

Quasi-linear correlation between shear and young's moduli in some polycrystalline ceramics

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Abstract

This work aims to investigate the correlation between the shear modulus and the Young's modulus of some pore-free polycrystalline ceramics. Our study shows that the shear modulus correlates quasi-linearly with the Young's modulus. The best fit was obtained using the linear model. The fit of the shear modulus G as a function of the Young modulus E obeys this linear expression: $G = 0.43E - 7.7$ (where both G and E are expressed in GPa). The coefficient of the correlation was found at around 0.994. Our expression was used to predict the shear modulus G of some other polycrystalline ceramics, especially: Dy_2O_3 , Er_2O_3 , and Y_2O_3 materials, which are estimated at around 65.6, 72.4, and 51.8 GPa, respectively. We attempt also to estimate the Vickers hardness of our materials of interest using an empirical expression of the literature. Unfortunately our predicted results are larger than those reported previously in the literature.

Keywords: Polycrystalline Ceramics; Young's Modulus; Shear Modulus; Quasi-Linear Correlation.

1. Introduction

Elastic deformation under external stress is one of the most important considerations in structural applications of solids [1]. Indeed, elastic properties of materials are commonly required in computer aided design and manufacturing techniques to simulate a product's behavior under variable conditions of external stress and temperature [1].

Using different theoretical approaches, semi-empirical and empirical models, many works [2-28] investigated and analyzed the relations between different parameters for several materials with different crystallographic structures.

Until now, reported literature showed very little attention has been paid to the investigation of the correlation between the different elastic moduli of polycrystalline ceramics and the relation between the different elastic moduli is still to be discussed in comparison with these reported for the porosity.

Using a reactive sintering method, Zhang et al. [18] synthesized porous chromium carbide (Cr_3C_2) ceramics, which exhibit a uniform and accessible porous structure comprised of three-dimensional connected struts. Young's modulus of the ceramics synthesized was measured by dynamic mechanical analysis (DMA). They also developed a semi-empirical model basing on the generalized mixture rule (GMR), to describe the porosity dependence of Young's modulus for this ceramic.

Wachtman et al. [19] reported the essential data on the ceramic materials in their book 'Mechanical properties of ceramics', which is constituted from 25 chapters. They explained in detail the mechanical behaviors of ceramic materials under stress and high temperatures. Shein et al. [20] have investigated the elastic constants C_{ij} and several other related properties of cubic binary phases of thorium based nonmetals, i.e. ThX ($X = C, N, O, P, As, Sb, S, Se$) using first-principles calculation with the full-potential linearized augmented plane wave (FLAPW) approach and the generalized gradient approximation (GGA) for the exchange-correlation potential. They found that the aggregate bulk modulus B of the polycrystalline ceramics decreases with increasing inter-atomic distances in the following sequences: $ThN \rightarrow ThP \rightarrow ThAs$ and $ThO \rightarrow ThS \rightarrow ThSe$, respectively.

Using first principles calculations based on density functional theory (DFT), Qi et al. [25] have investigated the crystal structure, electronic and elastic properties, and mechanical anisotropy of XB_2 ($X = V, Nb, Ta, Cr, Mo, W$) binary compounds. They found that the bonding behaviors of XB_2 ($X = V, Nb, Ta, Cr, Mo, W$) are the combination of covalent and metallic bonds.

Using Niobium-doped lead zirconate-titanate (PNZT), Biswas and Fulrath [29] investigated the effect of porosity on the mechanical properties of a polycrystalline ceramic. They found that the Young's modulus showed a linear relationship with increase in porosity.

In the present work, we investigate the correlation between the bulk modulus, the shear modulus G for some pore-free polycrystalline ceramic materials based on some data reported by Wachtman et al. [19].

2. Theory and discussion of the results

Young's modulus E is a key mechanical property of isotropic brittle materials such as polycrystalline ceramics [18]. Values for Young's modulus E and shear modulus G for some pore-free polycrystalline ceramics are given in Table 1 [19], which are in some cases averages of measurements from several authors. The values of the Young's modulus E and shear modulus G of pore-free polycrystalline ceramics summarized in Table 1 are plotted in Figure 1.

Table 1: Values of Young's and Shear Moduli of Pore-Free Polycrystalline Ceramics [19]

Material	Al ₂ O ₃	MgO	ThO ₂	TiO ₂	ZnO
E (GPa)	402.8	310.9	261.0	284.2	123.5
G (GPa)	163.0	133.4	100.6	111.5	45.6

The values of the Young's modulus E and the shear modulus G are also plotted in Figure 1. From Figure 1, we can observe clearly that the shear modulus G increases quasi-linearly with increase of the Young's modulus E . The data plotted in Figure 1 were fitted using the linear model, the obtained linear expression is given as follows: $G = 0.43 E - 7.7$, where both G and E are expressed in GPa. The linear coefficient of the correlation was found at around 0.994, while the average error on the estimation of the shear modulus G is only around 2.83%. Using the previous model, the errors on the estimation of the shear modulus G are around 1.54 % for Al₂O₃, 5.56 % for MgO, 3.91 % for ThO₂, 2.70 % for TiO₂ and 0.43 % for ZnO, respectively. So, the maximum error (5.56 %) was found for MgO, while the minimum error (0.43 %) was reported for ZnO material. It is very important to that a quasi-linear correlation between Young's and shear moduli was also obtained for several intermetallic compounds: X₃Ir (X = Ti, V, Cr, Nb and Mo) [5].

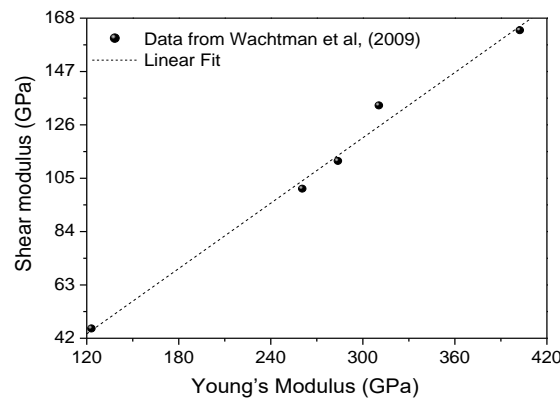


Fig. 1: Quasi-Linear Correlation between Young's and Shear Moduli in Pore-Free Polycrystalline Ceramics.

In the following section, we use our proposed model to predict the shear modulus G of some other polycrystalline ceramics based on their Young's modulus E values reported in the literature [19], [30], [31]. Using the C_{ij} single crystal of Ref [31] and Voigt–Reuss–Hill approximation [32], the obtained values of the shear modulus G for different polycrystalline ceramics are summarized in Table 2.

Table 2: Predicted Values of Shear Modulus for Some Ceramics, *Using C_{ij} Single Crystal of Ref [31] and Voigt–Reuss–Hill Approach [32].

Material	Dy ₂ O ₃	Er ₂ O ₃	Y ₂ O ₃	ZrO ₂	BeO	CaO
E (GPa)	170.5 [19]	186.3 [19]	138.3 [19]	230 [19]	379.2 [30]	197.7 [31]*
G (GPa)	65.6	72.4	51.8	91.2	155.4	77.3

There are different relationships frequently employed to predict the hardness H and the fracture toughness K_{IC} of the materials [33–37]. We calculate the Vickers hardness H_V of pore-free polycrystalline ceramics according to the following empirical expression: $H_V = 0.92(G/B)^{1.137}G^{0.708}$ [32], [38], where G is the shear modulus and B is the bulk modulus. Frequently, the elastic moduli of the polycrystalline materials are calculated [39–48]. Iuga et al. [45] have investigated the elastic constants C_{ij} of ceramic crystals using an Ab-initio calculation. For aggregate polycrystalline materials, the bulk modulus B is expressed as a function of the Young's modulus E and the shear modulus G as follows: $B = (1/3)EG/(3G-E)$ [19]. The obtained values of the bulk modulus B for different ceramics are summarized in Table 3, along the experimental one (210 GPa) for BeO material [49].

Table 3: Predicted Values of Bulk Modulus B for Some Polycrystalline Ceramics, ^afrom Ref [1], ^bfrom Ref [45], ^cfrom Ref [49].

Material	Dy ₂ O ₃	Er ₂ O ₃	Y ₂ O ₃	ZrO ₂	BeO	CaO
B (GPa)	141.55, 144 ^a	145.39, 160 ^a	140.33, 166 ^b	160.37, 170 ^a	226.06, 210 ^c	148.95, 112.3 ^b

The Poisson's ratio ν is expressed as a function of E and G as follows: $\nu = (E/2G) - 1$ [19]. The obtained values of the Poisson's ratio and H_V for different ceramics are reported in Table 4. In the case of MgO and Y₂O₃, we present a comparison of our calculations values with theoretical data of Iuga [45] on Poisson's ratio ν (Table 4). Furthermore, it is very clear that our obtained value (22.8 GPa) of the Vickers hardness for Al₂O₃ ceramic is very higher than the experimental one (13 GPa) reported by Wachtman et al [19].

Table 4: Predicted Values of Poisson's Ratio ν and Vickers Hardness H_V for Pore-Free Polycrystalline Ceramics, *from Ref [45].

Material	Al ₂ O ₃	MgO	ThO ₂	TiO ₂	ZnO	Dy ₂ O ₃	Er ₂ O ₃	Y ₂ O ₃	ZrO ₂	BeO	CaO
ν	0.22	0.23, 0.18*	0.25	0.24	0.36	0.30	0.29	0.34, 0.31*	0.26	0.22	0.28
H_V (GPa)	22.8	17.2	14.0	15.5	3.6	7.4	8.6	4.8	11.8	21.4	8.8

Our value (11.8 GPa) of H_V for ZrO₂ material is also higher than the experimental one (9 GPa) reported by Wachtman et al [19]. Our obtained value (21.4 GPa) of H_V for BeO material is also very higher than the predicted one (9.95 GPa) reported by Yang et al [50]. The calculated values of the Poisson's ratio ν and Vickers hardness H_V for ceramics are also plotted in Figure 2. We could see clearly that the

H_v decreases exponentially with increasing Poisson's ratio ν . Bioud and Benchiheb [51] studied the pressure effect on some physical properties of calcium oxide material, and found a correlation between the bulk modulus B and the microhardness H . Zhang et al. [18] have mentioned that the Young's modulus is correlated with most mechanical properties including tensile strength, hardness, bending strength, and fracture toughness, while Mezouar and Logzit [52] have studied the microhardness of $B_xAl_{1-x}Sb$ alloys using semi-empirical approach. It is well known that the Diamond is the hardest material on earth having H_v in the range of 70–150 GPa [53]. The values of H_v for ceramics are very smaller than that of the diamond, and higher than that (3.57 GPa) of GaSb [54].

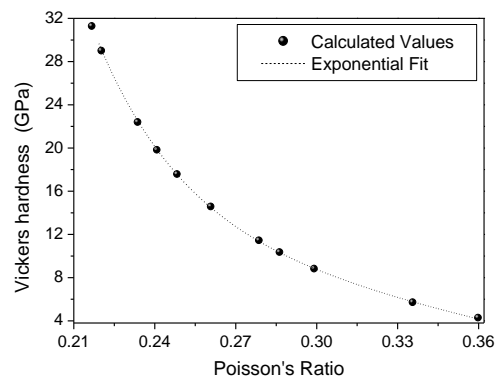


Fig. 2: Exponential Correlation between Poisson's Ratio ν and Vickers Hardness H_v in Pore-Free Polycrystalline Ceramics.

3. Conclusion

Based on some data reported previously in the literature, we have investigated the correlation between the shear modulus G and the Young's modulus E for some pore-free polycrystalline ceramics. Our investigation shows that the shear modulus correlates quasi-linearly with the Young's modulus. The best fit between G and E was obtained using the linear model as follows: $G = 0.43E - 7.7$ (where both G and E are expressed in GPa). The coefficient of the correlation was found at around 0.994, while the average error is only around 2.83%. Our expression was used to predict the shear modulus of some other isotropic polycrystalline ceramic materials. The shear modulus of Dy_2O_3 , Er_2O_3 , Y_2O_3 and CaO ceramics are estimated at around 65.6, 72.4, 51.8 and 77.3 GPa, respectively. We calculated also the Vickers hardness (H_v) of some polycrystalline ceramics; our obtained values are very larger than other experimental data of the literature. The calculated Vickers hardness H_v of polycrystalline ceramics decreases exponentially with increasing Poisson's ratio ν .

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