_{i=1}^{n} k_h\,\left(X_{ij}^T\beta\right)\begin{pmatrix} 1 \\ \frac{X_{ij}^T\beta}{h} \end{pmatrix}\,Y_i$$

Step (2): calculate

$$\hat{\beta} = \left\{\sum_{i,j}^{n} k_h\,\left(X_{ij}^T\beta\right)(\hat{b}_j)^2.\,X_{ij}\,X_{ij}^T\right\}^{-1}.\,\sum_{i,j}^{n} k_h\,\left(\,X_{ij}^T\beta\right)\hat{b}_j\,X_{ij}(\,Y_i - \hat{a}_j)\,\}$$

And

$$\hat{\beta}^{MAVE} = \text{sign}\,(\,\hat{\beta}_1)\,\frac{\hat{\beta}}{\|\hat{\beta}\|}$$

Step (3): Repeat step (1) and step (2) until the iteration process converges, the final vector β is The MAVE estimator for $\hat{\beta}^{(0)}$.

Xia and Tong (2006) proved that the. estimator produced by the algorithm can a chive root – n consistency and has the same. asymptotic distribution as the estimator of Hardle et al (1993)

4. Numerical studies

In This section we illustrate the performance of methods (MAVE-Biweight, MAVE –Epanechnikov and MAVE-Gaussian) by Three simulation studies and real data for Iraq stock exchange analysis based on (R-Package) .

4.1. Simulation

Experiment1. In this experiment we simulate 200 datasets consisting of sample size $n = 25{,}100$ observation from the following model.

$$Y = \sin\,(X^T\beta) + \epsilon$$

Where $X = (x_1, x_2, x_3, x_4, x_5)^T$, $\beta = (1,1,1,1,1)^T/\sqrt{5}$, x_i are iid ~$N(0, \sigma^2)$, $\epsilon \sim N(0, 1)$ and correlation between x_i and x_j , is $\rho^{|i-j|}$ for i ,j =1 , 2 , 3 , 4, 5 . With two values of ρ were explored 0.2 and 0.9 with ϵ and X are independent.

Experiment 2 in this experiment we simulate 200 datasets consisting of sample size n= 25,100 observation from the following model with homoscedastic errors.

$$Y = 2\sin\,(X^T\,\beta) + \exp\,(2X^T\,\beta) + 0.5\,\epsilon$$

Where $X=(x_1, x_2, x_3, x_4, x_5, x_6)^T$, $\beta=(1,1,1,1,1,0)^T/\sqrt{5}$, x_i are iid ~$N(0, \sigma^2)$, $\epsilon \sim N(0, 1)$ and correlation between xi and xj , is $\rho^{|i-j|}$ for i ,j =1 , 2 , 3 , 4, 5, 6 . With Two values of ρ were explored 0.2 and 0.9 with ϵ and X are independent.

Experiment 3. In this experiment we simulate 200 datasets consisting of sample size
n= 25,100 observation from the following model.

$$Y = 1+2(X^T\beta + 3)\,\log\,(3\,\left|X^T\beta\right| + 1) + \epsilon$$

Where$X=(x_1, x_2, x_3, x_4, x_5, x_6, x_7)^T$, $\beta = (0.4, -0.4, 0.8, -0.2, 0, 0, 0)^T$, x_i are iid ~$N(0, \sigma^2)$, $\epsilon \sim N(0, 1)$ and correlation between xi and xj , is $\rho^{|i-j|}$ for i ,j =1 , 2 , 3 , 4, 5, 6,7. With two values of ρ were explored 0.2 and 0.9 with ϵ and X are independent.

Table 2: Amse For Coefficients B Estimated by Methods (Mave-Biwieght, Mave-Epanechnikov and Mave – Gaussian) Based on the Model in Experiment 2 for Sample Size, $N = 25,100$ with $P = 0.2$, 0.9 and $P = 5$.

ρ	sample size	methods		$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$
			(Average)$\hat{\beta}$	0.43730	0.51004	0.42394	0.45107	0.40672
		MAVE-B	AMSE	0.13259	0.10707	0.12220	0.11685	0.11209
			(Average)$\hat{\beta}$	0.44845	0.44731	0.45129	0.46509	0.42287
0.2	25	MAVE-E	AMSE	0.05032	0.04615	0.04255	0.05129	0.04169
			(Average)$\hat{\beta}$	0.45179	0.44625	0.45506	0.46736	0.41379
		MAVE-G	AMSE	0.04670	0.04592	0.04180	0.05297	0.04708
			(Average)$\hat{\beta}$	0.54414	0.38704	0.32959	0.47440	0.46947
		MAVE-B	AMSE	0.15749	0.21327	0.24989	0.23489	0.14976
			(Average)$\hat{\beta}$	0.45664	0.44839	0.42537	0.44849	0.45642
0.9	25	MAVE-E	AMSE	0.09681	0.16980	0.17228	0.16512	0.10290
			(Average)$\hat{\beta}$	0.47592	0.42583	0.43109	0.44991	0.45155
		MAVE-G	AMSE	0.09500	0.18556	0.16934	0.17596	0.09634
			(Average)$\hat{\beta}$	0.41724	0.44376	0.46495	0.51013	0.39058
		MAVE-B	AMSE	0.14655	0.14493	0.11837	0.13704	0.12885
			(Average)$\hat{\beta}$	0.44295	0.44754	0.44658	0.46118	0.43745
0.2	100	MAVE-E	AMSE	0.01467	0.01120	0.01331	0.01666	0.01176
			(Average)$\hat{\beta}$	0.44437	0.44643	0.44704	0.45908	0.43888
		MAVE-G	AMSE	0.01440	0.01215	0.01341	0.01706	0.01175
			(Average)$\hat{\beta}$	0.42333	0.32393	0.58982	0.52445	0.30482
		MAVE-B	AMSE	0.14855	0.21529	0.24252	0.24470	0.15464
			(Average)$\hat{\beta}$	0.44599	0.44144	0.46386	0.45112	0.43303
0.9	100	MAVE-E	AMSE	0.04707	0.08485	0.07157	0.07803	0.04301
			(Average)$\hat{\beta}$	0.44425	0.44694	0.45591	0.45031	0.43844
		MAVE-G	AMSE	0.05032	0.08705	0.07595	0.08052	0.04305

Table 3: AMSE for Coefficients B Estimated by Methods (MAVE-Biwieght, MAVE-Epanechnikov and MAVE – Gaussian) Based on the Model in Experiment 1 for Sample Size N =25,100 with P = 0.2, 0.9 and P =6

ρ	sample size	methods		$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$
			(Average)$\hat{\beta}$	0.43530	0.45099	0.42675	0.45290	0.44194	0.15668
		MAVE-B	AMSE	0.01817	0.01774	0.01851	0.01959	0.01926	0.04949
			(Average)$\hat{\beta}$	0.45484	0.44343	0.44485	0.44736	0.44087	0.06392
0.2	25	MAVE-E	AMSE	0.00563	0.00781	0.00666	0.00645	0.00733	0.00947
			(Average)$\hat{\beta}$	0.45372	0.44389	0.44551	0.44700	0.44082	0.06685
		MAVE-G	AMSE	0.00546	0.00728	0.00647	0.00636	0.00726	0.00964
			(Average)$\hat{\beta}$	0.38431	0.52544	0.39215	0.47639	0.41046	0.16429
		MAVE-B	AMSE	0.08829	0.17495	0.14530	0.16629	0.13236	0.13349
			(Average)$\hat{\beta}$	0.45352	0.41453	0.48001	0.41590	0.46334	0.06624
0.9	25	MAVE-E	AMSE	0.03831	0.08195	0.06311	0.07267	0.04842	0.04065
			(Average)$\hat{\beta}$	0.45494	0.41421	0.47978	0.41314	0.46459	0.06869
		MAVE-G	AMSE	0.03676	0.08167	0.06137	0.07093	0.04758	0.03999
			(Average)$\hat{\beta}$	0.42820	0.45863	0.43980	0.44838	0.45589	0.06310
		MAVE-B	AMSE	0.02092	0.01977	0.02085	0.01914	0.02005	0.02731
			(Average)$\hat{\beta}$	0.44480	0.45143	0.44421	0.44791	0.44469	0.05145
0.2	100	MAVE-E	AMSE	0.00126	0.00107	0.00108	0.00113	0.00129	0.00331
			(Average)$\hat{\beta}$	0.44460	0.45117	0.44413	0.44790	0.44463	0.05651
		MAVE-G	AMSE	0.00122	0.00104	0.00105	0.00111	0.00125	0.00387
			(Average)$\hat{\beta}$	0.41465	0.54081	0.36691	0.32506	0.54070	0.05410
		MAVE-B	AMSE	0.10381	0.16461	0.16566	0.16215	0.17670	0.09891
			(Average)$\hat{\beta}$	0.44701	0.45064	0.43805	0.43993	0.45746	0.04886
0.9	100	MAVE-E	AMSE	0.00705	0.01266	0.01427	0.01577	0.01021	0.00814
			(Average)$\hat{\beta}$	0.44709	0.45005	0.43821	0.44084	0.45671	0.05082
		MAVE-G	AMSE	0.00694	0.01240	0.01398	0.01542	0.00993	0.00822

Table 4: The Average and Average Mean Squared Error (AMSE) for Coefficients B which are Estimated by Methods (MAVE-Biwieght, MAVE-Epanechnikov and MAVE – Gaussian) Based on the Model in Experiment 3 for Sample Size, N=25,100 with P = 0.2, 0.9 and P =7.

ρ	sample size	methods		$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$
0.2	25	MAVE-B	(Average)$\hat{\beta}$	0.50403	0.06336	.0.69527	0.15367	0.29558	0.27241	0.27079
			AMSE	0.11876	0.34889	0.27521	0.20426	0.15983	0.13213	0.13893
		MAVE-E	(Average)$\hat{\beta}$	0.47524	0.12630	0.67674	0.21132	0.28459	0.32911	0.25736
			AMSE	0.06281	0.39983	0.15202	0.25486	0.13386	0.16192	0.12006
		MAVE-G	(Average)$\hat{\beta}$	0.47489	0.11662	0.67567	0.20894	0.28846	0.33253	0.25865
			AMSE	0.06482	0.39051	0.15760	0.25638	0.13948	0.16481	0.12028
0.9	25	MAVE-B	(Average)$\hat{\beta}$	0.50862	-0.24808	0.80748	-0.10937	0.05614	0.10344	0.04374
			AMSE	0.08078	0.13219	0.10663	0.12197	0.09106	0.12756	0.07520
		MAVE-E	(Average)$\hat{\beta}$	0.42045	-0.31698	0.83119	-0.16678	0.04639	0.03083	0.03071
			AMSE	0.03681	0.07596	0.07010	0.05790	0.05514	0.06106	0.04176
		MAVE-G	(Average)$\hat{\beta}$	0.41418	-0.31556	0.83295	-0.17548	0.04734	0.03476	0.02835
			AMSE	0.03608	0.07927	0.05909	0.05150	0.05237	0.05539	0.03959
0.2	100	MAVE-B	(Average)$\hat{\beta}$	0.45937	-0.28840	0.82163	-0.12303	0.06299	0.07807	0.07423
			AMSE	0.08680	0.11735	0.30022	0.04941	0.02349	0.02198	0.02349
		MAVE-E	(Average)$\hat{\beta}$	0.46511	-0.21177	0.82594	-0.04414	0.13369	0.14034	0.13080
			AMSE	0.02069	0.12686	0.06916	0.07343	0.04367	0.04909	0.04142
		MAVE-G	(Average)$\hat{\beta}$	0.46472	-0.21250	0.82637	-0.04733	0.13311	0.13872	0.12955
			AMSE	0.02112	0.12354	0.07022	0.06934	0.04220	0.04775	0.04053
0.9	100	MAVE-B	(Average)$\hat{\beta}$	0.42525	-0.37670	0.80106	-0.18429	0.03120	0.02013	0.01374
			AMSE	0.02259	0.02473	0.01581	0.03841	0.03878	0.04224	0.02250
		MAVE-E	(Average)$\hat{\beta}$	0.41291	-0.38675	0.80296	-0.18630	0.01184	0.01157	0.01364
			AMSE	0.00193	0.00490	0.00195	0.00378	0.00296	0.00567	0.00249
		MAVE-G	(Average)$\hat{\beta}$	0.41506	-0.38257	0.80467	-0.18198	0.01506	0.01419	0.01754
			AMSE	0.00203	0.00538	0.00174	0.00448	0.00343	0.00552	0.00328

Table 5: AMSE for $\hat{G}$ ($X_i^t\hat{B}$) Estimated by Methods (MAVE-Biwieght, MAVE-Epanechnikov and MAVE – Gaussian) Sample Size N =25,100 with ρ = 0.2, 0.9

Model	p	methods	ρ =0.2,n=25		ρ =0.2 , n=100		ρ =0.9 ,n= 25		ρ =0.9,n=100	
			AMSE	AMAE	AMSE	AMAE	AMSE	AMAE	AMSE	AMAE
1	5	MAVE-B	0.009854	0.005652	0.009697	0.0055976	0.00971	0.00563	0.01021	0.00574
		MAVE-E	0.009851	0.005651	0.009699	0.0055977	0.00972	0.00559	0.00220	0.00534
		MAVE-G	0.009919	0.005658	0.00952	0.00555	0.00869	0.00530	0.00861	0.00571
2	6	MAVE-B	0.42850	0.02478	0.45480	0.02682	825.082	0.12472	121.548	0.13056
		MAVE-E	0.42837	0.02478	0.41450	0.02383	825.076	0.12478	121.465	0.11303
		MAVE-G	0.42306	0.02456	0.41636	0.02250	824.211	0.11190	115.234	0.09350
3	7	MAVE-B	0.27856	0.02699	0.20045	0.02272	0.17318	0.02077	0.13185	0.01840
		MAVE-E	0.27955	0.02706	0.20086	0.02272	0.17067	0.02065	0.12640	0.01815
		MAVE-G	0.25173	0.02570	0.18155	0.02186	0.19794	0.02570	0.13030	0.01844

According to the AMSE for the coefficients $\hat{\beta}$, from Table 2,3,and 4 .In general , and for different experiments we observed that in the majority of the estimated coefficients , and According to the AMSE and AMAE for the link function $\hat{g}$ ($X_i^T\hat{\beta}$) from table 5. The MAVE-G method have a lower AMSE and a lower AMAE than the MAVE-E and MAVE-B method.

4.2. Real data

To illustrate the performance of our methods from through analysis of the Iraq stock exchange data, the data set in our consist of n=27 companies, 21 banks sector, 4 Insurance sector and 2 investment sector. Through of average Interval (2008-2011) based on companies guide and it is a variable in the R- Package.

The response variable (y_i) is Earning per share (Iraqi Dinars (ID)) and The selected There are seven explanatory variables.

x_1- Share Turnover Ratio (%)

x_2- Owner ship Rate (%)

x_3- Interest Repetition (time)

x_4- Trading Rate (%)

x_5- Book Value (ID)

x_6- Annual closing price (ID)

x_7- Annual Average price

We estimate vector of parameters (β) and link function g (X^T β) for semi-parametric singl – index model between The Earning per share (y_i) and seven independent variables for predictor to earning per share in Iraq stock exchange from The following semi-parametric single – index model :

$$y_i = g(x_{i1}\beta_1 + x_{i2}\beta_2 + x_{i3}\beta_3 + x_{i4}\beta_4 + x_{i5}\beta_5 + x_{i6}\beta_6 + x_{i7}\beta_7) + \epsilon_i$$

According to Real data listed in the table below. Based on companies guide of Iraq stock exchange.

Table 6: A Real Data for Iraq Stock Exchange of Average Interval (2008-2011)

n	Yi	X1	X2	X3	X4	X5	X6	X7
1	0.095	12.028	43.525	21.6475	1.785	1.36275	1.315	1.41
2	0.20475	17.7275	14.7775	12.9375	1.1425	1.26125	2.48	2.46
3	0.0055	41.3725	52.24	156.3275	2.0125	1.005	0.94	0.96
4	0.2024	30.5175	14.975	9.8925	1.0725	1.385	1.9925	1.9475
5	0.1315	19.4175	33.5125	10.585	1.4475	1.2115	1.21	1.205
6	0.045	6.395	51.3475	64.7852	2.015	1.0975	0.8575	0.96
7	0.145	2.365	28.805	25.51	1.405	1.38	3.075	3.11375
8	0.15175	0.775	14.3	120.24	1.155	1.035	5.625	5.9625
9	0.02925	76.325	69.1925	111.5175	2.5275	1.1075	0.8575	0.9175
10	0.09925	11.16	33.59	14.0325	1.38	1.17375	1.145	1.1
11	0.22625	10.255	32.66	17.8325	1.3675	1.3605	2.1975	1.8225
12	0.2075	16.555	24.185	8	1.2525	1.275025	1.29	1.2125
13	0.185	14.705	29.0525	7.3475	1.3775	1.28975	1.215	1.2075
14	0.08075	3.7625	53.485	13.1875	2.03	1.12	0.9125	0.915
15	0.1925	24.82	24.6475	12.19	1.2575	1.3025	2.2025	2.48875
16	0.1855	2.345	26.365	11.3225	1.3175	1.39525	1.8425	1.8025
17	0.14725	9.125	46.27	7.07	1.7925	1.2035	1.005	1.06375
18	0.2955	7.265	51.61	18.4375	20.875	1.17125	1.3225	1.65625
19	0.145	25.39	34.1825	8.3375	1.49	1.375	1.575	1.465
20	0.08175	1.2875	32.045	4.2425	1.365	1.09925	0.39	0.43
21	0.26175	0.095	33.02	1.155	1.39	1.1075	0.2425	0.245
22	0.13725	24.675	80.5375	16.3325	5.6	1.485	1.255	1.27
23	0.08025	16.885	94.4175	103.56	19.1125	1.3765	2.4875	2.7625
24	0.10975	8.605	89.145	14.2925	8.4925	1.3625	1.055	1.125
25	0.001925	51.2875	91.7925	45.2225	14.97	1.053	1.21	1.275
26	0.09425	11.335	93.41	67.495	23.51	11.075	1.6625	1.6735
27	0.05	13.0178	91.6375	27.9075	11.9425	1.16925	1.0875	1.0225

Table 7: The Estimated Coefficients $\hat{B}$ for Single Index Model which are Estimated by Methods (MAVE-Biwieght, MAVE- Epanechnikov and MAVE-Gaussian) Based on Real Data

Methods	coefficients $\hat{\beta}$						
	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$
MAVE-B	0.04695	0.58040	0.15398	-0.43744	0.07417	-0.45523	0.48282
MAVE-E	0.08946	0.57626	0.23449	-0.43922	0.05470	-0.45003	0.45439
MAVE-G	0.10508	-0.64090	-0.13815	0.55173	-0.12048	-0.33928	0.35420

Table 8: The Values of MSE and MAE for Single Index Model which are Estimated by Methods (MAVE-Biwieght, MAVE- Epanechnikov and MAVE-Gaussian) Based on Real Data

Methods	MSE	MAE
MAVE-B	0.00229	0.03518
MAVE-E	0.00233	0.03623
MAVE-G	0.00210	0.03171

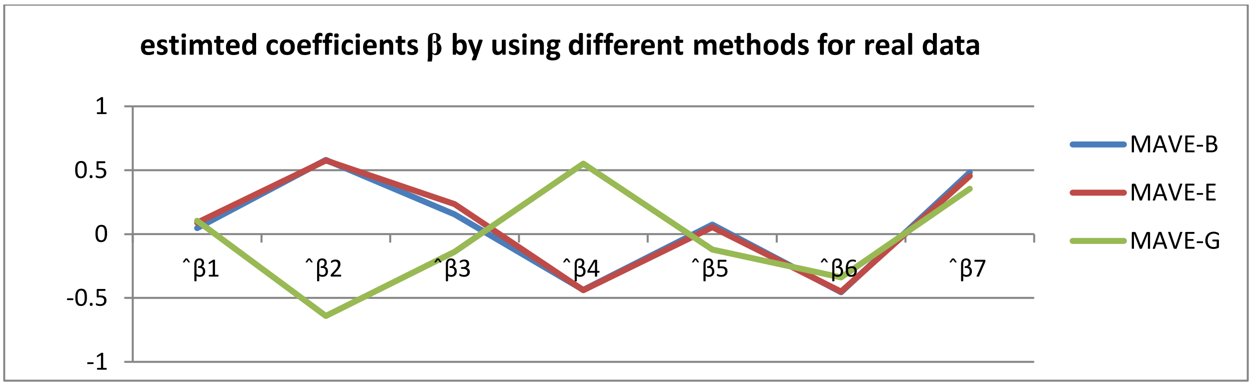

Fig. 1: Plot Explain the Estimated Coefficients B of Single – Index Model which are Estimated by the Different Methods (MAVE-B , MAVE-E and MAVE-G) Based on A Real Data.

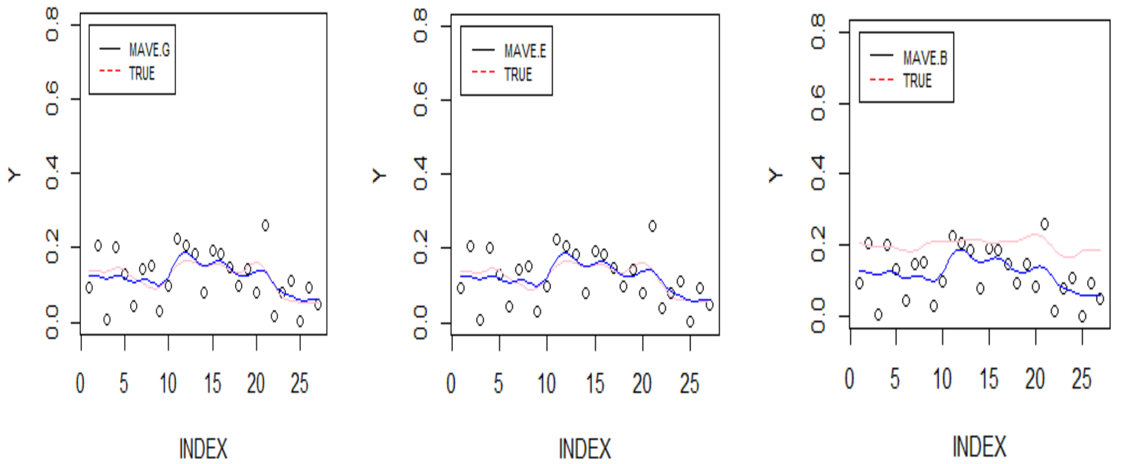

Fig. 2: Plot for the Smooth Estimated Curve $\widehat{G}\,(X_i^t\widehat{B})$ of Single – Index Model which are Estimated By Methods (MAVE-B, MAVE-E and MAVE-G) with Based on Areal Data.

According to the MSE and AME for link function $\hat{g}\,(X_i^T\hat{\beta})$ of single – index model from table 8 and figure 2. We find that the same results in simulation study are extended to practical study, the results of the real data example confirm the results of the simulation studies, i.e,the MAVE-G show a better performance than the MAVE-E and MAVE-B method .

5. Conclusions

We can be conclude that the MAVE method based on Gaussian kernel function perform well in comparison to the MAVE method based on Epanechnikov and Biweight kernel function for estimated parameters and link function of semi-parametric single–index model.

References

[1] Al – kenani, A., and Yu, K. (2013), "Penalized single Indexquantile regression ". International Journal of statistics and probability, vol.2, No.3, pp. 12-30.

[2] Akkus, O. (2011), "Xplore package for the popular parametric and semi-parametric single index models " .Journe of science ,vol.24 , No.4, pp. 753-762 .

[3] Hansen, W.E. (2009),"lecture Notes on Non-parametric ".university of Wisconsin,spring.

[4] Hardle, W., Hall, P., and Ichimura, H. (1993)," optimal smoothing in single index models ". The Annals of statistics, vol.21, pp. 157-178. https://doi.org/10.1214/aos/1176349020.

[5] Kong, E., Xia, Yi. (2007), "variable selection for the single index model".Biometrika94,pp.217-229. https://doi.org/10.1093/biomet/asm008.

[6] Kopytou, E., and Santalova, D. (2007), "Application of the single index model for forcasting of the inland conveyances "Recent advances in stochastic modeling and data analysis. Singupre, world scientific poblishing copte Ltd,pp.268-276. https://doi.org/10.1142/9789812709691_0033.

[7] Lai, P., Wang, Q., and Lion, H. (2011), "Bias-corrected GEE estimation and smooth–threshold GEE variable selection for single index models with clustered data ". Journal of multivariate analysis, vol.105,No.1,pp.422-432. https://doi.org/10.1016/j.jmva.2011.08.009.

[8] Leng, C.L., Xia, Y., & Xu, J. (2008), "An adaptive estimation method for semi-parametric models and dimension reduction ". Department of statistics and Applied probability National university of Singapore .Exploration of a nonlinear world, pp. 347-360.

[9] Li , R. , and Fan , J., (2004) , " New Estimation and model selection procedures for semi-parametric modeling in longitudinal Data A nalysis " , journal of the American statistical Association , vol .99 , No . 467, pp. 710-723.

[10] Naik, P.A., and Tsai, C.L. (2001), "single index model selections". Biometrika88,pp.821-832. https://doi.org/10.1093/biomet/88.3.821.

[11] Peng, H., and Huang, T. (2011), "penalized least squares for single index models ". Journal of statistical planning and inference 141, pp. 1362-1379. https://doi.org/10.1016/j.jspi.2010.10.003.

[12] Thomas, J.F. (2006), "Simulation study for single index model" .submitted to the Department of Mathematical sciences of Clemson University, in partial fulfillment for The requirements for The degree of Master of science in Mathematical sciences.

[13] Wang, G. (2015),"High / Ultra High Dimensional single – index models ". A Dissertation submitted to The Graduate Faculty of The University of Georgina in partial Fulfillment of The requirements for the Doctor PHD. Department of statistics, Athens, GEORGIA.

[14] Wang ,Q.,Zhang,T.,and Hardle,W.,(2014), " An Extended single – index model with missing response at random " . SFB 649, Humbodt, university Zu Berlin spandaure. http:// sfb649.wiwi.hu-berlin .de.

[15] Wang, T., Xu .P.,and Zhu , L.(2013) ," Penalized Minimum Average Variance Estimation " . Statistics sinica 23, pp.543-569. https://doi.org/10.5705/ss.2011.275.

[16] Xia, Y. (2006), "Asymptotic Distribution for Tow estimators of the Single – index model", National university of singopore, EconometricTheory,22,pp.1112–1137. https://doi.org/10.1017/S0266466606060531.