

Space time energy equivalence

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Abstract

Space–time–energy equivalence is the principle that everything that has space and time (in the presence of a constant force with an impact point) has an equivalent amount of energy, and vice versa. This equivalence is widespread in physics and astrophysics. Five examples (Stoney units, Planck units, Newton's law and Interaction of light rays, standard gravitational parameter and Coulomb's law) of the algebraic notation of this principle show its universality. Five examples (repetitions) of the same principle should justify the universality of the new principle and its use in physics and astrophysics. The proposed equivalence reveals the functional relationship between physical constants: Planck's constant, electric charge of an electron, vacuum permittivity and vacuum permeability. The realization of this equivalence will allow, through deepening the understanding of the nature of space–time, to take a fresh look at physics, when describing natural laws and principles.

Keywords: Energy; Space; Time; Equivalence; Mass; Constants; Gravitation; Units.

1. Introduction

The equivalence of space–time and energy, despite the wide use in describing the principles of physics and astrophysics, has not yet been formulated. Currently, classical and quantum mechanics, field theory are based on the principle of the space–time–energy equivalence. An initial description of the proposed equivalence is presented using examples of the equivalence of the distance (between two points of physical space with influencing of constant force) and energy, as the most simple and generally accepted. The consequence of this equivalence is the equivalence of time and energy, and vice versa. The article lists the most famous and simple examples of using this equivalence. The equivalence is widely used in physics, accompanied by other physical quantities and constants. For example: the ratio of energy unit and space unit in Stoney units and Planck units, despite of the differences in these units, gives one result equal to the ratio of the fourth power of the speed of light to the gravitational constant, namely, the limiting value of the force equal to the Planck force. This relationship is one example of writing a space–energy equivalence constant. At the basis of Newton's law of universal gravitation is also this equivalence. In the standard gravitational parameter for a beam of light, the ratio of the right and left sides of the equation is equal to the space–mass constant of equivalence. Considering mass–energy equivalence we have space–energy equivalence. Coulomb's law and Newton's law, written in Stoney and Planck units, is equal to the space–energy constant. In all five examples, the space–time–energy equivalence was originally written in a hidden form. The disclosures forms of notation, each separately, are not sufficient to formulate a new principle of space–time–energy equivalence. The totality of all five examples is sufficient for this. Space and time in a gravitational field, with magnetic permeability and permittivity, has a functional connection with mass and energy. The proposed equivalence is one of these functions. This equivalence opens up the possibility of numerical defining some physical constants in terms of others constants, for: Planck's constant, electric charge of an electron, vacuum permittivity and vacuum permeability. Based on known equivalences and by analogy with them, the equivalence coefficients of physical quantities are presented: electric charge and energy, temperature and energy. The next step in describing this equivalence is to give an example of the equivalence of the difference between two intervals of curvature of space and energy. It is taken into account that to make this difference, the Planck force is necessary. Ways have been outlined to describe the equivalence of area and energy, as well as volume and energy, as well as Planck density and energy.

1.1. History

In 1881, the work "On the physical units of nature" [1] was presented, in which no space–time–energy equivalence was found. Even if this could be done, one example would not be sufficient to formulate this equivalence.

In 1899 (1900) the article "About irreversible radiation processes" was presented [2], in which this equivalence was latently presented for the second time, independently of the first case.

Isaac Newton put forward the law of gravitation in 1687. This is one of the first uses of this equivalence (for rays of light) in the history of physics. [3]

Coulomb's law was formulated and published in 1785. This is the oldest application of this equivalence (writing in latent form) given here. Withal the standard gravitational parameter were introduced to the scientific world, which also contained latent records of this equivalence. Throughout history (from the end of the 18th century) we can see how all five examples of description and observation of the principle of space–time – energy equivalence, presented in this article, are found in physics and astrophysics right up to the present day.

2. Method

Due to the lack of satisfactory methods for studying and describing space and time, the original methods that were used in physics in past centuries are used in this work. This is an algebraic description of natural principles and, in cases of their repetition, the use of an axonomic generalization. Also, generally accepted encyclopaedic knowledge is taken as the basis for the description of the proposed equivalence.

2.1. First step

The amount of energy E can be calculated if the force F is multiplied by the distance L over which this force acts: [4] [5]

$$E = F_k L \quad (a)$$

If in this equation the force F_k does not change in magnitude and direction when the point of its application moves along the straight line L , then this force is the equivalence coefficient between L and E .

Equivalence, a relation like equality (equation). [6]

Equation is statement of equality between two expressions consisting of variables. [7]

For equivalence of space L and energy E , variables is L and E .

$$E = LF_p = Lk_{se} \quad (b)$$

In formula (b) there is a coefficient (constant: Planck force) $F_p = k_{se}$. Here k_{se} is the space–energy equivalence constant.

If we take as a coefficient not F but L , for example: Planck length l_p , then we get the equivalence of force F and energy E :

$$E = Fl_p \quad (c)$$

And vice versa

$$F = E \frac{1}{l_p} \quad (d)$$

If we take as a coefficient not F or L , but E , for example: the Planck energy E_p , then we obtain the equivalence of force F and the reciprocal of length $\frac{1}{l}$:

$$F = E_p \frac{1}{L} \quad (e)$$

And vice versa

$$L = E_p \frac{1}{F} \quad (f)$$

Further. One of the entries of formula (a):

$$E = LF_k \frac{v}{v} \quad (g)$$

Or

$$E = TvF_k \quad (h)$$

This is the equivalence of time T and energy E , while the equivalence coefficient is the product of constant speed v and constant force F_k . If F_k is the equivalence coefficient of space and energy, and $F_k = F_p = k_{se}$, and $v = c$, then

$$E = TcF_p = Tck_{se} \quad (i)$$

This occurs on the basis that where there is movement (along the length L) there is also a speed of movement v (c) along the same length L . The consequence of this is the time T .

2.2. Traditional use of this equivalence

The described equivalence has been used in physics since time immemorial. Therefore, there is no shortage of examples of such uses. Preference is given to the most ancient descriptions of such use. It is the authors of these examples who have the unconscious priority of describing this principle of the space–time – energy equivalence: Coulomb, Newton, Stony and Planck.

Natural units are known: Stoney units (1881) [1], and Planck units (1899–1900). [2]

In addition to Stony and Planck units, the International System of Units (SI). If we write space–time–energy equivalences (formula b) in each of these unit systems, we get confirmation of this principle:

$$E = Lk_{se} \quad (1)$$

and (formula i)

$$E = Tck_{se} \quad (2)$$

Here L is space, T is time, L and T are the equivalences of energy E , c is the speed of light, k_{se} is the space–energy constant of equivalence:

$$k_{se} = \frac{c^4}{G} \quad (3)$$

A similar disclosure of this equivalence is made on the examples of formulas: Newton's law, Coulomb's law and the standard gravitational parameter. These five examples are sufficient to justify this equivalence on the basis of an axonometric generalization.

3. Examples of equivalence formulas

The space–energy constant of equivalence k_{se} is measured in the newton (N) and, for example, is equal to the Planck unit of force F_p . This follows from the definition of mass–energy equivalence: $E = mc^2$ or

$$F_p L = mc^2 \quad (3.1)$$

Here $F_p L$ is energy E , equal to the product of force F_p and distance L . Hence

$$mc^2 = k_{se} L \quad (3.2)$$

Or

$$E = k_{se} L \quad (3.3)$$

Which corresponds to formula (1).

4. Remark

In physics, physical constants are known to have one of several values. For example, the speed of light has one of the values: $c^2 = \frac{1}{\epsilon_0 \mu_0}$. [8] [9]

Similarly, the Planck unit of force F_p has the following meaning: $F_p = \frac{c^4}{G}$, or $F_p = k_{se}$. It follows that $k_{se} = \frac{c^4}{G}$ is one of the main meanings. For areas of physics where the speed of light c and the gravitational constant G are physical constants, the space–energy constant of equivalence k_{se} is also a physical constant.

4.1. Natural units

If we write down the equivalence of space–time and energy in Natural units: Stoney units [1] and Planck units [2], we get confirmation of this principle.

4.1.1. Stoney units

Stoney units represent the following units: mass m_s , energy E_s , length L_s and time T_s : [1]

Stoney mass is $m_s = \sqrt{\frac{k_e e^2}{G}}$, Stoney energy is $E_s = m_s c^2$, Stoney length is $L_s = \sqrt{\frac{G k_e e^2}{c^4}}$, Stoney time is $T_s = \sqrt{\frac{G k_e e^2}{c^6}}$. [1]

Here k_e is the Coulomb constant, e is the electric charge of an electron, c is the speed of light in a vacuum, G is the gravitational constant. From here:

$$E_s = m_s c^2 = c^2 \sqrt{\frac{k_e e^2}{G}} = \frac{c^4}{G} \sqrt{\frac{G k_e e^2}{c^4}} = L_s k_{se} \quad (4)$$

Derivation from this formula (4):

$$E_s = L_s k_{se} \quad (5)$$

Also:

$$E_s = c^2 \sqrt{\frac{k_e e^2}{G}} = c \frac{c^4}{G} \sqrt{\frac{G k_e e^2}{c^6}} = T_s c k_{se} \quad (6)$$

Here $k_{se} = F_s$ is the space–energy constant of equivalence equal to the unit Stoney force.
From formula (5) it follows that

$$E_s = F_s L_s \quad (6.1)$$

Derivation from formula (6):

$$E_s = c T_s k_{se} \quad (6.2)$$

Formulas (4) and (6) confirm formulas (1) and (2).

4.1.2. Planck units

$$E_p = c^2 m_p = c^2 \sqrt{\frac{\hbar c}{G}} = \sqrt{\frac{\hbar c^5}{G}} = \sqrt{\frac{\hbar G c^4}{c^3 G}} = L_p k_{se} = c \sqrt{\frac{\hbar G c^4}{c^5 G}} = c T_p k_{se} \quad (7)$$

Here $L_p = \sqrt{\frac{\hbar G}{c^3}}$ is Planck length [2], $T_p = \sqrt{\frac{\hbar G}{c^5}}$ is Planck time, $m_p = \sqrt{\frac{\hbar c}{G}}$ is Planck mass, \hbar is the reduced Planck constant, E_p is Planck energy, $k_{se} = F_p$ is the constant of equivalence of space and energy, equal to the Planck unit of force, this also means:

$$E_p = F_p L_p \quad (7.1)$$

This corresponds to Planck units. Derivation from formula (7):

$$E_p = L_p k_{se} = c T_p k_{se} \quad (8)$$

Formula (8) confirm formulas (1) and (2).

4.1.3. Derivation from stoney and planck units

Formulas for writing the equivalence of space, time and energy (6.1), (7.1), as well as (6.2) and (8) are presented for the case of a one–dimensional functional dependence, i.e., the proposed equivalences are written in Stoney units (Planck units) for one unit of measurement:

$$E_{s(p)} = F_p \times 1 L_{s(p)} \quad (8.1)$$

And

$$E_{s(p)} = c 1 T_{s(p)} F_p \quad (8.2)$$

For cases when $L = n L_{s(p)}$ and $T = n T_{s(p)}$ the proposed formulas will take the form:

$$E = n E_{s(p)} = F_p \times n L_{s(p)} \quad (8.3)$$

here n is a multiplier between L and $L_{s(p)}$, and n also between T and $T_{s(p)}$. And

$$E = n E_{s(p)} = c n T_{s(p)} F_p \quad (8.4)$$

Formulas (8.1) – (8.4) are universal and suitable for writing the proposed equivalence for cases with any values of space, time and energy.

4.2. Equivalence of Planck and stoney units of measurement

Planck units and Stoney units equivalence coefficient is \hat{k} , it is the dimensionless constant:

$$\hat{k} = \frac{L_s}{L_p} = \sqrt{\frac{k_e e^2}{\hbar c}} \quad (9)$$

This refers to primary physical quantities in Planck and Stoney units:

$$X_s = \hat{k} X_p \quad (9.1)$$

Here X_p is physical quantity in Planck units, X_s is physical quantity in Stoney units. \hat{k} is equivalence coefficient of Planck units and Stoney units. This means that (Planck units) – (Stoney units) is equivalence.

Hence, Stoney temperature is T_s^o :

$$T_s^o = k T_p^o = \sqrt{\frac{k_e c^4 e^2}{G k_b^2}} \quad (10)$$

Here $T_p^o = \sqrt{\frac{hc^5}{G k_b^2}}$ is Planck temperature. If we consider the equivalence coefficient of Stoney units and Planck units:

$$k_2 = \frac{X_p}{X_s} = \sqrt{\frac{hc}{k_e e^2}} \quad (10.1)$$

and if we accept Planck units as the main one, then we get the equivalence:

$$X_p = k_2 X_s \quad (10.2)$$

Planck electric charge is Q_p :

$$Q_p = k_2 Q_s = \sqrt{\frac{hc}{k_e}} \quad (11)$$

Here $Q_s = e$ is Stoney electric charge. This way Stoney units and Planck units can complete each other.

4.2.1. Equivalences of physical constants

Another suggested entry of the equivalence coefficient of physical constants is κ , it is the dimensionless constant. From formula (9):

$$\kappa = \sqrt{2k} \quad (12)$$

Or

$$\kappa = \sqrt{\frac{e^2}{hc \epsilon_0}} \quad (12.1)$$

Or

$$\kappa^2 = 2\alpha \quad (12.2)$$

Here α is the fine-structure constant (the Sommerfeld constant). [10] [11]

From (9) which it follows that $k = \sqrt{\alpha}$. Continuation:

$$\kappa = \sqrt{\frac{e^2}{h} \sqrt{\frac{\mu_0}{\epsilon_0}}} \quad (13)$$

Or

$$\kappa = \sqrt{\frac{Z_0}{R_K}} \quad (13.1)$$

Here Z_0 is the vacuum impedance or impedance in free space, [12]

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad (13.2)$$

Or from formula (12.1)

$$\kappa = \sqrt{\frac{1}{c \epsilon_0 R_K}} \quad (14)$$

Here R_K is the von Klitzing Constant, [13]

$$R_K = \frac{h}{e^2} \quad (14.1)$$

Therefore, from formula (13):

$$\kappa = \sqrt{\frac{1}{R_K} \sqrt{\frac{\mu_0}{\epsilon_0}}} \quad (15)$$

And κ opens formulas for each of h , e , ϵ_0 and μ_0 through the rest from them:

$$h = \frac{e^2}{\kappa^2} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{Z_0 e^2}{\alpha} \quad (16)$$

Or

$$h = \frac{e^2}{\kappa^2 c \varepsilon_0} = \frac{e^2}{\alpha c \varepsilon_0} \quad (16.1)$$

From here

$$e = \sqrt{h \kappa^2 \sqrt{\frac{\varepsilon_0}{\mu_0}}} = \sqrt{\frac{h \alpha}{Z_0}} = \sqrt{\kappa^2 h c \varepsilon_0} \quad (17)$$

Also

$$\varepsilon_0 = \frac{e^2}{\kappa^2 c h} = \frac{1}{\alpha c R_K} \quad (18)$$

From formula (17):

$$\mu_0 = \frac{\kappa^4 h^2 \varepsilon_0}{e^4} \quad (19)$$

Or

$$\mu_0 = \kappa^4 R_K^2 \varepsilon_0 = \alpha^2 R_K^2 \varepsilon_0 \quad (19.1)$$

From formulas from (9) to (19.1) it follows that these physical constants are equivalent to each other:

$$\kappa - \alpha - h - k_e - c - Z_0 - R_K - e - \varepsilon_0 - \mu_0 \text{ are equivalence} \quad (20)$$

This opens the way for refining the numerical values of physical constants, and once again confirms the equivalence of space and energy. From formulas from (12.1) to (15) and (9) it follows that the constant κ is coefficient of (Stoney units) – (Planck units) equivalence:

$$\kappa = \sqrt{2\alpha} = \sqrt{\frac{e^2}{h c \varepsilon_0}} = \sqrt{\frac{Z_0}{R_K}} = \sqrt{\frac{1}{c \varepsilon_0 R_K}} = \sqrt{\frac{1}{R_K \sqrt{\frac{\mu_0}{\varepsilon_0}}}} = \sqrt{\frac{k_e e^2}{h c}} \quad (20.1)$$

This is another direction in physics using the proposed equivalence, namely the quantitative determination of the strength of electromagnetic interaction between elementary charged particles.

4.3. Interaction of light rays

The forces of attraction and repulsion between adjacent light beams have been experimentally revealed. [14]

The goal here is to describe the equivalence of space–time and energy, so a simple algebraic description of the experimentally established phenomenon of the interaction of light rays is presented here, since it is in the extremely simple description that the proposed equivalence is obvious. When moving to a full description of this phenomenon, we observe a complication of the formulas.

In addition to the gravitational one, in the interaction of light rays there are a number of other interactions that depend on the electromagnetic components of these rays. Only gravitational interactions are considered here.

4.3.1. Newton's law of universal gravity

The principle of gravitational interaction of two masses M_1 and M_2 . Newton's law of universal gravity [3]:

$$F = G \frac{M_1 M_2}{R^2} \quad (21)$$

Or

$$\Delta E \approx G \frac{M_1 M_2 \Delta R}{R^2} \quad (21.1)$$

here R is the distance between the masses M_1 and M_2 , ΔR is the elementary part of the radius R , on which the energy ΔE is concentrated, special case $\Delta R = R$, ΔE is the interaction energy of M_1 and M_2 on the interval ΔR .

If the mass of photons is $M_1 = M_2 = m_{ph}$ and is equal to the relativistic mass m of the energy of gravitational interaction of these photons:

$$\Delta E = mc^2 \quad [15]$$

$$M_1 = M_2 = m \quad (22)$$

And

$$R = \Delta R = L \quad (23)$$

Then from formula (21) for relativistic energy E and relativistic mass m :

$$m = G \frac{m m L}{c^2 L^2} \quad (23.1)$$

Or

$$1 = G \frac{m}{L c^2} \quad (23.2)$$

Or

$$L = \frac{G m}{c^2} \quad (24)$$

Or

$$E = L \frac{c^4}{G} = k_{se} L \quad (25)$$

Which corresponds to the space–energy equivalence for relativistic energy E and relativistic mass m , in a reference frame where the system is moving, see formula (1). The basis of this equation is the principle of equivalence of gravitational and inertial mass. [15] [16]
If conditions (22) and (23) are not satisfied, then the equivalence of (24) and (25) takes the form of formulas taking into account the differences of conditions (22) and (23).

4.3.2. Gravitational interaction of two rays of light

This derivation of the formula of the proposed equivalence based on the principle of the attraction of two rays of light, in which two masses interact, equivalent to the energy of these two rays. [14]

Two parallel beams of light, located from each other at a distance of the wavelength of this light λ :

$$\lambda = \frac{c}{\nu} \quad (25.1)$$

here ν is radiation frequency.

These rays of light create a gravitational interaction F_{ph} between themselves (Newton's law of universal gravity):

$$F_{ph} = G \frac{m_{ph}^2}{\lambda^2} \quad (25.2)$$

If the photon motion energy F_{ph} is equal to the interaction energy of two photons during their motion $F\lambda$, then

$$F\lambda = \lambda m_{ph} c^2 = G \frac{m_{ph}^2}{\lambda^2} \quad (25.2.1)$$

From here

$$m_{ph} = \frac{c^2 \lambda}{G} \quad (25.2.2)$$

Or

$$E = \lambda \frac{c^4}{G} \quad (25.3)$$

Which corresponds to formula (25). Taking into account (25.1) we obtain:

$$E = \frac{c^5}{\nu G} \quad (25.4)$$

Which corresponds to formula (6.1), or from formula (25.2):

$$F_{ph} = G \frac{E_{ph}^2}{c^4 \lambda^2} = G \frac{h^2 \nu^2}{c^4 \lambda^2} \quad (25.5)$$

Here $m_{ph} = \frac{E_{ph}}{c^2}$ is photon mass, $E_{ph} = h\nu$ is Planck–Einstein relation. Considering that $E = F\lambda$ we get:

$$E = h\nu = G \frac{h^2 \nu^2}{c^4 \lambda} \quad (25.6)$$

This corresponds to formula (25.3), that is

$$1 = G \frac{h \nu}{c^4 \lambda} \quad (25.7)$$

Or

$$E = \frac{\lambda c^4}{G} \quad (25.8)$$

That is formula (25.3).

Wavelength λ is a characteristic of the spatial wave structure, and space with wavelength has a functional dependence, and this formula is an analogue of the proposed equivalence (in turn, the next equivalence).

In this way we can obtain the proposed equivalence, but if any of the stated conditions are not met, then we will obtain an equation containing this equivalence. For example: if the distance L between photons is not equal to the wavelength λ , but n times greater than this wavelength:

$$L = n\lambda \quad (25.9)$$

then we get the simplified equation:

$$\frac{E}{n} = \frac{\lambda c^4}{G} \quad (25.10)$$

4.3.3. Gravitational deflection of light

Another example of confirmation of proposed formula for the equivalence of space–time – energy is the deflection of a light beam in a gravitational field. [17 - 19]

A photon (of mass m_{ph}) of a light beam moving in the gravitational field of a celestial object of mass M , taken here as a material point, at a distance R from this material point, is acted upon by a force F :

$$F = G \frac{m_{ph} M}{R^2} \quad (26)$$

From here

$$E = G \frac{h\nu M}{Rc^2} \quad (26.1)$$

And

$$1 = G \frac{M}{Rc^2} \quad (26.2)$$

Here

$$M = \frac{Rc^2}{G} \quad (26.3)$$

Or

$$E = R \frac{c^4}{G} \quad (26.4)$$

We obtain this equivalence in the ideal case when the energy of photon motion is equal to the energy of gravitational interaction of this photon with the mass of the celestial body. In all other cases, we will not obtain an equivalence, but an equation for the interaction of a photon with a celestial body, and the equivalence will have to be calculated from this equation.

4.4. Standard gravitational parameter

The standard gravitational parameter μ celestial object of mass m is known: [20]

$$\mu = Gm \quad (27)$$

This parameter μ also matters:

$$\mu = Rv^2 \quad (27.1)$$

here R is the orbit radius, v is the orbital speed.

This parameter μ from the formula (27.1) for a beam of the light ray matters:

$$\mu = Lc^2 \quad (28)$$

for ray of light or electromagnetic radiation and for the circular orbit with orbital radius L according to formulas (27) and (28):

$$Gm = Lc^2 \quad (28.1)$$

From which follows the space–mass equivalence:

$$L - m \text{ is equivalence} \quad (29)$$

Or

$$m = \frac{Lc^2}{G} \quad (29.1)$$

And

$$T - m \text{ is equivalence} \quad (29.2)$$

or

$$m = \frac{Tc^3}{G} \quad (30)$$

Based on this and on mass–energy equivalence [15] is the space–energy equivalence:

$$L - E \text{ is equivalence} \quad (30.1)$$

Or

$$E = \frac{Lc^4}{G} = Lk_{se} \quad (31)$$

And

$$T - E \text{ is equivalence} \quad (32)$$

Or

$$E = \frac{Tc^5}{G} = cTk_{se} \quad (32.1)$$

The mass–energy equivalence: $E = mc^2$. [15]

Has a natural reflection – and vice versa: $m = \frac{E}{c^2}$. [21]

Therefore, energy also has the equivalences of space L and time T :

$$L = \frac{E}{k_{se}} \text{ is energy–space equivalence} \quad (31.1)$$

$$T = \frac{E}{ck_{se}} \text{ is energy–time equivalence} \quad (32.2)$$

That is, energy by formulas (31.1) and (32.1) has time and space, which confirms formulas (1) and (2).

4.5. Forces in stoney and Planck units

Forces, physical vector quantities written in Stoney and Planck units, are equal to the space-energy constant. Examples: Coulomb's law and Newton's law.

4.5.1. For coulomb's law

If we write Coulomb's law: $F_{q-s} = k_e \frac{Q_1 Q_2}{L_s^2}$. [22] [23]

Here F_{q-s} is Coulomb force in Stoney units, L_s is the electrostatic Stoney space.

Then, in Stoney units, the Coulomb force of interaction between two electrons e will show us the space–energy constant k_{se} :

$$F_{q-s} = k_e \frac{e^2}{L_s^2} = k_e \frac{e^2 c^4}{G k_e e^2} = \frac{c^4}{G} = k_{se} \quad (33)$$

And in Planck units between two electric charges Q_p :

$$F_{q-p} = k_e \frac{Q_p^2}{L_p^2} = k_e \frac{\hbar c c^3}{k_e \hbar G} = \frac{c^4}{G} = k_{se} \quad (33.1)$$

Here Q_p is Planck electric charge, formula (11), k_{se} is the space–energy constant, F_{q-p} is Coulomb force to Planck units, L_p is Planck electrostatic space.

Formulas (34.2) and (34.3) once again confirm the proposed equivalence.

4.5.2. For newton's law

If we write Newton's law: [3]

$$F_{m-s} = G \frac{m_1 m_2}{L_s^2} \quad (33.4)$$

Here F_{m-s} is Newton force in Stoney units, L_s is Stoney electrostatic space. Or

$$F_{m-s} = G \frac{k_e e^2 c^4}{G G k_e e^2} = \frac{c^4}{G} \quad (33.5)$$

Newton force in Planck units:

$$F_{m-p} = G \frac{m_p^2}{L_p^2} = G \frac{h c c^3}{G h G} = \frac{c^4}{G} \quad (33.6)$$

here F_{m-p} is Newton force in Planck units. That is

$$F_{m-s} = F_{m-p} = F_{q-s} = F_{q-p} = \frac{c^4}{G} = k_{se} \quad (34)$$

5. Results

All the above examples show that the space–time–energy equivalence in all cases has the same proportionality factor, namely the space–energy constant k_{se} :

$$\frac{E_s}{L_s} = \frac{E_p}{L_p} = \frac{E}{L} = F_s = F_p = F = k_{se} \quad (34.1)$$

Here F_s is Stoney force, F_p is Planck force, F is the force of the SI.

The equivalence coefficient of space and energy k_{se} is equal to the Planck force F_p . [2]

Formula (34.1) is composed of formulas (1), (3), (4), (5), (7), (8), (25), (31), (31.1), and (34).

The equivalence of space–time and energy are independent of the unit's system, in which we consider natural phenomena. These unit's systems indicate the presence of such equivalence and the presence of physical units common for other to unit's systems.

6. Accompanying history

65 years ago, on the basis of astrophysics observations, the origin of the main energy of stars was established, a particular case of which is the time–energy equivalence. [24]

40 years ago, based on the Geometrized unit system, the space–energy equivalence coefficient was established. [25]

20 years ago, on the basis of the unity of physical quantities, the space–time–energy equivalence was established. [26]

Two years ago, the proposed equivalence was formulated and described. An attempt was made to determine the prospects for the development of physics taking into account the use of this equivalence. [27]

7. Analogies

The Special Theory of Relativity describes the basics of nuclear energy release. By analogy with these already known principles, we can describe the basics of space–time energy release and expand the list of equivalence coefficients, both known and expected.

7.1. Special theory of relativity

- 1) Time dilation. Between time $\Delta T'$ and time ΔT of rest: [28]

$$\Delta T' = \gamma \Delta T \quad (35)$$

Here γ is Lorentz factor. [30]

- 2) Length contraction. Between the length $\Delta x'$ of an object and the resting length Δx of the same object: [30] [31]

$$\Delta x' = \frac{\Delta x}{\gamma} \quad (36)$$

- 3) Relativistic mass. The relativistic mass m of a moving object depends on the rest mass m_0 : [32] [33]

$$m = \gamma m_0 \quad (37)$$

- 4) Relativistic energy. Formulas: total energy E , [34] [35]

$$E = mc^2 = \gamma m_0 c^2 \quad (38)$$

Relativistic kinetic energy E_k :

$$E_k = (\gamma - 1)m_0 c^2 \quad (39)$$

7.1.1. One-dimensional space

By analogy with the difference between the mass of a moving object and the rest mass of this object, as well as the difference between the length of a moving object and the rest length of this object (an integral part is the curvature of space), which follows from the above analogues (36) – (39), the difference also follows between two intervals of space curvature Δl_0 :

$$\Delta l_0 = \frac{GM}{c^2} = \frac{\mu}{c^2} = \frac{c^2 m}{k_{se}} = \frac{E_0}{k_{se}} \quad (34.2)$$

Here μ is standard gravitational parameter of a celestial object .

The above examples indicate the equivalence of space–time – energy in one–dimensional space, as well as between the curvature of space and energy in this space.

7.1.2. Relativistic conclusion

In formulas (35), (36), (37), (38) and (39), the Lorentz factor γ is a proportionality coefficient in the special theory of relativity, indicating time dilation, space contraction, and changes in mass and energy. In these four formulas, the Lorentz factor γ indicates the analogy in changes in four physical quantities: space x , time T , energy E and mass m . This means that, by analogy with the equivalences of mass and energy, we can observe similar equivalences between all these physical quantities:

$$T-L-m-E \text{ are equivalences} \quad (40)$$

7.2. Equivalence coefficients of physical quantities

Here is a list of equivalence coefficients for physical quantities, both known and under consideration, as well as those equivalences that have yet to be proven.

- 1) c^2 is the coefficient in mass–energy equivalence [15]:

$$E = mc^2$$

- 2) $\frac{1}{c^2}$ is the coefficient in energy–mass equivalence [21]:

$$m = \frac{E}{c^2}$$

- 3) c is the coefficient in time–space equivalence:

$$L = Tc$$

- 4) $\frac{c^4}{G}$ is the coefficient in space–energy equivalence, formulas (1), (3), (3.3), (4), (5), (7.1) and (31):

$$E = L \frac{c^4}{G}$$

- 5) $\frac{c^5}{G}$ is the coefficient in time–energy equivalence, formulas (6), (6.1), (8) and (37) :

$$E = T \frac{c^5}{G}$$

- 6) h is the coefficient of (radiation frequency ν) – energy E equivalence, formula of Planck–Einstein relation (25.6) [9]:

$$E = h\nu \quad (41)$$

- 7) ch is the coefficient of (light wavelength λ) – energy E equivalence:

$$E = ch \frac{1}{\lambda} \quad (42)$$

- 8) $\frac{G}{c^4}$ is the coefficient in energy–space equivalence, formula (31.1):

$$L = E \frac{G}{c^4}$$

- 9) $\frac{G}{c^5}$ is the coefficient in energy–time equivalence, formula (32.1):

$$T = E \frac{G}{c^5}$$

- 10) k_B is the coefficient in temperature–energy equivalence:

$$E = \frac{3}{2} k_B T^o \quad (43)$$

Here k_B is the Boltzmann constant, E is the mean kinetic energy for an individual particle, T^o is the thermodynamic temperature of the bulk quantity of the substance (on the Kelvin scale). [36]

1) $\sqrt{\frac{c^{11}}{\hbar G^3}}$ is the coefficient of equivalent of Planck area (L^2) and energy E : [2]

$$E = L^2 \sqrt{\frac{c^{11}}{\hbar G^3}} \quad (44)$$

2) $\frac{c^7}{G^2 \hbar}$ is the coefficient of equivalent of Planck volume (L^3) and energy E : [2]

$$E = L^3 \frac{c^7}{G^2 \hbar} \quad (45)$$

3) $\sqrt{\frac{\hbar^3 G^3}{c^5}}$ is the coefficient of equivalence of Planck density ρ and energy E : [2]

$$E = \rho \sqrt{\frac{\hbar^3 G^3}{c^5}} \quad (46)$$

4) $8\pi \frac{G}{c^4}$ is the coefficient of {the energy–momentum tensor (also referred to as the stress–energy tensor) $F_{\mu\nu}$ }–(the geometry of space–time $G_{\mu\nu}$) equivalence: [37] [38]

$$G_{\mu\nu} = 8\pi \frac{G}{c^4} F_{\mu\nu} \quad (47)$$

5) $\frac{2G}{c^2}$ is the coefficient of (mass of a celestial object M) – (the Schwarzschild radius r_s) equivalence: [39] [40]

$$r_s = \frac{2G}{c^2} M \quad (48)$$

6) mg – coefficient of equivalence of space height L in a gravitational field with free fall acceleration g for a body with mass m and potential energy E : [41]

$$E = mgL \quad (49)$$

And vice versa

$$L = \frac{1}{mg} E \quad (50)$$

7) μN is coefficient (the product of the sliding friction coefficient μ and the support reaction force equal to the pressing force N) of the equivalence of the length and friction energy E : [42]

$$E = \mu N L \quad (51)$$

And vice versa

$$L = \frac{E}{\mu N} \quad (52)$$

Here are presented only some equivalences coefficients that belong to the corresponding pairs of equivalent physical quantities.

8. Conclusion

On the one hand, the article presents the age–old glory of the equivalence of space–time and energy, on the other hand, it follows the difficulty of understanding this equivalence. One of the reasons for this discrepancy is the unsuitability of known scientific methods to describe this equivalence, which makes it difficult for us to understand, what is time and what is space?

We can observe space–time – energy equivalences in all areas of physics and astrophysics. As examples of such observations, algebraic expansions are given in two Natural units: Stoney units and Planck units, as well as example with the Newton's law of universal gravitation, standard gravitational parameter for a beam of light, and Coulomb's law. Writing down the known principles of physics for limiting values: the speed of light, the Planck force, leads to writing the proposed equivalence.

In 1905, Albert Einstein published a work on the special theory of relativity, constructed deductively, and based on the axiomatic method. [43]

This work also uses the axiomatic method. These equivalences are formulated in various examples from physics, which corresponds to the deductive construction and confirms the existence of this equivalence in nature.

Summarizing what has been done, we can the sequence time–space–mass–energy equivalences:

$$T-L-m-E \text{ are equivalences} \quad (53)$$

Or

$$\frac{Tc^5}{G} = \frac{Lc^4}{G} = m c^2 = E \quad (54)$$

From here

$$L = EG (\mu_0 \varepsilon_0)^2 \quad (55)$$

Or

$$L = m G \mu_0 \varepsilon_0 \quad (56)$$

Space in a gravitational field G , with vacuum magnetic permeability μ_0 and permittivity ε_0 is equivalent to energy E and mass m . In this case, the coefficient $(G\mu_0 \varepsilon_0)$ is directly proportional to space L and inversely proportional to mass m .

This one is the space–time – energy equivalence is a consequence of being in a gravitational field and in a medium with magnetic and electrical qualities.

The possibility of using this equivalence, formulas (12) – (20), when calculating some physical constants through others: Planck's constant, electric charge of an electron, vacuum permittivity and vacuum permeability, also confirms the existence of the proposed equivalence in nature.

The coefficient k_{se} in the proposed equivalence has the value of constants: Planck force F_p (7.1) or Planck constant h (41). The possibilities of this equivalence coefficient do not end there. For example, for the conditions described by classical mechanics, formula (a), any force F , constant in time and throughout the entire space L , is an equivalence coefficient in the proposed principle: $E = FL$.

From formulas (7.1), (41) and (42) it is clear that the proposed equivalence with the coefficient F_p is the reciprocal of a fraction of the same equivalence with the coefficient h . Energy values equivalent to space–time in gravitational and electrostatic fields are the reciprocal fraction of energy equivalent to wavelength in electromagnetic radiation.

In all cases of space–energy equivalences considered in this article, the space–energy equivalence constant (constant force) has a point of application throughout the entire space considered. The presence of energy in space indicates the presence of forces generating this energy. The presence of forces and variable energy indicates the presence and even the creation of space. A force with a moving point of application in space generates energy.

The work presents examples with gravitational and electrostatic energies and forces, which correspond to gravitational and electrostatic space, respectively. When describing the proposed equivalences, one should take into account the types of spaces: gravitational (25), electrostatic (33), electromagnetic (42), Planck area (44), Planck volume (45), geometry of space–time (47), and Schwarzschild radius (48). In all these examples, space is represented in the form of length, distance, a segment of space, area, volume, as well as in the form of other physical quantities, the names of which usually coincide with the names of energies and forces equivalent to the quantity of this space.

In each such example of the equivalence of space and energy, there is a separate spatial–energy equivalence constant, the name of which may also coincide with a specific example of the name of space. In each such example, the constant has a specific meaning. In this work, preference is given to the constant - the Planck force. The peculiarity of this constant is that it occurs more often than other values of constants when describing principles in physics.

Forces (points of their action) moving through spaces have speeds of movement. These speeds also have the names of the energies and forces accompanying these speeds. The times of such movements also have names similar to the names of the energies and forces acting during these times.

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