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Research paper

Equivalence of electric charge and energy

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Abstract

The equivalence of electric charge and energy is the principle that everything that has an electric charge has an equivalent amount of energy and vice versa. Main methods used: conversion of natural units, algebra, analogy. The equivalence of electric charge and energy, despite the wide use in describing the principles of physics and astrophysics, has not yet been formulated. In this work, the formula for the equivalence of electric charge and energy is presented. This is done on the basis of known measurement systems, parameters and principles of physics.
Five examples (Stoney units, Planck units, Newton's law of universal gravitation, Coulomb's law and Ampere's law) of the algebraic notation of this principle show its universality. Five examples should justify the universality of the new principle and its use in physics, astrophysics and technology.
New physical Units are proposed for mass and electric charge has been proposed, each of which is suitable for measuring both mass and electric charge. The paper gives perspectives of using the proposed equivalence: starting calculation of mass excess, refinement of orbits of celestial, and Einstein field equation.

Keywords: *Energy; Equivalence; Electric Charge; Gravitational Constant; Coulomb Constant; Stoney Units; Planck Units; Mass Excess; Tensor.*

1. Introduction

The article lists five of the most famous and simple examples of the use of this equivalence: Stoney units, Planck units, Newton's law of universal gravitation and Coulomb's law, as well as the analogues of the proposed principle: analogy of the mass–energy equivalence, analogy of the of space–time–energy equivalence. The given principles are averaged as axioms. Based on the Axiomatic Method, a conclusion is made about the presence of this equivalence in nature.
The perspective of using this equivalence is given: ten examples from different fields of physics and technology:

1) Additional standard gravitational parameter and additional standard electrical parameter.
2) Units of measurement of mass and electric charge of a body and their energies.
3) Electrical standard parameter.
4) Gravitational–electric standard parameter.
5) Refinement of the orbits of celestial objects.
6) The way to determine the mass excess.
7) Analogues of the energy–momentum tensor in Einstein field equations.
8) Two the energy tensors of electric field and current, for identical celestial objects and for one dominant celestial object.
9) The analogue of the Schwarzschild radius for an electrically charged celestial body.
10) Equivalence of physical constants: Dirac constant, speed of light in vacuum, electric charge of an electron, Coulomb constant. The value of each of these constants is determined by the values of these other constants.

1.1. History

The first attempt to connect the concept of electromagnetic mass and energy was made in the work of Joseph John Thomson (1856 – 1940), which appeared in 1881. Thomson not only introduces the concept of electromagnetic mass, but also represented this mass as part from the inert mass of a charged body. [1]
The unity of the nature of mass and electric charge is known. 2003 [2]

2. Methods

To record the proposed equivalence, natural units are used: Stoney units and Planck units, as well as the International System of Units and the laws of Newton, Coulomb and Ampere (after their algebraic processing).

If we write the equivalence of electric charge and energy in each of these systems of units and in each formulas for describing these laws, then we will receive confirmation of this proposed principle.
The laws of attraction of masses and electric charges among themselves, the analogy of these principles and the joint recording of the formulas of these principles, provide support for the proposed principle.
The equivalence of physical constants indicates the universality of the proposed principle.
The Axiomatic Method, according to which the connection of the equivalence of electric charge and energy with known physical principles proves the existence of this principle in nature.

3. Examples of the equivalence formula

3.1. Natural units

If we write down the mass– (electric charge)–energy equivalences (electric—energy equivalence) in Natural units: Stoney units [3] and Planck units [4] [5], we get confirmation of this principle.

3.1.1. Stoney units

Stoney energy is $E_s = m_s c^2$, Stoney mass is $m_s = \sqrt{\frac{k_e e^2}{G}}$, charge is $Q_s = e$. From here

$$E_s = m_s c^2 = c^2\sqrt{\frac{k_e e^2}{G}} = e c^2\sqrt{\frac{k_e}{G}} = c^2 Q_s k_{qe} = Q_s k_{ee\text{-}Q} \tag{1}$$

From here

$$k_{qe} = \sqrt{\frac{k_e}{G}} \tag{2}$$

Here k_{qe} is coefficient of the electric charge–mass equivalence:

$$c^2 k_{qe} = k_{ee\text{-}Q} \tag{3}$$

Here $k_{ee\text{-}Q}$ is coefficient of the electric charge–energy equivalence.
This is one way to calculate the equivalence coefficient of electric charge and energy (mass).

3.1.2. Planck units

Planck energy is $E_p = \sqrt{\frac{\hbar c^5}{G}}$, Planck mass is $m_p = \sqrt{\frac{\hbar c}{G}}$, Planck charge is $Q_p = \sqrt{\frac{\hbar c}{k_e}}$. From here

$$E_p = m_p c^2 = c^2\sqrt{\frac{\hbar c k_e}{k_e G}} = c^2 Q_p k_{qe} = k_{ee\text{-}Q} Q_p \tag{4}$$

Or

$$E_p = E_p\sqrt{\frac{k_e}{k_e}} = \sqrt{\frac{\hbar c^5 k_e}{k_e G}} = c^2 Q_p\sqrt{\frac{k_e}{G}} = k_{ee\text{-}Q} Q_p \tag{5}$$

Here $k_{ee\text{-}Q}$ is coefficient of the equivalence of electric charge Q and energy E. And vice versa:

$$Q_p = \frac{E_p}{k_{ee-Q}} \tag{6}$$

Here Q_p is the relativistic electric charge. This is another way to calculate the equivalence of electric charge and energy (mass) proposed here and vice versa.

3.2. Another definition of electric energy equivalence

3.2.1. Newton's law of universal gravitation

Every point mass M_1 attracts every other point mass M_2 by a force F_G acting along the line R intersecting these two points: [6]

$$F_G = G\frac{M_1 M_2}{R^2} \tag{7.0}$$

If

$$M_1 = M_2 = M \tag{7}$$

then this law looks like this:

$$F_G = \frac{(\sqrt{G}\,M)(\sqrt{G}\,M)}{R^2} \tag{8.0}$$

Or

$$F_G = \frac{\left(M\sqrt{G}\right)^2}{R^2} \tag{8}$$

From here

$$\mu_m = M\sqrt{G} \tag{9}$$

Here μ_m is an additional standard gravitational parameter for the conditions of equality of interacting masses (7).

3.2.2. Similarly, for coulomb's law

The magnitude of the force F_q of attraction or repulsion between two point charges Q_1 and Q_2 equal in magnitude is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them R^2. [7]

$$F_q = k_e \frac{Q_1\,Q_2}{R^2} \tag{10.0}$$

Or, if

$$Q_1 = Q_2 = Q \tag{10}$$

So

$$F_q = \frac{\left(Q\sqrt{k_e}\right)^2}{R^2} \tag{10.1}$$

From here

$$\mu_q = \sqrt{k_e}\,Q \tag{11}$$

Here μ_q is an additional standard electrical parameter for conditions (10). If

$$F_G = F_Q \tag{12}$$

Based on (9), (11) and (13) the equivalence of additional standard parameters is

$$M\sqrt{G} = Q\sqrt{k_e} \tag{13}$$

From here

$$M = Q\sqrt{\frac{k_e}{G}} \tag{14}$$

Which corresponds to formulas (1) and (2), further

$$Mc^2 = E = Q\sqrt{\frac{k_e}{G}}c^2 = Q\,k_{ee\text{-}Q} \tag{15}$$

This is the third way to calculate the equivalence of electric charge and energy (mass) proposed here and vice versa.

3.2.3. Ampère's law

Ampere's law is known. [8 - 11]
This is the law of mechanical interaction by force F of two currents I_1 and I_2 flowing in conductors L_1 and L_2 located at distance R from each other:

$$F_A = \frac{\mu_o\,I_1\,I_2\,L_1\,L_2}{4\pi\,R^2} \tag{16}$$

If

$$I_1 = I_2 = I \tag{17}$$

And

$$L_1 = L_2 = R = L \tag{17.1}$$

so

$$F_A = \frac{\mu_o I^2}{4\pi} \tag{18}$$

Or

$$E = \frac{\mu_o I^2 L}{4\pi} \tag{19}$$

Here L is elementary space, on which energy E is concentrated. From here:

$$E = \frac{\mu_o Q^2 c^2}{4\pi L} \tag{19.1}$$

Or

$$E = k_e \frac{Q^2}{L} \tag{20}$$

And

$$m = k_e \frac{Q^2}{L c^2} \tag{20.1}$$

For

$L = L_S$

$$m = Q\sqrt{\frac{k_e}{G}} \tag{21}$$

And

$$E = c^2 Q\sqrt{\frac{k_e}{G}} \tag{21.1}$$

It should be noted that if conditions (17) and (17.1) are not met, then formulas (18) – (21.1) take on the form of the Ampere law formula (16).
This is the fourth way to calculate the equivalence of electric charge and energy (mass) and vice versa.

3.2.4. Ampere's law in natural units

If we consider Ampere's law within one unit of measurement, using natural units, then we will also receive confirmation of the proposed equivalence. For Stoney units:

$$E_s = \frac{\mu_o I_S^2 L_S^3}{4\pi L_S^2} = \frac{\mu_o Q_S^2 L_S}{4\pi T_S^2} = \frac{\mu_o e^2 \sqrt{k_e G e^2}}{4\pi c^2} \frac{c^6}{G e^2 k_e} = c^2 e\sqrt{\frac{k_e}{G}} \tag{22}$$

For Planck units:

$$E_p = \frac{\mu_o I_P^2 L_P^3}{4\pi L_P^2} = \frac{\mu_o c^3}{4\pi \sqrt{k_e G}} \sqrt{\frac{\hbar G}{c^3}} = c^2\sqrt{c\frac{\hbar}{G}} = c^2 Q_p\sqrt{\frac{k_e}{G}} \tag{23}$$

This is the fifth way to calculate the equivalence of electric charge and energy (mass).

4. Perspectives

The proposed equivalence will expand our understanding of the physics of nature.
New physical quantities

4.1. Mass and electric charge measurements

Taking into account the properties of the environment: G and k_e, we get the opportunity to measure the electric charge Q in the unit of mass in M (kg) (6.1), (13) and (14):
For

$$\dim Q\sqrt{\frac{k_e}{G}} = \mathrm{M}$$

As well as the measurement of mass m in the unit of electric charge in TI (C) :
For

$$\dim m\sqrt{\frac{G}{k_e}} = \mathrm{TI}$$

In systems of measurement where G is equal to k_e, the units of mass and electric charge will be the same, i.e. dim m=dim Q

4.2. Based on standard parameters

Formulas (13) and (14) indicates to the unit of measurement *Щ* :

$$Щ = m\sqrt{G} = Q\sqrt{k_e} \tag{24}$$

Here *Щ* is for measurement a mass m :

$$m = \frac{Щ}{\sqrt{G}} \tag{25}$$

And also for measuring the electric charge Q:

$$Q = \frac{Щ}{\sqrt{k_e}} \tag{26}$$

dim *Щ* = $\mathrm{M}^{1/2}\ \mathrm{L}^{3/2}\ \mathrm{T}^{-1}$

dim *Щ* is unit dimension of electrical mass: dim $m\sqrt{G}$ and dim $Q\sqrt{k_e}$.

The measurement of two different physical quantities in one unit of measurement is already known. By mass—energy equivalence, the electronvolt (unit E) corresponds to a unit of mass. By analogy with this, *Щ* can be used to measure the energy E . *Щ* —E equivalence:

$$\mathrm{E} = \frac{Щ\, c^2}{\sqrt{G}} \tag{27}$$

This formula is suitable for measuring the energy of both mass and electric charge.
In addition, from formula (24) it follows standard gravitational parameter μ_m:

$$\mu_m = Щ\sqrt{G} \tag{28}$$

From formula (13), by multiplying both parts of the equation by $\sqrt{G}$, it follows that the standard gravitational parameter μ of a celestial body with mass m is equivalent to the standard electrical parameter μ_Q of a celestial body with electric charge Q :

$$\mu - \mu_Q \text{ equivalence} \tag{29}$$

Or

$$mG = Q\sqrt{Gk_e} \tag{30}$$

And

$$\mu_{M+Q} = \mu + \mu_Q = mG + Q\sqrt{Gk_e} \tag{31}$$

Here μ_{M+Q} is gravity-electric standard parameter (standard astrophysical parameter) of a celestial body with mass m and electric charge Q . The equivalent parameters μ and μ_Q (denoted by the letter *Ш*) can be used to measure the physical quantities electrical mass m with electric charge Q , as well as *Щ* (24) and (25), for energy measurement E :

$$E = \frac{Ш\, c^2}{G} \tag{32}$$

Herewith dim $Ш = L^3\ T^{-2}$
Formula (32) is suitable for measuring the energy of both mass and electric charge.

4.3. The way to determine the mass excess

4.3.1. For two rest masses

Principles of interaction of two masses M_1 and M_2 [6]:

$$F = G\frac{M_1 M_2}{R^2} \tag{34.0}$$

Or

$$\Delta E_+ \approx G\frac{M_1\, M_2\, \Delta R}{R^2} \tag{34}$$

Here R is the distance between the masses M_1 and M_2, ΔR is elementary part of radius R, on which the energy ΔE_+ is concentrated, that is ΔE_+ is the interaction energy M_1 and M_2 on interspace e ΔR. Relativistic mass of the interaction of two rest masses M_1 and M_2

$$\Delta m_+ \approx G\frac{M_1\, M_2\, \Delta R}{R^2 c^2} \tag{35}$$

Here Δm_+ is the excess mass equivalent to the energy ΔE_+ from the interaction of the masses M_1 and M_2 on interspace ΔR.

4.3.2. For two electric charges

The principle the interaction of two electric charges Q_1 and Q_2 at a distance R: [7]

$$F = \frac{Q_1\, Q_2}{K\, \varepsilon_o\, R^2}$$

Or

$$\Delta E_{+q} \approx \frac{Q_1\, Q_2\, \Delta R}{K\, \varepsilon_o\, R^2} \tag{36}$$

Here ΔE_{+q} is the interaction energy Q_1 and Q_2 on interspace ΔR, R is the distance between the Q_1 and Q_2, K=4π
From this we obtain the following equivalence $\Delta E_{+q} - \Delta m_{+q}$:

$$\Delta m_{+q} \approx \frac{Q_1\, Q_2\, \Delta R}{K\, \varepsilon_o\, R^2\ c^2} \tag{37}$$

Or

$$\Delta m_{+q} \approx \frac{Q_1\, Q_2\, \mu_o\, \Delta R}{K\, R^2} \tag{38}$$

Here Δm_{+q} is excess mass of relativistic energy ΔE_{+q} from the interaction of electric charges Q_1 and Q_2 on interspace ΔR.

4.3.3. On the mass excess

Part ΔM of the mass excess of hydrogen ΔM_H can be explained by the presence of the sum of proton–electron pair equivalences: the mass excess from the interaction of Q_1 and Q_2 (38) – ΔM_q, as well as M_1 and M_2 (35) is ΔM_m:

$$\Delta M = \Delta M_q + \Delta M_m \tag{39.0}$$

Here $\Delta M_m = m_+$ (35) and $\Delta M_q = m_{+q}$ (38):

$$\Delta M \approx \frac{Q_e\, Q_p\, \mu_o\, \Delta R}{K\, R_e^2} + G\frac{M_e\, M_p\, \Delta R}{R_{e^2}\, c^2} \tag{39}$$

Where Q_e is electric charge of an electron, Q_p is electric charge of a proton, R_e is hydrogen radius, K is proportionality coefficient.
ΔM is less than ΔM_H, that is, the excess mass ΔM_H consists not only of the sum of the equivalent mass ΔM_q and M_m, there is still a component in ΔM_H, larger in magnitude than ΔM.

4.4. Analogues of the energy–momentum tensor and Einstein field equations

Einstein field equations [12] for local scale:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{40.0}$$

Here, the stress–energy tensor $T_{\mu\nu}$ is equivalence to the energy tensor of electric field and current $T_{\mu_{Q\nu}}$

$$T_{\mu\nu} - T_{\mu_{Q\nu}} \text{ equivalence} \tag{40}$$

Or

$$T_{\mu\nu} = \frac{k_e}{G} T_{\mu_{Q\nu}} \tag{41}$$

$T_{\mu_{Q\nu}}$ is for the case when an electric field is created around a celestial body with $Q \to \infty$ (or this Q significantly exceeds the magnitude of the electric charges of other celestial bodies surrounding this Q, so much so that they can be neglected).
For the case when the field is created by a cluster of identical celestial bodies or a uniform nebula, $T_{\mu_{q\nu}}$ is the energy tensor of electric field and current for a homogeneous medium:

$$T_{\mu\nu} - T_{\mu_{q\nu}} \text{ equivalence} \tag{42}$$

And

$$T_{\mu\nu} = \sqrt{\frac{k_e}{G}} T_{\mu_{q\nu}} \tag{43}$$

Then the Einstein equation for the electric field will take two forms:

$$G_{\mu\nu} = \frac{\kappa_g k_e}{c^4} T_{\mu_{Q\nu}} \tag{44}$$

And

$$G_{\mu\nu} = \frac{\kappa_g \sqrt{Gk_e}}{c^4} T_{\mu_{q\nu}} \tag{45}$$

Here κ_g is proportionality coefficient, $\kappa_g = 8\pi$. The proposed equation of the electric field makes it possible to determine the dependence of electric charges and fields with the components of the space–time curvature.

4.5. Schwarzschild radius analogy

The Schwarzschild radius is known: [13]

$$r_s = \frac{2GM}{c^2} \tag{46.0}$$

Based on the electric charge–energy–mass equivalences (14), we represent the Schwarzschild radius r_s for mass M as the analogous radius r_q for electric charge Q:

$$r_q = \frac{2Q\sqrt{Gk_e}}{c^2} \tag{46}$$

5. Calculation of physical constants

From formulas (1) and (5) it follows that

$$E_s - m_s - Q_s \text{ equivalence} \tag{46.1}$$

And

$$E_p - m_p - Q_p \text{ equivalence} \tag{46.2}$$

As well as

$$\frac{E_p}{E_s} = \frac{m_p}{m_s} = \frac{Q_p}{Q_s} = ƙ \tag{46.3}$$

From here

$$ƙ = \sqrt{\frac{\hbar c}{k_e e^2}} = \sqrt{\alpha} \tag{47}$$

Here $ƙ$ is constant of equivalence of Planck and Stoney units, ħ is Dirac constant, c is speed of light in vacuum, e is electric charge of an electron, k_e is Coulomb constant, α is fine-structure constant. Therefore:

$$\hbar = \frac{ƙ^2 \, k_e \, e^2}{c} \tag{48}$$

$$c = \frac{ƙ^2 \, k_e \, e^2}{\hbar} \tag{49}$$

$$k_e = \frac{c \, \hbar}{ƙ^2 \, e^2} \tag{50}$$

$$e = \sqrt{\frac{\hbar \, c}{ƙ^2 \, k_e}} \tag{51}$$

Which corresponds to the derivation of physical constants based on Space–Time–Energy Equivalences [14, formulas 25–29].

6. Conclusion

We can observe the equivalence of electric charge and energy in all areas of physics and astrophysics. As examples of such observations, algebraic expansions in two natural units are given: Stoney units and Planck units (1), (4) and (5), the example with Newton's law of universal gravitation and Coulomb's laws (14), (15), and Ampere's law (21), (21.1), (22) and (23). Writing down the known principles of physics for limiting quantities: the speed of light, the Planck force leads to writing the proposed equivalence.
In 1905, Albert Einstein published a work on special relativity, a deductive theory based on the axiomatic method. [15]
This work also uses the axiomatic method. The proposed equivalence is formulated using various examples from physics, which corresponds to the deductive construction and confirms the existence of this equivalence in nature.
Electric charge–energy equivalence is the principle that anything having electric charge Q has an equivalent amount of energy E:

$$E = Q\, c^2 \sqrt{\frac{k_e}{G}} = Q\, k_{qe} c^2 = Q\, k_{ee\text{-}Q} \tag{22}$$

And vice versa

$$Q = \frac{E}{c^2}\sqrt{\frac{G}{k_e}} = \frac{E}{k_{qe}c^2} = \frac{E}{k_{ee-Q}} \tag{23}$$

Or E—Q equivalence and vice versa Q—E equivalence.
Summarizing what has been done (taking into account the time–space–mass–energy equivalences [14, (53) and (54)]), we can construct an extended equivalence sequence:

Q–m–E–T–L — equivalences (52)

Or

$$c^2 Q \sqrt{\frac{k_e}{G}} = mc^2 = E = \frac{Tc^5}{G} = \frac{Lc^4}{G} \tag{53}$$

This extended equivalence (53) indicates that the systems of natural units of Planck and Stoney (1), (4) and (5) were created on the basis of the equivalence of physical quantities (52), as well as the equivalence of physical constants (49) – (51) and [14, formulas (16) – (20)]. Now this dependence (52), [14, (20)] is just a confirmation of equivalences in physics, which is another confirmation of the proposed equivalence. From here

$$L = \frac{Q}{c^2}\sqrt{G k_e} \tag{54}$$

Or

$$L = Q\mu_0 \sqrt{G\varepsilon_0} \tag{55}$$

A space in a gravitational field G with a vacuum magnetic permeability μ_0 and dielectric constant ε_0 is equivalent to an electric charge Q. In this case, the coefficient ($\mu_0 \sqrt{G\varepsilon_0}$) is directly proportional to the space L and inversely proportional to the electric charge Q.
This electric charge – energy equivalence is a consequence of space being in a gravitational field and in an environment with magnetic and electrical properties.
The proposed equivalence opens up the possibility for us to calculate excess mass in physics, describe electrical standard parameters in astrophysics, write down electrical analogues of the energy–momentum tensor and Einstein field equations, and also present an electrical analogue of the Schwarzschild radius.
Based on gravitational and electrical additional standard parameters, a standard astrophysical parameter and two unified units of measurement of mass and electric charge are proposed.

Using this equivalence, it became possible to calculate some physical constants through others: the Dirac constant, the speed of light in a vacuum, the electric charge of an electron, the Coulomb Constant, which also confirms the existence of the proposed equivalence in nature.

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