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Research paper

Graphical representation of a real valued function in the interval where it gives imaginary value

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Abstract

We are introduced to the graph of a real valued function $y = f(x)$. But when values of $f(x)$ are imaginary numbers at some points of real axis, we have no graphical representation of the function at those points. This article demonstrates a model where we construct a 3D graph of the function at those points where values of the function are imaginary numbers.

Keywords: *Complex Plane; Graph of a Function; Imaginary Number; Real Valued Function; Three-Dimensional Rectangular Coordinate System.*

1. Introduction

Graph of function is very important to understand the behaviour of a function. Graphs can provide valuable visual information about a function. Everywhere graph of function is used. In mathematics graph of function is widely used to describe others mathematical phenomenon. If f is a real-valued function of a real variable, then the graph of f in the xy-plane is defined to be the graph of the equation $y = f(x)$. [1] If value of a function at some points is imaginary number, then the graph of the equation $y = f(x)$ doesn't show this imaginary part. In this article we are trying to have a visual presentation of the part of a function where it gives imaginary number. As example, $\ln x$, $\sqrt{x}$ functions have no graph on the negative x axis, since $\ln x$, $\sqrt{x}$ functions give imaginary number when x is a negative number. Here, we represent graphs of $\ln x$, $\sqrt{x}$ and $\sec^{-1} x$ functions on the interval where these functions give imaginary value along with usual graph.

2. Materials and methods

At first, we take the three-dimensional rectangular coordinate system to describe this model. At the point $x = x_0$ where value of the function is a complex number, at that point we consider a complex plane (we may denote this complex plane by $CP_{x=x_0}$) that is perpendicular to the x-axis. Real axis of the complex plane $CP_{x=x_0}$ is the straight line $x = x_0$ in the xy plane that is parallel to the y axis and the imaginary axis of the complex plane $CP_{x=x_0}$ is the perpendicular line to the xy plane at the point $(x_0, 0, 0)$ that is parallel to the z-axis. Part of the real axis of the complex plane $CP_{x=x_0}$ in the side of the positive y-axis is considered as positive real axis and part of the real axis of the complex plane $CP_{x=x_0}$ in the side of the negative y-axis is considered as negative real axis. Positive and negative imaginary axes of the complex plane $CP_{x=x_0}$ are determined same as z-axis. Origin of the complex plane $CP_{x=x_0}$ is on the x-axis at the point $(x_0, 0, 0)$. If $f(x_0) = u + iv$, then the point (x_0, u, v) is a point on the graph of the function. The graph created by combining all these points may be called imaginary part of the graph of the function f(x) and the as usual graph may be called real part of the graph. Real part of the graph of a function always stays in the xy plane. But imaginary part always stays outside of the xy plane.

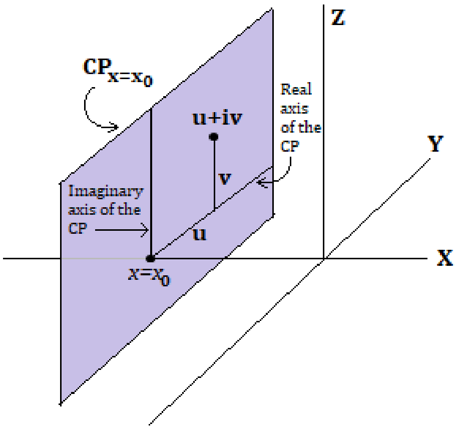

Fig. 1: Complex Plane $CP_{X=X_0}$ at $X = X_0$.

3. Results and discussions

3.1. Graph of ln x function

We know that, [2]

$\ln x = \ln|x| + i\pi = u + iv \qquad x \in (-\infty, 0)$

Therefore, $u = \ln\ |x|$ and $v = \pi$

Parametric equations of the graph of ln x function:
Parametric equations of the real part:

$x(t) = t;\ \ y(t) = \ln t\,;\ \ z(t) = 0; \quad t \in (0, \infty)$

Real part is created by combining all the points of the set: $\{(t, \ln t\,, 0)\ |\ t \in (0, \infty)\}$.

Parametric equations of imaginary part:

$x(t) = t; \quad y(t) = \ln|t|\,; \quad z(t) = \pi; \quad t \in (-\infty, 0)$

Imaginary part is created by combining all the points of the set: $\{(t, \ln|t|\,, \pi)\ \ |\ t \in (-\infty, 0)\}$.
The following graph of ln x function is generated by Mathematica. The code of Mathematica is given below-

```
a = ParametricPlot3D[{t, Log[t],0}, {t, 0,5},  PlotStyle → Blue];

b = ParametricPlot3D[{t, Log[Abs[−t]], π}, {t, −5,0},  PlotStyle → Red];

Show[{a, b}, PlotRange → {{−5,5}, {−5,3}, {0,5}}, Axes → True, AxesOrigin → {0,0,0}, Ticks → {{−5, −4, −3, −2, −1,0,1,2,3,4,5},
{−5, −4, −3, −2, −1,1,2,3}, {1,2,3,4,5}}, TicksStyle → Directive[Black, 20]]
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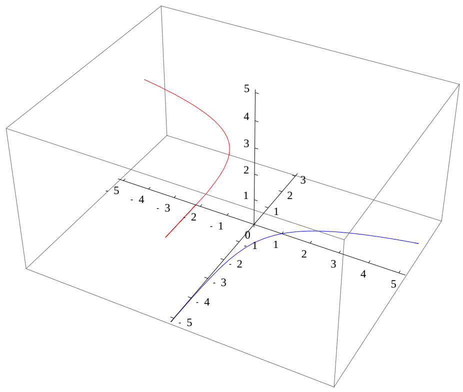

Fig. 2: Graph of ln x Function.

Blue curve is the real part of the graph of the ln x function where outputs of ln x are real numbers. The red one is the imaginary part of the graph of ln x function where outputs of ln x are imaginary numbers. Axis with ticks from −5 to 5 is the x-axis, axis with ticks from −5 to 3 is the y-axis and axis with ticks from 0 to 5 is the z-axis. Imaginary part lies in the plane $z = \pi$.

Other views of the graph of ln x function:

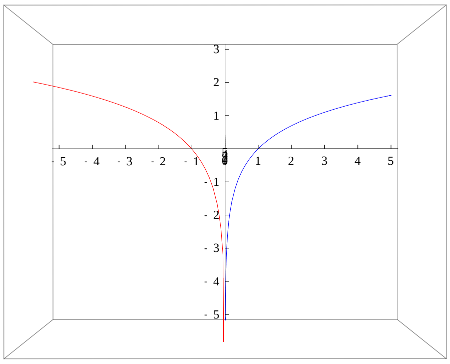

Fig. 3: Top View of the Graph of ln x Function.

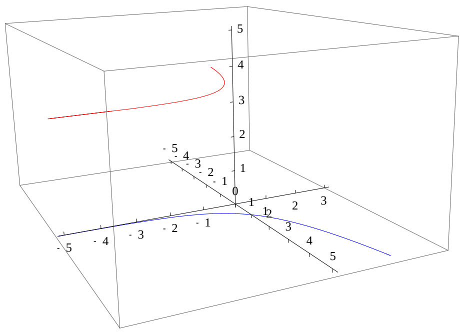

Fig. 4: Side View of the Graph of ln x Function.

3.2. Graph of $\sqrt{x}$ function

$\sqrt{x} = i\sqrt{|x|} = u + iv \qquad x \in (-\infty, 0)$

Therefore, $u = 0$ and $v = \sqrt{|x|}$.

Parametric equations of the graph of $\sqrt{x}$ function:
Parametric equations of the real part:

$x(t) = t;\ \ y(t) = \sqrt{t};\ \ z(t) = 0; \quad t \in [0, \infty)$

Real part is created by combining all the points of the set: $\{(t, \sqrt{t}, 0) \mid t \in [0, \infty)\}$.

Parametric equations of imaginary part:

$x(t) = t; \quad y(t) = 0; \quad z(t) = \sqrt{|t|}; \quad t \in (-\infty, 0)$

Imaginary part is created by combining all the points of the set: $\{(t, 0, \sqrt{|t|}) \mid t \in (-\infty, 0)\}$.
The following graph of $\sqrt{x}$ function is generated by Mathematica. The code of Mathematica is given below-

$a = \text{ParametricPlot3D}[\{t, \sqrt{t}, 0\}, \{t, 0,5\},\ \text{PlotStyle} \to \text{Blue}];$

$b = \text{ParametricPlot3D}[\{t, 0, \sqrt{\text{Abs}[t]}\}, \{t, 0, -5\},\ \text{PlotStyle} \to \text{Red}];$

$Show[\{a, b\}, PlotRange \to \{\{-5,5\}, \{-5,3\}, \{0,5\}\}, Axes \to True, AxesOrigin \to \{0,0,0\},\ Ticks \to \{\{-5, -4, -3, -2, -1,0,1,2,3,4,5\}, \{-5, -4, -3, -2, -1,1,2,3\}, \{1,2,3,4,5\}\}, TicksStyle \to Directive[Black, 20]]$

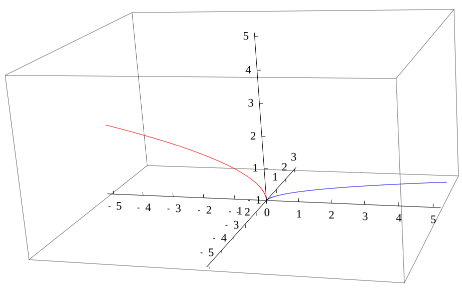

Fig. 5: Graph of $\sqrt{x}$ Function.

Blue curve is the real part of the graph of the $\sqrt{x}$ function where outputs of $\sqrt{x}$ are real numbers. The red one is the imaginary part of the graph of $\sqrt{x}$ function where outputs of $\sqrt{x}$ are imaginary numbers. Axis with ticks from -5 to 5 is the x-axis, axis with ticks from -5 to 3 is the y-axis and axis with ticks from 0 to 5 is the z-axis. Imaginary part lies in the plane $y = 0$.

Other views of the graph of $\sqrt{x}$ function:

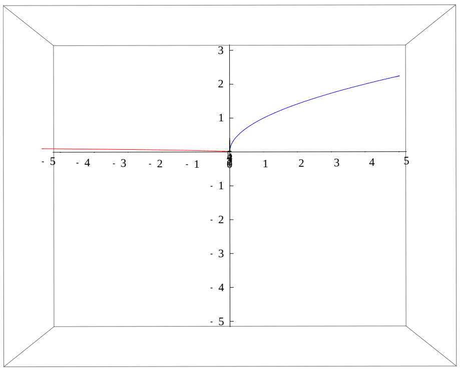

Fig. 6: Top View of the Graph of $\sqrt{x}$ Function.

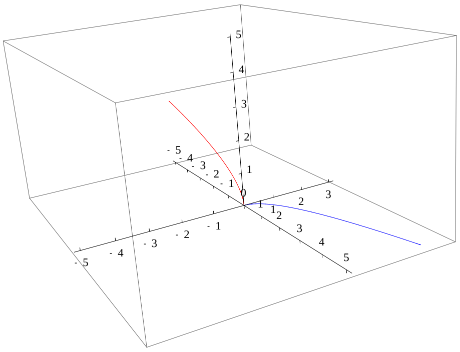

Fig. 7: Side View of the Graph of $\sqrt{x}$ Function.

3.3. Graph of $\sec^{-1} x$ function

Actually, this function is the source of my interest. Since, it has no graph in the interval $(-1,1)$.
We know that, for $x \in (0,1)$

if $\sec^{-1} x = ai$ then $\sec^{-1}(-x) = \pi - ai$.

Now, using the proposed model graph of $\sec^{-1} x$ function is generated by Mathematica which is given below along with code-

$a = ParametricPlot3D[\{t, ArcSec[t],0\}, \{t, -5,5\},\ PlotStyle \to Blue];$

$b = ParametricPlot3D[\{t, 0, Im[ArcSec[t]]\}, \{t, 0,1\},\ PlotStyle \to Red];$

$c = ParametricPlot3D[\{t, \pi, Im[ArcSec[t]]\}, \{t, -1,0\},\ PlotStyle \to Red];$

$Show[\{a, b, c\}, PlotRange \to \{\{-5,5\}, \{-5,5\}, \{-5,5\}\}, Axes \to True, AxesOrigin \to \{0,0,0\}, Ticks \to \{\{-5, -4, -3, -2, -1,0,1,2,3,4,5\},$
$\{-5, -4, -3, -2, -1,1,2,3\}, \{1,2,3,4,5\}\}, TicksStyle \to Directive[Black, 20]]$

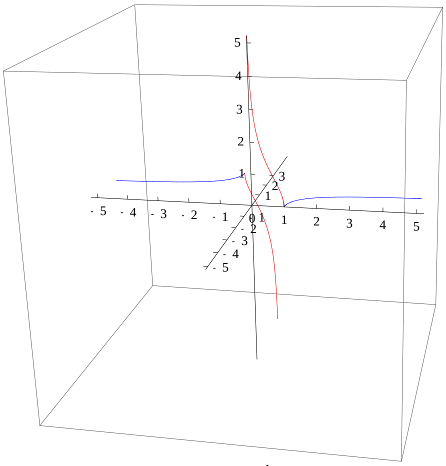

Fig. 8: Graph of $\sec^{-1} x$ Function.

Blue curve is the real part of the graph of the $\sec^{-1} x$ function where outputs of $\sec^{-1} x$ are real numbers. The red one is the imaginary part of the graph of $\sec^{-1} x$ function where outputs of $\sec^{-1} x$ are imaginary numbers. Axis with ticks from -5 to 5 is the x-axis, axis with ticks from -5 to 3 is the y-axis and axis with ticks from 0 to 5 is the z-axis. Imaginary part in the interval $(0,1)$ lies in the plane $y = 0$ and imaginary part in the interval $(-1,0)$ lies in the plane $y = \pi$. From the graph we can see that when x tends to 0 from right hand side, then value of the function tends to $+\infty$ along with positive z axis in the plane $y = 0$ and when x tends to 0 from left hand side, then value of the function tends to $-\infty$ along with negative z axis in the plane $y = \pi$.

Other views of the graph of $\sec^{-1} x$ function:

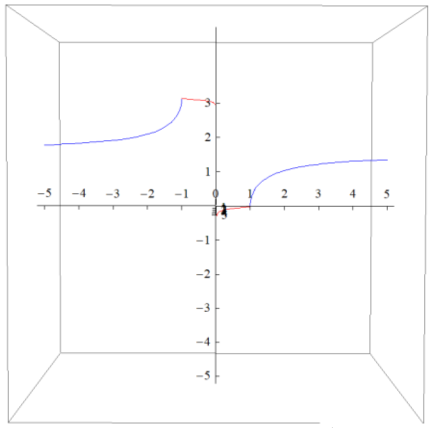

Fig. 9: Top View of the Graph of $\sec^{-1} x$ Function.

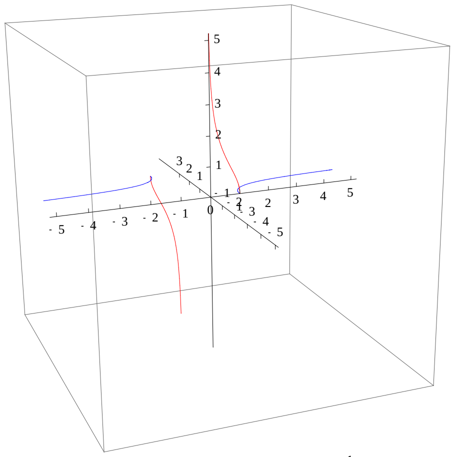

Fig. 10: Side View of the Graph of $\sec^{-1} x$ Function.

4. Conclusions

By this model we can create graph of any function with imaginary output at some points of x axis and can analysis behavior of the function in this portion. For the function $y = f(x)$, simply we can use parametric plot of the parametric equations-

$$x = t; \quad y = \mathrm{Re}\big(f(t)\big); \quad z = \mathrm{Im}\big(f(t)\big).$$

Acknowledgements

My thinking is started from seeing the graph of $\sec^{-1} x$ function. I see that there is no graph of this function from -1 to 1. Why? Because, in this interval values of this function are imaginary numbers. Then a question is arisen in my mind that is there no way to represent this portion of this function graphically? At last, I find this technique by the grace of Almighty.

References

[1] Howard Anton, Irl Bivens, Stephen Davis, *Calculus Early Transcendentals*, 10th Edition, John Wiley & Sons, Inc., New York, USA, 2012, page# 04.

[2] James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications*, 8th Edition, McGraw Hill, New York, USA, 2004, page# 93.