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Research paper

New temperature dependent models for estimating global solar radiation across the midland climatic zone of Nigeria

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Abstract

Authentic information of the availability of global solar radiation is significant to agro/hydro meteorologists, atmospheric Physicists and solar energy engineers for the purpose of local and international marketing, designs and manufacturing of solar equipment. In this study, five new proposed temperature dependent models were evaluated using measured monthly average daily global solar radiation, maximum and minimum temperature meteorological data during the period of thirty one years (1980-2010). The new models were compared with three existing temperature dependent models (Chen et al., Hargreaves and Samani and Garcia) using seven different statistical validation indicators of coefficient of determination (R^2), Mean Bias Error (MBE), Root Mean Square Error (RMSE), Mean Percentage Error (MPE), t – test, Nash – Sutcliffe Equation (NSE) and Index of Agreement (IA) to ascertain the suitability of global solar radiation estimation in five different locations (Zaria, Bauchi, Jos, Minna and Yola) situated in the Midland climatic zone of Nigeria. In each location, the result shows that a new empirical regression model was found more accurate when compared to the existing models and are therefore recommended for estimating global solar radiation in the location and regions with similar climatic information where only temperature data are available. The evaluated existing Hargreaves and Samani and Garcia temperature based models for Jos were compared to those available in literature and was found more suitable for estimating global solar radiation for the location. The comparison between the measured and estimated temperature dependent models depicts slight overestimation and underestimation in some months with good fitting in the studied locations. However, the recommended models give the best fitting.

Keywords: *Global Solar Radiation; Meteorological Data; Midland Climatic Zone; Statistical Validation Indicators; Temperature Dependent Models.*

1. Introduction

Solar Energy is the cleanest and the most abundant renewable energy in the world [1]. Solar radiation data is the most important resource needed for solar energy system design. Solar radiation is the most important source of energy on earth because it plays a major role in sustaining all the activities and processes that support life of both plants and animals on earth [2].Solar radiation data are needed in a variety of technological areas: agriculture, engineering, forestry, meteorology, water resources management, and the designing and sizing of solar energy systems. Among the various professionals that use solar radiation data, solar energy devices design experts are more concerned about the accuracy of the data since efficient design, sizing, and performance of solar energy devices depend on the accuracy of the available insolation data of the site [2]. Due to unavailability of solar radiation data in many locations, the solar energy system design expert must be familiar with the various models used to estimate global solar radiation so as to determine the amount of global solar radiation available at any location from the available meteorological parameters [2].

Several models have been proposed to estimate global solar radiation. Ångström [3] was the first scientist known to suggest a simple linear relationship to estimate global solar radiation. Page [4] presents a linear regression model used in correlating the global solar radiation data with relative sunshine duration, which is a modified Ångström type model. Sanusi and Abisoye [5] presents and evaluates the behavior of three empirical models (Hargreaves – Samani model, Hargreaves model with linear regression and Hargreaves model with power regression) based on the difference between maximum and minimum temperature at Ibadan. The data used in their study were obtained from the International Institute of Tropical Agriculture (IITA), station of Ibadan, located within the rainforest climatic zone of Western Nigeria. The data obtained covered a period of 6 years (2001 – 2006). The best performing model was found to be Hargreaves model with linear regression and followed by Hargreaves model with power regression and original Hargreaves model. Therefore, they concluded that the Hargreaves model with linear regression was recommended for predicting global solar radiation at Ibadan and stations with similar geographical information.

Huashan et al. [6] proposed new model based on Hargreaves and Samani (HS) method for the estimation of global solar radiation at 65 meteorological stations in China. The new model was compared with the HS model and its two modification (Samani model and Chen model), using statistical error tests of MPE, MBE, RMSE and NSE. According to them, it was reported that the new model is more accurate and robust than the HS, Samani and Chen models in all climatic regions, especially in the humid region and therefore recommended for solar radiation estimation in regions where air temperature data are available in China. Girma [7] compare several existing sunshine and temperature based models using data on sunshine hours, minimum and maximum temperatures obtained from the Ethiopian Institute of Agricultural Research: Tepi National Spices Research Centre to estimate the global solar radiation for Tepi located in South West of Ethiopia. The monthly averages sunshine hour for 4 years (2013 – 2016) and monthly averages maximum and minimum temperatures for 5 years (2012 – 2016). The monthly averages daily global solar radiation from the Archives of National Aeronautics and Space Administration (NASA) for 22 years (July, 1983 – June, 2005) was utilized in his study. According to him, from the sunshine based models, the Samuel (polynomial) and the Newland (logarithm) models are appropriate for Tepi while the Chen et al model from the temperature based models is more appropriate for Tepi.

The purpose of this study is to develop new temperature dependent models and to compare these models with the existing temperature dependent models with a view to recommend the most suitable model for estimating global solar radiation in each of the five studied areas (Zaria, Bauchi, Jos, Minna and Yola) situated in the Midland climatic zone of Nigeria.

2. Methodology

2.1. Acquisition of data

It was mentioned according to the World Meteorological Organization [8] and Ojo and Adeyemi [9] that to ensure the optimal climate modeling, data series should be a minimum of thirty years long. Based on this, the measured monthly average daily global solar radiation, maximum and minimum temperature meteorological data during the period of thirty one years (1980-2010) was used in this study. The meteorological data were obtained from the Nigerian Meteorological Agency (NIMET), Oshodi, Lagos, Nigeria. Twenty five (25) (1980 – 2004) years data was used for developing the empirical models while six (6) years (2005 – 2010) data was used for validation of the models. The stations that are located within the Midland climatic zones of the study areas are shown in Fig. 1.

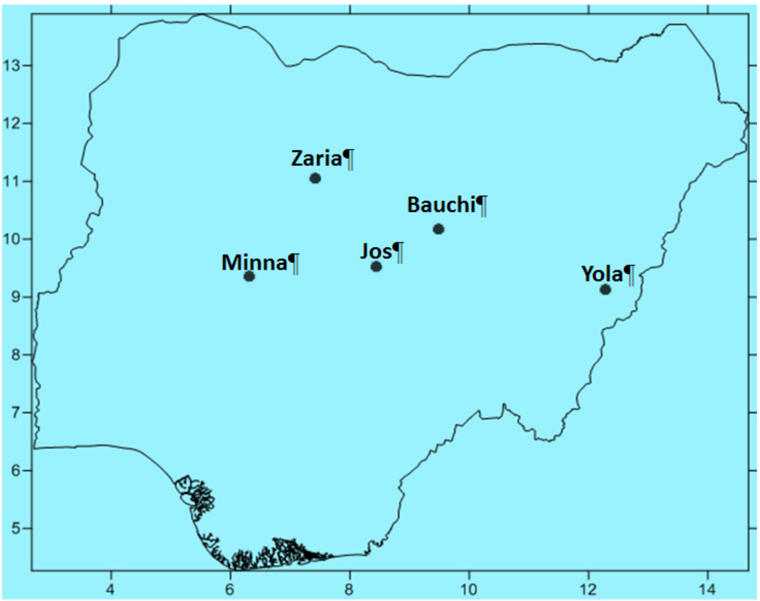


Fig. 1: A Map of Nigeria Showing the Midland Climatic Zone and the Studied Locations.

2.2. Regression analysis

The monthly average daily extraterrestrial radiation on a horizontal surface (H_o) in MJ/m^2/day can be calculated for days giving average of each month from the following equation [10 – 11]:

$$H_o = \left(\frac{24}{\pi}\right) I_{sc}\left[1 + 0.033\cos\left(\frac{360n}{365}\right)\right]\left[\cos\varphi\cos\delta\sin\omega_s + \left(\frac{2\pi\omega_s}{360}\right)\sin\varphi\sin\delta\right] \quad (1)$$

where I_{sc} is the solar constant (=1367 Wm^{-2}), φ is the latitude of the site, δ is the solar declination and ω_s is the mean sunrise hour angle for the given month and n is the number of days of the year starting from 1st of January to 31st of December.

The solar declination, δ and the mean sunrise hour angle, ω_s can be calculated using the following equation [10 – 11]:

$$\delta = 23.45\sin\left\{360\left(\frac{284+n}{365}\right)\right\} \quad (2)$$

$$\omega_s = \cos^{-1}(-\tan\varphi\tan\delta) \quad (3)$$

For a given month, the maximum possible sunshine duration (monthly average day length (S_o)) in hours can be computed [10 – 11] by

$$S_o = \frac{2}{15}\omega_s \quad (4)$$

The clearness index (K_T) is defined as the ratio of the observed/measured horizontal terrestrial solar radiation H, to the calculated/predicted/estimated horizontal extraterrestrial solar radiation H_o [12].

$$K_T = \frac{H}{H_o} \qquad (5)$$

where H is the monthly average daily global solar radiation on a horizontal surface (MJ/m^2/day). In this study, H_o and S_o were computed for each month using equations (1) and (4) respectively.

Table 1: Temperature Based Regression Models Proposed in Literature that was used in this Study

Model No.	Model Type	Regression equation	Source
1	Logarithmic	$\frac{H}{H_0} = a_2 + b_2 \ln\Delta T$	[13]
2	Linear exponent	$\frac{H}{H_0} = a_3 + b_3 \Delta T^{0.5}$	[14]
3	Linear	$\frac{H}{H_0} = a_4 + b_4\left(\frac{\Delta T}{S_0}\right)$	[15]

where H, H_0 and S_0 are as previously defined. ΔT is the difference between the monthly average daily maximum and minimum temperature i.e.,$T_{max} - T_{min}$

The constants $a_{2,}, a_3, a_4, b_2, b_3$ and b_4 in Table 1 are empirical coefficients determined by regression analysis and the other terms are the model correlated parameters. The models are basically the three widely used temperature dependent models and has been found suitable in all climatic conditions. The five proposed temperature dependent models by the authors in this study are given in Table 2.

Table 2: Temperature Based Regression Model Proposed in this Study

Model No.	Model Type	Regression equation
1	Linear (exponential 1)	$\frac{H}{H_o} = a + b(\Delta T)^{0.5} + cexp[(\Delta T)^{0.5}]$
2	Quadratic Logarithmic	$\frac{H}{H_o} = a + (\Delta T) + c(\Delta T)^2 + dln(\Delta T)$
3	Quadratic exponential	$\frac{H}{H_o} = a + b(\Delta T) + c(\Delta T)^2 + dexp(\Delta T)$
4	Linear (exponential 2)	$\frac{H}{H_o} = a + \mathrm{b}\left(\frac{\Delta T}{S_0}\right) + cexp\left(\frac{\Delta T}{S_0}\right)$
5	Linear logarithmic	$\frac{H}{H_o} = a + b\left(\frac{\Delta T}{S_0}\right) + cln\left(\frac{\Delta T}{S_0}\right)$

The models in Table 2 are proposed for this study in form of mathematical equations that relate the clearness index as the dependent variable and temperature as the independent variables. The proposed temperature dependent models are based on the modification of the existing models. The essence of modification is to find out if it improves the accuracy of the existing models.

The accuracy or validation of the estimated values was statistically tested by computing the MBE, RMSE, MPE, t-test, NSE and the IA, similarly, R^2 was determined for each of the models.The expressions for the MBE, RMSE and MPE as stated according to El-Sebaii and Trabea [16] are given as follows.

$$MBE = \frac{1}{n}\sum_{i=1}^{n}\left(H_{i,cal} - H_{i,mea}\right) \qquad (6)$$

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}\left(H_{i,cal} - H_{i,mea}\right)^2\right]^{\frac{1}{2}} \qquad (7)$$

$$MPE = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{H_{i,mea}-H_{i,cal}}{H_{i,mea}}\right) * 100 \qquad (8)$$

The t-test defined by student [17] in one of the tests for mean values, the random variable t with n-1 degrees of freedom may be written as follows.

$$t = \left[\frac{(n-1)(MBE)^2}{(RMSE)^2-(MBE)^2}\right]^{\frac{1}{2}} \qquad (9)$$

The Nash-Sutcliffe equation (NSE) is given by the expression

$$NSE = 1 - \frac{\sum_1^n\left(H_{i,mea}-H_{i,cal}\right)^2}{\sum_1^n\left(H_{i,mea}-\bar{H}_{i,meas}\right)^2} \qquad (10)$$

The Index of Agreement (IA) is given as

$$IA = 1 - \frac{\sum_{i=1}^{n}\left(H_{i,cal}-H_{i,mea}\right)^2}{\sum_{i=1}^{n}\left(\left|H_{i,cal}-\bar{H}_{i,mea}\right|+\left|H_{i,mea}-\bar{H}_{i,mea}\right|\right)^2} \qquad (11)$$

From equations (6) – (11) $H_{i,mea}$, $H_{i,cal}$ and n are respectively the i^{th} measured and i^{th} calculated values of daily global solar radiation and the total number of observations, also $\bar{H}_{i,mea}$ is the mean measured global radiation.

Chen et al. [13] have recommended that a zero value for MBE is ideal and a low RMSE and MPE are desirable. The smaller the value of the MBE, MPE and RMSE the better is the model's performance, a positive MPE and MBE values provide the averages amount of overestimation in the calculated values, while the negative values gives underestimation. The percentage error between -10% and $+10\%$ is considered acceptable [18]. The smaller the value of t the better is the performance. High value of R^2, NSE and IA are desirable. The MBE and the RMSE are in $MJm^{-2}day^{-1}$, while R^2, MPE, NSE and IA are in percentage (%), the t – test is non dimensional.

3. Results and discussion

3.1. Temperature based models for Zaria

The evaluated regression equations for the existing temperature dependent models for Zaria based on Table 1 are

$$\frac{H}{H_0} = -0.210 + 0.331\, ln\, \Delta T \qquad (12a)$$

$$\frac{H}{H_0} = -0.0572 + 0.192\, Sqrt\, \Delta T \qquad (12b)$$

$$\frac{H}{H_0} = 0.314 + 0.291\, \frac{\Delta T}{S_0} \qquad (12c)$$

The proposed evaluated regression equations for the temperature dependent models for Zaria based on Table 2 are

$$\frac{H}{H_0} = -0.438 + 0.350\ Sqrt\, \Delta T - 0.00479\, exp\, Sqrt\,(\Delta T) \qquad (12d)$$

$$\frac{H}{H_0} = -2.05 - 0.280\, \Delta T + 0.0053\, (\Delta T)^2 + 2.12\, ln\, \Delta T \qquad (12e)$$

$$\frac{H}{H_0} = -0.174 + 0.107\, \Delta T - 0.00341\, (\Delta T)^2 + 0.0\, exp\, \Delta T \qquad (12f)$$

$$\frac{H}{H_0} = 0.305 + 0.881 \frac{\Delta T}{S_0} - 0.207\, exp \left(\frac{\Delta T}{S_0}\right) \qquad (12g)$$

$$\frac{H}{H_0} = 0.939 - 0.315 \frac{\Delta T}{S_0} + 0.606\, ln \left(\frac{\Delta T}{S_0}\right) \qquad (12h)$$

Table 3: A) Validation of the Temperature Based Models for Zaria under Different Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.12a	95.1	-0.0165	0.5712	0.0763	0.0961	99.1219	99.7807
E.12b	94.4	-0.0362	0.6099	0.1430	0.1972	98.9987	99.7486
E.12c	94.5	0.0103	0.6147	-0.1458	0.0556	98.9831	99.7442
Author’s Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.12d	95.5	0.049	0.5594	-0.1919	0.2915	99.1578	99.7926
E.12e	95.7	0.0199	0.533	-0.0638	0.1241	99.2356	99.8107
E.12f	95.6	-0.379	0.7557	1.7165	1.9228	98.4630	99.5917
E.12g	97.3	0.0386	0.4353	-0.1642	0.2950	99.4900	99.8738
E.12h	97.5	0.0139	0.4045	-0.0438	0.1137	99.5596	99.8906

Table 3: B) Ranking of the Evaluated Temperature Based Models for Zaria as Per Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.12a	6	3	5	3	2	5	5	24
E.12b	8	5	6	4	5	6	6	34
E.12c	7	1	7	5	1	7	7	28
Author’s Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.12d	5	7	4	7	6	4	4	33
E.12e	3	4	3	2	4	3	3	19
E.12f	4	8	8	8	8	8	8	44
E.12g	2	6	2	6	7	2	2	25
E.12h	1	2	1	1	3	1	1	9

Table 3A) and 3B) gives the summary of the various statistical tests adopted and ranking of the temperature based models for Zaria. Based on the R^2 the model, equation 12h (author’s model) has the highest value with 97.5 % and is judged the best model. Based on the MBE, the model, equation 12c (existing model) has the lowest value with overestimation of 0.0103 $MJm^{-2}day^{-1}$ in the estimated value and is judged the best model. Based on the RMSE the model, equation 12h (author’s model) has the lowest value with 0.4045 $MJm^{-2}day^{-1}$ and is judged the best model. Based on the MPE, despite the observed overestimation and underestimation exhibited by some of the existing and author’s models they are fall within the acceptable range $(MPE \leq \pm 10\%)$ with the model, equation 12h (author’s model) having the lowest value with underestimation of 0.0438 % in the estimated value and is judged the best model. The study site was statistically tested at the $(1-\alpha)$ confidence levels of significance of 95% and 99%. For the critical t-value, i.e., at α level of significance and degree of freedom, the calculated t-value must be less than the critical value $(t_{critical} = 2.20, df = 11, p < 0.05)$ for 95% and $(t_{critical} = 3.12, df = 11, p < 0.01)$ for 99%. It was observed that the $t_{cal} < t_{critical}$values for all the models under consideration. The $t -$ test shows that all models are significant at 95% and 99% confidence levels. However, the model, equation 12c (existing model) has the lowest value with 0.0556 and is judged the best model. Based on the NSE and IA the models, equation 12h (author’s model) has the highest value with 99.5596 % and 99.8906 % and is judged the best model.
The ranking of the existing and author’s models {Table 3B)} was done based on the validation of the models {Table 3A)}. The total ranks acquired by the different models were in the range 9 to 44. Based on the overall results for the temperature dependent models for

Zaria, the model, equation 12h (author's model) is reported as the best and therefore most suitable for estimating global solar radiation in this location as compared to other evaluated temperature dependent models.

Fig. 2A shows the comparison between the measured and estimated global solar radiation for Zaria. It can be seen that the estimated temperature based models underestimated the measured global solar radiation in the months of September, October and December and overestimated the measured in the month of August. The figure show that there is a remarkable underestimation of the model (equation 12f) in the month from January to March and from November to December as compared to the measured and other estimated temperature based global solar radiation models. The pattern of variation of the estimated models follows similar pattern with the measured indicating good fitting despite slight overestimation and underestimation in their estimated values. However, the model (equation 12h) gives the best fitting.

3.2. Temperature based models for Bauchi

The evaluated regression equations for the existing temperature dependent models for Bauchi based on Table 1 are

$$\frac{H}{H_0} = -0.0376 + 0.240\, ln\, \Delta T \qquad (13a)$$

$$\frac{H}{H_0} = 0.0940 + 0.134\, Sqrt\, \Delta T \qquad (13b)$$

$$\frac{H}{H_0} = 0.364 + 0.192\, \frac{\Delta T}{S_0} \qquad (13c)$$

The proposed evaluated regression equations for the temperature dependent models for Bauchi based on Table 2 are

$$\frac{H}{H_0} = -0.511 + 0.371\ Sqrt\, \Delta T - 0.00628\, exp\, Sqrt\, (\Delta T) \qquad (13d)$$

$$\frac{H}{H_0} = -3.13 - 0.389\, \Delta T + 0.00664\, (\Delta T)^2 + 2.99\, ln\, \Delta T \qquad (13e)$$

$$\frac{H}{H_0} = -0.428 + 0.147\, \Delta T - 0.00522\, (\Delta T)^2 + 0.0\, exp\, \Delta T \qquad (13f)$$

$$\frac{H}{H_0} = 0.295 + 0.886 \frac{\Delta T}{S_0} - 0.222\, exp \left(\frac{\Delta T}{S_0}\right) \qquad (13g)$$

$$\frac{H}{H_0} = 1.15 - 0.573\, \frac{\Delta T}{S_0} + 0.829\ ln \left(\frac{\Delta T}{S_0}\right) \qquad (13h)$$

Table 4: A) Validation of the Temperature Based Models for Bauchi under Different Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.13a	88.9	-0.0444	0.6846	0.0426	0.2153	97.9621	99.4731
E.13b	87.3	0.0394	0.7375	-0.3959	0.1776	97.6348	99.3964
E.13c	83.3	-0.0086	0.8558	-0.2360	0.0332	96.8147	99.1693
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.13d	92.4	-0.0126	0.5502	0.0044	0.0763	98.6833	99.6682
E.13e	93.3	0.352	0.6269	-1.7860	2.2504	98.2907	99.5985
E.13f	94.2	-0.6977	1.3403	3.3221	2.0221	92.1877	97.7441
E.13g	92.6	0.0232	0.5450	-0.1771	0.1413	98.7085	99.6765
E.13h	93.8	-0.1549	0.5106	0.7121	1.0557	98.8660	99.7061

Table 4: B) Ranking of the Evaluated Temperature Based Models for Bauchi as Per Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.13a	6	5	5	2	5	5	5	28
E.13b	7	4	6	5	4	6	6	32
E.13c	8	1	7	4	2	7	7	29
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.13d	5	2	3	1	1	3	3	15
E.13e	3	7	4	7	8	4	4	33
E.13f	1	8	8	8	7	8	8	40
E.13g	4	3	2	3	3	2	2	17
E.13h	2	6	1	6	6	1	1	22

Table 4A) and 4B) gives the summary of the various statistical tests adopted and ranking of the temperature based models for Bauchi. Based on the R^2 the model, equation 13f (author's model) has the highest value with 94.2 % and is judged the best model. Based on the MBE, the model, equation 13c (existing model) has the lowest value with underestimation of 0.0086 $MJm^{-2}day^{-1}$ in the estimated value and is judged the best model. Based on the RMSE the model, equation 13h (author's model) has the lowest value with 0.5106 $MJm^{-2}day^{-1}$ and is judged the best model. Based on the MPE, despite the observed overestimation and underestimation exhibited by some of the existing and author's models they are fall within the acceptable range ($MPE \leq \pm 10\%$) with the model, equation 13d (author's model) having the lowest value with overestimation of 0.0044 % in the estimated value and is judged the best model. The study site was statistically tested at the $(1-\alpha)$ confidence levels of significance of 95% and 99%. For the critical t-value, i.e., at α level of significance and degree of freedom, the calculated t-value must be less than the critical value ($t_{critical} = 2.20, df = 11, p < 0.05$) for 95% and ($t_{critical} = 3.12, df = 11, p < 0.01$) for 99%. It was observed that the $t_{cal} < t_{critical}$ values for all the models under consideration. The $t-$test shows that all models are significant at 95% and 99% confidence levels, except for the model, equation 13e (author's model) that is not significant at 95% confidence levels. However, the model, equation 13d (author's model) has the lowest value with 0.0763 and is judged the best model. Based on the NSE and IA the model, equation 13h (author's model) has the highest value with 98.8660 % and 99.7061 % and is judged the best model.

The ranking of the existing and author's models {Table 4B)} was done based on the validation of the models {Table 4A)}. The total ranks acquired by the different models were in the range 15 to 40. Based on the overall results for the temperature based models for Bauchi, the model, equation 13d (author's model) is reported as the best and therefore most suitable for estimating global solar radiation in this location as compared to other evaluated temperature based models.

Fig. 2A shows the comparison between the measured and estimated global solar radiation for Bauchi. The figure revealed that the estimated temperature based models underestimated the measured global solar radiation in the months from September to November and overestimated the measured in the month of July. Notably, is the model (equation 13f) that depicts large underestimation in its estimated value in the month from January to March and November and December; this is an indication that the model may not suitable for global solar radiation estimation in the location as it was testify on Table 4A) and 4B).

3.3 Temperature based models for Jos

The evaluated regression equation for the existing temperature dependent models for Jos based on Table 1 are

$$\frac{H}{H_0} = -0.315 + 0.376\, ln\, \Delta T \qquad (14a)$$

$$\frac{H}{H_0} = -0.153 + 0.221\, Sqrt\, \Delta T \qquad (14b)$$

$$\frac{H}{H_0} = 0.255 + 0.348\, \frac{\Delta T}{S_0} \qquad (14c)$$

The proposed evaluated regression equations for the temperature dependent models for Jos based on Table 2 are

$$\frac{H}{H_0} = 0.037 + 0.142\ Sqrt\, \Delta T + 0.00240\, exp\, Sqrt\, (\Delta T) \qquad (14d)$$

$$\frac{H}{H_0} = 0.94 + 0.160\, \Delta T - 0.00261\, (\Delta T)^2 - 0.76\, ln\, \Delta T \qquad (14e)$$

$$\frac{H}{H_0} = 0.456 - 0.0133\, \Delta T + 0.00211\, (\Delta T)^2 - 0.0\, exp\, \Delta T \qquad (14f)$$

$$\frac{H}{H_0} = 0.255 + 0.362\, \frac{\Delta T}{S_0} - 0.0050\, exp\left(\frac{\Delta T}{S_0}\right) \qquad (14g)$$

$$\frac{H}{H_0} = 0.244 + 0.359\, \frac{\Delta T}{S_0} - 0.011\, ln\left(\frac{\Delta T}{S_0}\right) \qquad (14h)$$

Table 5: A) Validation of the Temperature Based Models for Jos under Different Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.14a	94.2	0.0510	0.8974	-0.2019	0.1889	97.8594	99.4736
E.14b	94.8	0.0469	0.8364	-0.1916	0.1864	98.1406	99.5421
E.14c	96.9	0.0210	0.6349	-0.0925	0.1098	98.9287	99.7341
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.14d	95.1	0.0868	0.8084	-0.3998	0.3581	98.2632	99.574
E.14e	95.2	-0.2792	0.8377	1.3225	1.1726	98.1350	99.5169
E.14f	95.8	0.6006	1.2851	-2.5067	1.7531	95.6105	99.0387
E.14g	96.9	0.0143	0.6350	-0.0607	0.0749	98.9282	99.7337
E.14h	96.9	0.0379	0.6356	-0.1747	0.1981	98.9264	99.7342

Table 5: B) Ranking of the Evaluated Temperature Based Models for Jos as Per Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.14a	6	5	7	5	4	7	7	34
E.14b	5	4	5	4	3	5	5	26
E.14c	1	2	1	2	2	1	2	9
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.14d	4	6	4	6	6	4	4	30
E.14e	3	7	6	7	7	6	6	36
E.14f	2	8	8	8	8	8	8	42
E.14g	1	1	2	1	1	2	3	8
E.14h	1	3	3	3	5	3	1	18

Table 5A) and 5B) gives the summary of the various statistical tests adopted and ranking of the temperature based models for Jos. Based on the R^2 the model, equation 14c (existing model), equation 14g (author's model) and equation 14h (author's model) has the highest value with 96.9 % and are judged the best model. Based on the MBE, the model, equation 14g (author's model) has the lowest value with overestimation of 0.0143 $MJm^{-2}day^{-1}$ in the estimated value and is judged the best model. Based on the RMSE the model, equation 14c (existing model) has the lowest value with 0.6349 $MJm^{-2}day^{-1}$ and is judged the best model. Based on the MPE, despite the observed overestimation and underestimation exhibited by some of the existing and author's models they are fall within the acceptable range $(MPE \leq \pm 10\%)$ with the model, equation 14g (author's model) having the lowest value with underestimation of 0.0607 % in the estimated value and is judged the best model. The study site was statistically tested at the $(1-\alpha)$ confidence levels of significance of 95% and 99%. For the critical t-value, i.e., at α level of significance and degree of freedom, the calculated t-value must be less than the critical value $(t_{critical} = 2.20, df = 11, p < 0.05)$ for 95% and $(t_{critical} = 3.12, df = 11, p < 0.01)$ for 99%. It was observed that the $t_{cal} <$

$t_{critical}$values for all the models under consideration. The $t -$ test shows that all models are significant at 95% and 99% confidence levels. However, the model, equation 14g (author's model) has the lowest value with 0.0749 and is judged the best model. Based on the NSE the model, equation 14c (existing model) has the highest value with 98.9287 % and is judged the best model. Based on the IA the model, equation 14h (author's model) has the highest value with 99.7342 % and is judged the best model.

The ranking of the existing and researcher's models {Table 5B)} was done based on the validation of the models {Table 5A)}. The total ranks acquired by the different models were in the range 8 to 42. Based on the overall results for the temperature based models for Jos, the model, equation 14g (researcher's model) is reported as the best and therefore most suitable for estimating global solar radiation in this location as compared to other evaluated temperature based models.

The Hargreaves and Samani and the Garcia temperature dependent models result obtained for Jos in this study were compared to that carried out by Ogolo [19]. The model equation with its empirical constants are given in equation 14b and 14c while the empirical constants given by Ogolo [19] are 0.129 and 0.166 for Hargreaves and Samani model; 0.359 and 0.333 for Garcia model. In this study the MBE, RMSE, MPE and IA for Hargreaves and Samani model are found to be 0.0469 $MJm^{-2}day^{-1}$, 0.8364 $MJm^{-2}day^{-1}$, -0.1916 % and 99.5421 % respectively while for Garcia model the MBE, RMSE, MPE and IA are found to be 0.0210 $MJm^{-2}day^{-1}$, 0.6349 $MJm^{-2}day^{-1}$, -0.0925 % and 99.7341 % respectively while the MBE, RMSE, MPE and IA given by those of Ogolo [19] for Hargreaves and Samani model are 4.51 $MJm^{-2}day^{-1}$, 22.38 $MJm^{-2}day^{-1}$, 17.58 % and 87.0 % respectively, for Garcia model, MBE, RMSE, MPE and IA are 4.43 $MJm^{-2}day^{-1}$, 20.89 $MJm^{-2}day^{-1}$, 17.19 %and 93.0 % respectively. Thus, this is evident that the model in this study performed better than his model based on the statistical test result. However, the model equation 14g (author's model) is reported as the best and therefore most suitable for estimating global solar radiation in this location as compared to other evaluated temperature based models.

Fig. 2C shows the comparison between the measured and estimated temperature based global solar radiation for Jos. The estimated models underestimated the measured global solar radiation in the months of June, September, October and November and overestimated the measured in the months from February to May. The model (equation 14a) underestimated the measured and other estimated models in the months of July and August and overestimated in the months of April and May. Notably, is the model (equation 14f) that shows large overestimation over the measured and other estimated temperature dependent models in the month from January to March and slight overestimation in December. The pattern of variation of the estimated models follows similar pattern with the measured indicating good fitting despite slight overestimation and underestimation in their estimated values. However, the model (equation 14g) gives the best fitting.

3.4. Temperature based models for Minna

The evaluated regression equation for the existing temperature dependent models for Minna based on Table 1 are

$$\frac{H}{H_0} = -0.149 + 0.294\, ln\, \Delta T \quad (15a)$$

$$\frac{H}{H_0} = -0.0332 + 0.176\, Sqrt\, \Delta T \quad (15b)$$

$$\frac{H}{H_0} = 0.292 + 0.275\, \frac{\Delta T}{S_0} \quad (15c)$$

The proposed evaluated regression equations for the temperature dependent models for Minna based on Table 2 are

$$\frac{H}{H_0} = -0.785 + 0.505\ Sqrt\, \Delta T - 0.0113\, exp\, Sqrt\, (\Delta T) \quad (15d)$$

$$\frac{H}{H_0} = -2.82 - 0.437\, \Delta T + 0.0084\, (\Delta T)^2 + 3.00\, ln\, \Delta T \quad (15e)$$

$$\frac{H}{H_0} = -0.541 + 0.183\, \Delta T - 0.00733\, (\Delta T)^2 + 0.0\, exp\, \Delta T \quad (15f)$$

$$\frac{H}{H_0} = 0.346 + 1.35\, \frac{\Delta T}{S_0} - 0.403\, exp\left(\frac{\Delta T}{S_0}\right) \quad (15g)$$

$$\frac{H}{H_0} = 1.33 - 0.730\, \frac{\Delta T}{S_0} + 0.933\, ln\left(\frac{\Delta T}{S_0}\right) \quad (15h)$$

Table 6: A) Validation of the Temperature Based Models for Minna under Different Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.15a	86.5	0.0231	1.02	-0.5609	0.0753	94.2813	98.4756
E.15b	84.2	-0.0009	1.115	-0.5135	0.0027	93.1666	98.1495
E.15c	79.7	-0.0217	1.279	-0.5478	0.0562	91.0085	97.5035
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.15d	91.2	-0.0115	0.804	-0.1159	0.0476	96.4468	99.0858
E.15e	92.5	0.1847	0.7666	-1.0772	0.8235	96.7696	99.206
E.15f	92.9	-0.0298	0.7459	-0.0036	0.1326	96.9418	99.2101
E.15g	90.7	-0.1375	0.8419	0.4974	0.5489	96.1039	98.9652
E.15h	92	-0.0719	0.767	0.2168	0.3121	96.7662	99.1594

Table 6: B) Ranking of the Evaluated Temperature Based Models for Minna as Per Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.15a	6	4	6	7	4	6	6	33
E.15b	7	1	7	5	1	7	7	28
E.15c	8	3	8	6	3	8	8	36
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.15d	4	2	4	2	2	4	4	18

E.15e	2	8	2	8	8	2	2	30
E.15f	1	5	1	1	5	1	1	14
E.15g	5	7	5	4	7	5	5	33
E.15h	3	6	3	3	6	3	3	24

Table 6A) and 6B) gives the summary of the various statistical tests adopted and ranking of the temperature based models for Minna. Based on the R^2 the model, equation 15f (author's model) has the highest value with 92.9 % and is judged the best model. Based on the MBE, the model, equation 15b (existing model) has the lowest value with underestimation of 0.0009 $MJm^{-2}day^{-1}$ in the estimated value and is judged the best model, the second is the model. Based on the RMSE the model, equation 15f (author's model) has the lowest value with 0.7459 $MJm^{-2}day^{-1}$ and is judged the best model. Based on the MPE, despite the observed overestimation and underestimation exhibited by some of the existing and author's models they are fall within the acceptable range $(MPE \leq \pm 10\%)$the model, equation 15f (author's model) having the lowest value with underestimation of 0.0036 % in the estimated value and is judged the best model. The study site was statistically tested at the $(1-\alpha)$ confidence levels of significance of 95% and 99%. For the critical t-value, i.e., at α level of significance and degree of freedom, the calculated t-value must be less than the critical value $(t_{critical} = 2.20, df = 11, p < 0.05)$ for 95% and $(t_{critical} = 3.12, df = 11, p < 0.01)$ for 99%. It was observed that the $t_{cal} < t_{critical}$values for all the models under consideration. The $t-$ test shows that all models are significant at 95% and 99% confidence levels. However, the model, equation 15b (existing model) has the lowest value with 0.0027 and is judged the best model. Based on the NSE and IA the model, equation 15f (author's model) has the highest value with 96.9418 % and 99.2101 % and is judged the best model.

The ranking of the existing and author's models {Table 6B)} was done based on the validation of the models {Table 6A)}. The total ranks acquired by the different models were in the range 14 to 36. Based on the overall results for the temperature based models for Minna, the model, equation 15f (author's model) is reported as the best and therefore most suitable for estimating global solar radiation in this location as compared to other evaluated temperature based models.

Fig. 2D shows the comparison between the measured and estimated temperature based global solar radiation for Minna. The figure shows that the estimated models underestimated the measured global solar radiation in the months of February, October and November and overestimated the measured in the months of January and August. The model (equation 15c) overestimated the measured and other estimated models in the month of July, August and December and show underestimation in the months from March to May.

3.5. Temperature based models for Yola

The evaluated regression equation for the existing temperature dependent models for Yola based on Table 1 are

$$\frac{H}{H_0} = -0.0423 + 0.271\, ln\, \Delta T \qquad (16a)$$

$$\frac{H}{H_0} = 0.0900 + 0.155\, Sqrt\, \Delta T \qquad (16b)$$

$$\frac{H}{H_0} = 0.385 + 0.236\, \frac{\Delta T}{S_0} \qquad (16c)$$

The proposed evaluated regression equations for the temperature dependent models for Yola based on Table 2 are

$$\frac{\mathrm{H}}{\mathrm{H_0}} = -0.274 + 0303\ \mathrm{Sqrt}\,\Delta\mathrm{T} - 0.00414\,\exp\mathrm{Sqrt}\,(\Delta\mathrm{T}) \qquad (16d)$$

$$\frac{\mathrm{H}}{\mathrm{H_0}} = 0.67 + 0.177\,\Delta\mathrm{T} - 0.00407\,(\Delta\mathrm{T})^2 - 0.63\ln\Delta\mathrm{T} \qquad (16e)$$

$$\frac{\mathrm{H}}{\mathrm{H_0}} = 0.072 + 0.0702\,\Delta\mathrm{T} - 0.00189\,(\Delta\mathrm{T})^2 - 0.0\exp\Delta\mathrm{T} \qquad (16f)$$

$$\frac{\mathrm{H}}{\mathrm{H_0}} = 0.348 + 0.842\frac{\Delta\mathrm{T}}{\mathrm{S_0}} - 0.199\,\exp\left(\frac{\Delta\mathrm{T}}{\mathrm{S_0}}\right) \qquad (16g)$$

$$\frac{\mathrm{H}}{\mathrm{H_0}} = 1.01 - 0.363\,\frac{\Delta\mathrm{T}}{\mathrm{S_0}} + 0.625\ln\left(\frac{\Delta\mathrm{T}}{\mathrm{S_0}}\right) \qquad (16h)$$

Table 7: A) Validation of the Temperature Based Models for Yola under Different Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.16a	91.7	-0.0483	0.8164	0.0378	0.1964	98.6152	99.6468
E.16b	90.5	-0.0142	0.8652	-0.1492	0.0546	98.4449	99.604
E.16c	88.8	-0.0377	0.9427	-0.1118	0.1328	98.1538	99.5254
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA
E.16d	92.7	0.0046	0.7684	-0.1287	0.0201	98.7734	99.6916
E.16e	92.8	-0.0222	0.7624	-0.0019	0.0967	98.7925	99.6955
E.16f	92.8	-0.0240	0.7669	-0.0004	0.1037	98.7782	99.6916
E.16g	95	0.0427	0.6384	-0.2588	0.2223	99.1534	99.7889
E.16h	95	-0.09	0.6411	0.3294	0.4703	99.1462	99.7829

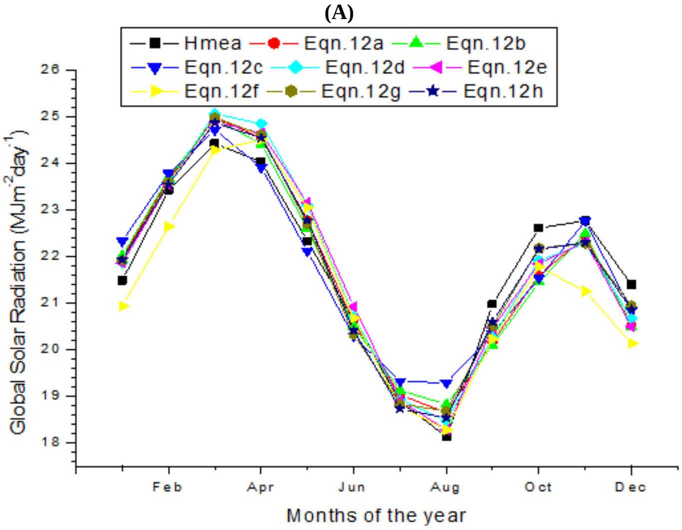
Table 7: B) Ranking of the Evaluated Temperature Based Models for Yola as Per Statistical Test

Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.16a	4	7	6	3	6	6	5	32
E.16b	5	2	7	6	2	7	6	29
E.16c	6	5	8	4	5	8	7	36
Author's Models	R^2	MBE	RMSE	MPE	t	NSE	IA	Total Rank
E.16d	3	1	5	5	1	5	4	20
E.16e	2	3	3	2	3	3	3	16

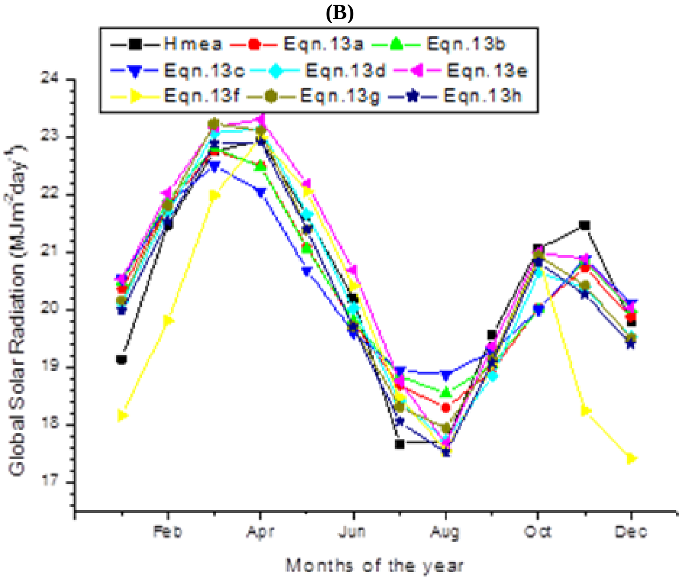
E.16f	2	4	4	1	4	4	4	19
E.16g	1	6	1	7	7	1	1	23
E.16h	1	8	2	8	8	2	2	29

Table 7A) and 7B) gives the summary of the various statistical tests adopted and ranking of the temperature based models for Yola. Based on the R^2 the model, equation 16g (author's model) and equation 16h (author's model) has the highest value with 95.0 % and are judged the best model. Based on the MBE, the model, equation 16d (author's model) has the lowest value with overestimation of 0.0046 $MJm^{-2}day^{-1}$ in the estimated value and is judged the best model. Based on the RMSE the model, equation 16g (author's model) has the lowest value with 0.6384 $MJm^{-2}day^{-1}$ and is judged the best model. Based on the MPE, despite the observed overestimation and underestimation exhibited by some of the existing and author's models they are fall within the acceptable range ($MPE \leq \pm 10\%$) with the model, equation 16f (author's model) having the lowest value with underestimation of 0.0004 % in the estimated value and is judged the best model. The study site was statistically tested at the $(1-\alpha)$ confidence levels of significance of 95% and 99%. For the critical t-value, i.e., at α level of significance and degree of freedom, the calculated t-value must be less than the critical value ($t_{critical} = 2.20, df = 11, p < 0.05$) for 95% and ($t_{critical} = 3.12, df = 11, p < 0.01$) for 99%. It was observed that the $t_{cal} < t_{critical}$ values for all the models under consideration. The $t-$test shows that all models are significant at 95% and 99% confidence levels. However, the model, equation 16d (author's model) has the lowest value with 0.0201 and is judged the best model. Based on the NSE and IA the model, equation 16g (author's model) has the highest value with 99.1534 % and 99.7889 % and is judged the best models.
The ranking of the existing and author's models {Table 7B)} was done based on the validation of the models {Table 7A)}. The total ranks acquired by the different models were in the range 16 to 36. Based on the overall results for the temperature based models for Yola, the model, equation 16e (author's model) is reported as the best and therefore most suitable for estimating global solar radiation in this location as compared to other evaluated temperature based models.
Fig. 2E shows the comparison between the measured and estimated temperature based global solar radiation for Yola. The estimated models underestimated the measured global solar radiation in the months from September to November and overestimated the measured in the months of January, July and December. The model (equation 16c) underestimated the measured and other estimated models in the months from February to June; the model also shows overestimation in the months of January, August and December. The model (equation 16h) underestimated the measured and other estimated temperature dependent models in the month of August. The pattern of variation of the estimated models follows similar pattern with the measured indicating good fitting despite slight overestimation and underestimation in their estimated values. However, the model (equation 16e) gives the best fitting.

(A)



(B)



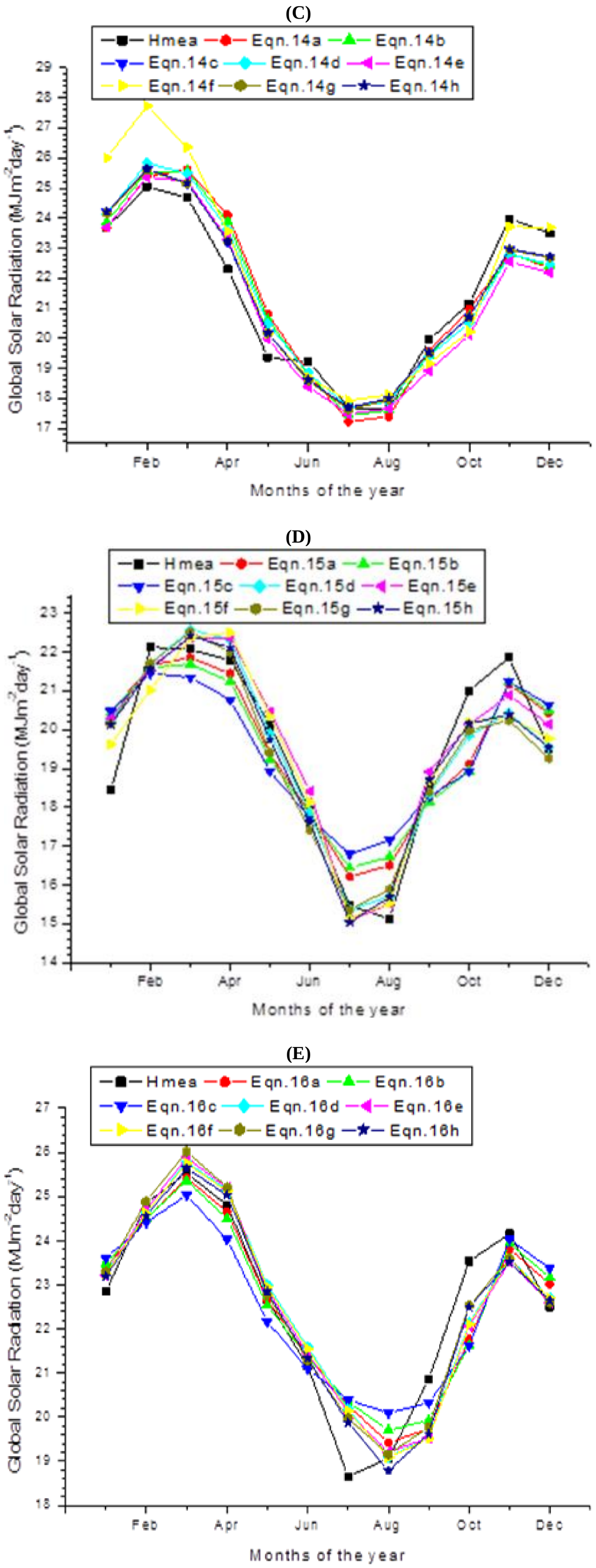


Fig. 2: Comparision between Measured and Estimated Global Solar Radiation for Midland Zone (A) Zaria (B) Bauchi (C) Jos (D) Minna (E) Yola.

4. Conclusion

This study emanated from the fact that air temperature data are commonly measured at many meteorological stations around the world and the need to obtain new models is a research that is still ongoing. In view of this, five new modified temperature dependent models were developed and compared with three existing temperature dependent models (Chen et al., Hargreaves and Samani and Garcia) for the estimation of global solar radiation in five different locations situated in the Midland climatic zone of Nigeria using measured monthly average daily global solar radiation, maximum and minimum temperature meteorological data during the period of thirty one years (1980 – 2010). Based on the seven different statistical validation indices of coefficient of determination (R^2), Mean Bias Error (MBE), Root Mean Square Error (RMSE), Mean Percentage Error (MPE), t – test, Nash – Sutcliffe Equation (NSE) and Index of Agreement (IA) used for the comparison; the result show that the five new modified temperature dependent models were found more suitable in each of the studied locations. For Zaria, the linear logarithmic temperature dependent model was found more accurate. For Bauchi, the linear (exponential 1) temperature dependent model was found more accurate. For Jos, the linear (exponential 2) temperature dependent model was found more accurate. For Minna, the quadratic exponential temperature dependent model was found more accurate. For Yola, the quadratic logarithmic temperature dependent model was found more accurate. The evaluated existing Hargreaves and Samani and Garcia temperature based models for Jos were compared to those available in literature and was found more suitable for global solar radiation estimation.

Admittedly, the five new modified temperature dependent models developed in this study are site dependent, as such when used in locations other than the location where the model was developed, there would be need to calibrate the regression coefficients against the local data for the location under investigation. The pattern of variation of the estimated temperature dependent models follows similar trend with the measured indicating good fitting, even though they exhibit overestimation and underestimation in their estimated values in some months. However, the quadratic exponential temperature dependent model for Bauchi shows obvious underestimation in its estimated value in the month from January to March and November to December. The quadratic exponential temperature dependent model for Jos shows obvious overestimation in its estimated value in the month from January to March and slight overestimation in December.

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References

[1] B. M. Olomiyesan, O. D. Oyedum, P. E. Ugwuoke, J. A. Ezenwora, A.G. Ibrahim, Solar Energy for Power Generation: A Review of Solar Radiation Measurement Processes and Global Solar Radiation Modelling Techniques. Nigerian Journal of Solar Energy, Vol. 26 (2015) Pp 1 – 8. https://doi.org/10.1155/2016/8197389.

[2] B. M. Olomiyesan, O. D. Oyedum, Comparative Study of Ground Measured, Satellite-Derived, and Estimated Global Solar Radiation Data in Nigeria. Hindawi Publishing Corporation. Journal of Solar Energy Volume 2016 (2016) Article ID 8197389, 7 pages https://doi.org/10.1155/2016/8197389.

[3] A. Ångström, Solar and terrestrial radiation. Quarterly Journal of the Royal Meteorological society, 50 (1924) 121-125.

[4] J. K. Page, The estimation of monthly mean values of daily total short – wave radiation on vertical and inclined surfaces from sunshine records for latitude $40°N$ - $40°S$. Proceeding of the UN Conference on New Sources of Energy, Rome, 4 (1964) 378 – 390.

[5] Y. K. Sanusi, S. G. Abisoye, Estimation of Solar Radiation at Ibadan, Nigeria, Journal of Emerging Trends in Engineering and Applied Sciences (JETEAS), 2, 4 (2011) 701 – 705.

[6] L. Huashan, C. Fei, W. Xianlong, M. Weibin, A Temperature-Based Model for Estimating Monthly Average Daily Global Solar Radiation in China. Hindawi Publishing Corporation. The Scientific World Journal, Volume 2014 (2014) 1 – 9. https://doi.org/10.1155/2014/128754.

[7] D. N. Girma, Estimation of Monthly Average Daily Solar Radiation from Meteorological Parameters: Sunshine Hours and Measured Temperature in Tepi, Ethiopia. International Journal of Energy and Environmental Science. Vol. 3, No. 1 (2018) 19-26.https://doi.org/10.11648/j.ijees.20180301.12.

[8] WMO, A Note on Climatological Normal. Technical Note. World Meteorological Organization, Geneva, Switzerland. (1967).

[9] O. S. Ojo, B. Adeyemi, Estimation of Solar Radiation using Air Temperature and Geographical Coordinate over Nigeria, The Pacific Journal of Science and Technology, 15, 2 (2014) 78 – 88.

[10] M. Iqbal, An introduction to solar radiation, first ed. Academic Press, New York. (1983)

[11] S. Zekai, Solar energy fundamentals and modeling techniques: atmosphere, Environment, climate change and renewable energy, first ed. Springer, London. (2008)

[12] E. O. Falayi, A. B. Rabiu, R. O. Teliat, Correlations to estimate monthly mean of daily diffuse solar radiation in some selected cities in Nigeria, Pelagia Research Library, 2, 4 (2011) 480-490.

[13] R. Chen, K. Ersi, J. Yang, S. Lu, W. Zhao, Validation of five global radiation Models with measured daily data in China. Energy Conversion and Management, 45 (2004) 1759-1769.https://doi.org/10.1016/j.enconman.2003.09.019.

[14] G. Hargreaves, Z. Samani, Estimating potential evapotranspiration. Journal of Irrigation and Drainage Engineering. ASCE, 108 (1982) 225-230.

[15] J. V. Garcia, PrincipiosF'isicos de la Climatolog'ia. Ediciones UNALM (Universidad Nacional Agraria La Molina: Lima, Peru) (1994)

[16] A. El-Sebaii, A. Trabea, Estimation of Global Solar Radiation on Horizontal Surfaces Over Egypt, Egypt. J. Solids, 28, 1 (2005) 163-175.

[17] P. R. Bevington, Data reduction and error analysis for the physical sciences, first ed. McGraw Hill Book Co., New York (1969)

[18] H. O. Merges, C. Ertekin, M. H. Sonmete, Evaluation of global solar radiation Models for Konya, Turkey. Energy Conversion and Management, 47 (2006) 3149-3173.https://doi.org/10.1016/j.enconman.2006.02.015.

[19] E. O. Ogolo, Evaluating the performance of some predictive models for estimating global solar radiation across varying climatic conditions in Nigeria. Indian Journal of Radio & space Physics, 39 (2010) 121-131.