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Research paper

Perturbation method for laminar flow of viscous incompressible fluid between two porous boundary walls

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Abstract

The present study focuses on the investigation of the velocity profile of viscous fluid between two parallel porous walls while fluid is being ejected from both channel walls at the same rate. The distributions of velocity, pressure and shear stress were determined theoretically for incompressible, steady, fully developed laminar flow past a porous boundary wall. Due to the pressure difference across the porous wall, there is ejection or injection into the wall. The velocity specified as the boundary condition at the wall has been used to explore fluid flow phenomena in porous tubes and ducts. The variable wall velocity is based on the pressure differential across the wall, the fluid characteristics, the thickness of the wall, and the permeability of the structure. An appropriate wall condition must take these factors into account. This paper discusses a Reynolds number solution. We seek formulas for the velocity component. The perturbation approach is used to solve the governing differential equation and the general solution of the velocity is obtained for higher terms of Re. Fifth term of perturbation is used in the perturbation method. The velocity of the fluid is analyzed for different values of Re for a boundary condition. Finally, the result is analysis graphically for the range of Re from .1 to 50. In addition, another strategy is used to keep the Reynolds number constant when the diameter of the hole in the porous wall changes, i.e., changing the value of λ.

Keywords: *Laminar Flow; Perturbation Method; Reynolds Number; Dimensionless Parameter; Newtonian Fluid.*

1. Introduction

The problem of laminar flow in channels or paths with porous boundary walls has numerous applications in various branches of engineering and technology, such as boundary layer control, transpiration cooling problems, extraction of plastic sheets, cooling of infinite metallic plates in a cool bath, liquid film condensation process and fields of glass and polymer industry. Due to the vast range of applications to particular engineering issues, the fluid flow via porous surfaces, pipes, and channels has studied. It has been analytically examined and used to lower the wall temperature of combustion chambers in rocket motors and to cool reentry bodies by injecting coolant through porous walls. It has proven possible to control boundary layers by draining fluid via porous surfaces. Other engineering issues that have seen widespread use of cellular surfaces include wave attenuation in fluidic circuits, gaseous diffusion processes, and jet engine noise reduction. The solution for the velocity profile and pressure distribution in a porous channel were firstly proposed by Berman (1953) for small suction value assuming no sliding at the porous wall. This was later extended to large suction and injection values by White et al. (1958) and Terrill (1964). Beavers and Joseph (1967) demonstrated the existence of shear velocities at porous boundaries, in their mass flow experiments. Analytical expressions for the velocity profile and axial pressure drop were reported by Singh and Laurence (1979) for small values of wall suction allowing fluid to slide at porous walls. Kaviany (1991) provides an excellent review of previous work in related areas. They presented the solution of the two-dimensional Navier-Stokes equation for shear flow in a channel with two equal porous walls at any arbitrary wall speed allowing for both suction and injection forces. J.L. Lage, M.J.S de Lemos, and D.A. (2002) Nield research on transport phenomena in porous media which was based on the studies of Antohe, B.V and Lage, J.L. (1997) , Bear, J.(1972) where they studied on turbulence model for incompressible flow in porous medium and Dynamics of fluid in porous medium. Later on, Ochoa Tapia, J.A and Whitaker, S (1995) studied on the momentum transfer at the boundary between a porous medium. The limitations of the Brinkman-Forchheimer equation in modeling flow in a saturated porous medium was studied by Nield, D.A. (1991) also Convection in porous media was studied by Nield, D.A. and Bejan, A. (1999).
The flow of a viscous fluid through a porous channel with a rectangular cross-section, when the Reynolds number is low, was investigated and a disturbance solution assuming equal normal velocities of the walls was obtained. Berman (1953). Vafai K and S. J. Kimi (1990) studied the fluid mechanics of the interface region between the porous medium and the liquid layer - an exact solution. Saffman P.G. (1971) studied boundary conditions on the surface of porous media. Das S.S. and UK Tripathy (2010) studied the effect of periodic suction on three-dimensional flow and heat transfer through a vertical foam plate embedded in a porous medium. Terrill (1964) studied laminar flow

in a uniform porous channel with large injection volumes. Terrill (1968) studied laminar flow with large injection volumes through uniform, parallel porous walls of varying permeability. Kohler, J.P.(1973) research on the laminar flow through a porous walled channel and Raptis A and C. Perdikis(1987) on free-convective flow through porous media. Later S. Chellam and Mark R. Wiesner, (1993) and Kuznetsov., A.V. (1997) work on laminar flow with slip in channels with uniformly porous walls and fluid flow in the interface region between a porous medium and a clear fluid in channels partially filled with porous medium. After that S. Krishnambal (2002), Avramenko, A. A, A.V. Kuznetsov, B.I. Basok and D.G. Blinov (2005) studied on laminar flow between two parallel porous walls with variable permeability and wall slip, and stability of laminar flow in a parallel-plate channel filled with a saturated porous medium. Later on, R. Yadav (2016) Numerically analysis the laminar flow of viscous fluid between two porous bounding walls. Most assessments of viscous flows in open-end porous tubes and channels have been constrained by a priori specification of the normal wall velocity and/or wall shear stress. By expanding series solutions, Yuan and Finkelstein, for instance, were able to find solutions for tubes and channels. For both internal and external flows over porous surfaces, numerical solutions were reported in the study by Eckert, Donoughe, and Moore. These investigations yielded a variable shear stress at the wall; the internal channel and pipe flows revealed a shear drop with suction (outflow) and an increase with injection (inflow). Terrill and Thomas took into account laminar flow in a pipe with equally distributed pores and constant normal wall velocity. It was shown that dual solutions were possible and that the issue could be simplified to the solution of two ordinary, nonlinear differential equations.

In this analysis, we studied the effect of Reynolds number on fluid velocity and considered that fluid moves in a porous double-walled channel in the presence of viscous fluid, with different Reynolds numbers for the higher term of function with perturbation method. Figure 1 and 2 represent the laminar flow and porous boundary wall respectively. We have observed that although there are many works done in this filed but at most fourth term of perturbation method is use for finding the solution of the required problem. But here we have used higher term of perturbation (fifth) for finding the solution in perturbation method. Although there are many graphical representation and discussion of the result, we have taken various values of Re for conducting the graphical representation and desiccation and tried to analysis increase its perimeter little by little and take a wide range of Re to conduct our research. Furthermore, we also try to understand the relation of Reynolds number and velocity of porous wall by changing the diameter of the hole in porous wall as we take Reynolds number as constant.

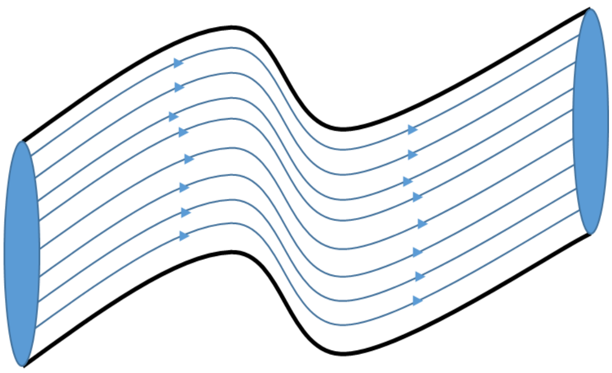

Fig. 1: Laminar Flow.

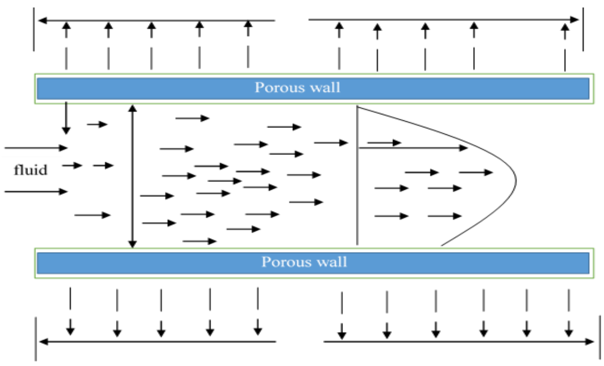

Fig. 2: Porous Boundary Wall.

2. Mathematical formulation

Let us consider the viscous flow of a Newtonian fluid between two parallel porous plates. The length of the channel is considered to be L, the distance between the plates is 2h, and the velocities components u and v represent the x- and y-directional of the flow, respectively.
The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

The equation of momentum is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

The boundary conditions are $u(x,h)=0, u(x,-h)=0, v(x,h)=V$ and $v(x,-h)=-V$
Where V is the velocity of suction at the walls of the channel. Let the dimensionless parameter be $\lambda = y/h$ and the equation 1-3 becomes

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \lambda}=0 \tag{4}$$

$$u\frac{\partial u}{\partial x}+\frac{v}{h}\frac{\partial u}{\partial \lambda}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \lambda^2}\right) \tag{5}$$

$$u\frac{\partial v}{\partial x}+\frac{v}{h}\frac{\partial v}{\partial \lambda}=-\frac{1}{\rho h}\frac{\partial p}{\partial \lambda}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \lambda^2}\right) \tag{6}$$

Where v denotes kinematic viscosity and p denotes pressure.
Introducing the stream function $\psi(x,y)$, we know that

$$u=\frac{\partial \psi}{\partial y} \ \& \ v=-\frac{\partial \psi}{\partial x} \tag{7}$$

Putting the value of dimensionless variable in the equation 7 we get

$$u=\frac{1}{h}\frac{\partial \psi}{\partial \lambda} \ \& \ v=-\frac{\partial \psi}{\partial x} \tag{8}$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x,\lambda)=(hu_0-v_w x)f(\lambda) \tag{9}$$

Now from equation 8 and 9

$$u=\frac{1}{h}(hu_0-v_w x)f'(\lambda) \ \& \ v=-v_w f(\lambda)$$

Now differentiating u and v with respect to x and λ we get

$$\frac{\partial u}{\partial x}=-\frac{1}{h}v_w f'(\lambda)\,;\frac{\partial^2 u}{\partial x^2}=0;\frac{\partial^2 u}{\partial x\,\partial \lambda}=-\frac{1}{h}v_w f''(\lambda)\,;\frac{\partial u}{\partial \lambda}=\frac{1}{h}(hu_0-v_w x)f''(\lambda);$$

$$\frac{\partial^2 u}{\partial \lambda^2}=\frac{1}{h}(hu_0-v_w x)f'''(\lambda);$$

$$\frac{\partial v}{\partial x}=0\,;\frac{\partial^2 v}{\partial x^2}=0;\frac{\partial^2 v}{\partial x\,\partial \lambda}=0\,;\frac{\partial v}{\partial \lambda}=-v_w f'(\lambda);\frac{\partial^2 v}{\partial \lambda^2}=-v_w f''(\lambda);$$

Multiplying equation 5 with $\frac{1}{h}$ and differentiating with respect to λ we get

$$\frac{1}{h}\frac{\partial u}{\partial \lambda}\frac{\partial u}{\partial x}+\frac{u}{h}\frac{\partial^2 u}{\partial x\,\partial \lambda}+\frac{1}{h^2}\frac{\partial v}{\partial \lambda}\frac{\partial u}{\partial \lambda}+\frac{1}{h^2}v\frac{\partial^2 u}{\partial \lambda^2}=-\frac{1}{\rho h}\frac{\partial^2 p}{\partial x\,\partial \lambda}+\frac{\nu}{h}\frac{\partial}{\partial \lambda}\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \lambda^2}\right) \tag{10}$$

Differentiating equation 6 with respect to x we get

$$\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+u\frac{\partial^2 v}{\partial x^2}+\frac{1}{h}\frac{\partial v}{\partial x}\frac{\partial v}{\partial \lambda}+\frac{v}{h}\frac{\partial^2 v}{\partial x\,\partial \lambda}=-\frac{1}{\rho h}\frac{\partial^2 p}{\partial x\,\partial \lambda}+\nu\frac{\partial}{\partial x}\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \lambda^2}\right) \qquad 11$$

Subtracting equation 10 from equation 11 and putting the values of

$\frac{\partial u}{\partial x},\frac{\partial^2 u}{\partial x^2},\frac{\partial^2 u}{\partial x\,\partial \lambda},\frac{\partial u}{\partial \lambda},\frac{\partial^2 u}{\partial \lambda^2},\frac{\partial v}{\partial x},\frac{\partial^2 v}{\partial x^2},\frac{\partial^2 v}{\partial x\,\partial \lambda},\frac{\partial v}{\partial \lambda},\frac{\partial^2 v}{\partial \lambda^2}$ we get

$$\frac{\partial}{\partial \lambda}\left[\frac{v_w}{h}\left(f'^2-ff''\right)+\frac{\nu}{h^2}f'''\right]=0 \tag{12}$$

Integrating the above equation (12)

$$\frac{v_w}{h}\left(f'^2-ff''\right)+\frac{\nu}{h^2}f'''=A$$

$$\frac{v_w}{h}\frac{h^2}{\nu}\left(f'^2-ff''\right)+\frac{\nu}{h^2}\frac{h^2}{\nu}f'''=A$$

$$\frac{v_w h}{\nu}\left(f'^2-ff''\right)+f'''=A \tag{13}$$

Putting $Re=\frac{v_w h}{\nu}$ in equation 13 we get

$$Re\left(f'^2-ff''\right)+f'''=A \tag{14}$$

In the preceding example, the family of Frankel-Skan equation is used where Re represent the Reynolds number. The perturbation method is used to solve the non-linear differential equation mentioned above. The boundary requirement for $f(\lambda)$ are

$$f(1) = 1, f(-1) = -1 \text{ and } f'(1) = f'(-1) = 0 \quad (15)$$

Hence the solution of the equation of motion and equation of continuity are given by a nonlinear third order differential equation (13), subject to the boundary conditions (15).

3. Method of solution

For the non-linear ordinary differential equation (14), we find the analytical solution of the differential equation subject to condition (15) which can be numerically integrated in general. However, for special case when Re and h are small, the approximate analytical results can be obtained by using regular perturbation approach. In this situation f may be expanded in the form

$$f(\lambda) = f_0(\lambda) + Ref_1(\lambda) + Re^2 f_2(\lambda) + Re^3 f_3(\lambda) + Re^4 f_4(\lambda) + \cdots = \sum_{n=0}^{\infty} Re^n f_n(\lambda) \quad (16)$$

And

$$A = A_0 + A_1 Re + A_2 Re^2 + A_3 Re^3 + A_4 Re^4 + \cdots = \sum_{n=0}^{\infty} A_n Re^n \text{ Where } n>0 \quad (17)$$

Where boundary condition are

$$f_0(-1) = -1, f_0(1) = 1, f_0'(1) = 0\,, f_0'(-1) = 0 \quad (18)$$

$$f_n(0) = 0 = f_n'(0), f_n(1) = 0 = f_n'(1) \quad (19)$$

Putting the value of$f(\lambda)$ & A in above equation (14), we get

$$Re[(f_0'(\lambda) + Ref_1'(\lambda) + Re^2 f_2'(\lambda) + Re^3 f_3'(\lambda) + Re^4 f_4'(\lambda) + \cdots)^2 - (f_0(\lambda) + Ref_1(\lambda) + Re^2 f_2(\lambda) + Re^3 f_3(\lambda) + Re^4 f_4(\lambda) + \cdots)(f_0''(\lambda) + Ref_1''(\lambda) + Re^2 f_2''(\lambda) + Re^3 f_3''(\lambda) + Re^4 f_4''(\lambda) + \cdots)] + f_0'''(\lambda) + Ref_1'''(\lambda) + Re^2 f_2'''(\lambda) + Re^3 f_3'''(\lambda) + Re^4 f_4'''(\lambda) + Re^5 f_5'''(\lambda) + \cdots = A_0 + A_1 Re + A_2 Re^2 + A_3 Re^3 + A_4 Re^4 + A_5 Re^5 \ldots$$

Or

$$Re[(f_0'(\lambda) + Ref_1'(\lambda) + Re^2 f_2'(\lambda) + Re^3 f_3'(\lambda) + Re^4 f_4'(\lambda) + \cdots)(f_0'(\lambda) + Ref_1'(\lambda) + Re^2 f_2'(\lambda) + Re^3 f_3'(\lambda) + Re^4 f_4'(\lambda) + \cdots) - (f_0(\lambda) + Ref_1(\lambda) + Re^2 f_2(\lambda) + Re^3 f_3(\lambda) + Re^4 f_4(\lambda) + \cdots)(f_0''(\lambda) + Ref_1''(\lambda) + Re^2 f_2''(\lambda) + Re^3 f_3''(\lambda) + Re^4 f_4''(\lambda) + \cdots)] + f_0'''(\lambda) + Ref_1'''(\lambda) + Re^2 f_2'''(\lambda) + Re^3 f_3'''(\lambda) + Re^4 f_4'''(\lambda) + Re^5 f_5'''(\lambda) = A_0 + A_1 Re + A_2 Re^2 + A_3 Re^3 + A_4 Re^4 + A_5 Re^5 \quad (20)$$

Taking the coefficient of $Re^0, Re^1, Re^2, Re^3, Re^4, Re^5$
We get

$$f_0''' = A_0 \quad (21)$$

$$f_1''' = A_1 - {f_0'}^2 + f_0 f_1'' \quad (22)$$

$$f_2''' = A_2 - 2f_0' f_1' + f_1 f_0'' + f_0 f_1'' \quad (23)$$

$$f_3''' = A_3 - {f_1'}^2 - 2f_0' f_1' + f_2 f_0'' + f_1 f_1'' + f_0 f_2'' \quad (24)$$

$$f_4''' = A_4 - 2f_0' f_3' - 2f_1' f_2' + f_0'' f_3 + f_0 f_3'' + f_1'' f_2 + f_1 f_2'' \quad (25)$$

$$f_5'''(\lambda) = A_5 - 2f_0' f_4' - 2f_1' f_3' - {f_2'}^2 + f_0 f_4'' + f_1 f_3'' + f_2 f_2'' + f_3 f_1'' + f_4 f_0'' \quad (26)$$

Integrating equation 21 and using boundary condition we get

$$f_0'' = A_0 \lambda + c1$$

$$f_0' = \frac{A_0 \lambda^2}{2} + c1\lambda + c2 \quad (27)$$

Now $f_0'(-1) = 0$ & $f_0'(1) = 0$

So $\frac{A_0}{2} - c1 + c2 = 0$ & $\frac{A_0}{2} + c1 + c2 = 0$

Solving above equation we get

$c2 = -\frac{A_0}{2}$ and $c1 = 0$

Putting these values in equation 27 we get

$$f_0' = \frac{A_0}{2}\lambda^2 - \frac{A_0}{2}$$

Again, integrating the above equation we get

$$f_0 = \frac{A_0\lambda^3}{6} - \frac{A_0\lambda}{2} + c3 \quad (28)$$

Now $f_0(1) = 1 \; and \; f_0(-1) = -1$

So $-1 = -\frac{A_0}{6} + \frac{A_0}{2} + c3$ & $1 = \frac{A_0}{6} - \frac{A_0}{2} + c3$

Solving the above equation we get

$c3 = 0$ & $A_0 = -3$

Putting these values in equation 27 we get

$$f_0(\lambda) = \frac{\lambda}{2}(3 - \lambda^2) = 1.3\lambda - .5\lambda^3 \quad (29)$$

Using similar process, we can achieve f_1, f_2, f_3, f_4

$$f_1(\lambda) = -.0143\lambda^2 + .01786\lambda^3 - 0.00357143\lambda^7 \quad (30)$$

$$f_2(\lambda) = .00039392\lambda^2 - .00191144\lambda^3 + .0017788\lambda^4 - .0002383\lambda^6 + .000255\lambda^7 - .000298\lambda^9 + .00001082\lambda^{11} \quad (31)$$

$$f_3(\lambda) = -.0022742\lambda^2 + .0015501\lambda^3 + .00362425\lambda^4 - .002983\lambda^5 - .000452\lambda^6 + .0008155\lambda^7 - .0003971\lambda^8 + .0003721\lambda^9 + .00008753\lambda^{10} - .0004417\lambda^{11} + .0000688\lambda^{13} - .0000002626\lambda^{15} \quad (32)$$

$$f_4(\lambda) = .00164225\lambda^2 - .00251\lambda^3 + .0002844\lambda^4 + .000233\lambda^5 + .000506\lambda^6 - .00018971\lambda^7 + .000000348\lambda^8 + .00005641\lambda^9 - .0000324\lambda^{10} + .0000283\lambda^{11} + .0000102\lambda^{12} - .0000373\lambda^{13} - .0000018544\lambda^{14} + .00000899\lambda^{15} - .00000076\lambda^{17} + .000000002871\lambda^{19} \quad (33)$$

Now for the co-efficient of Re^5

$$f_5'''(\lambda) = A_5 - 2f_0'f_4' - 2f_1'f_3' - {f_2'}^2 + f_0f_4'' + f_1f_3'' + f_2f_2'' + f_3f_1'' + f_4f_0'' \quad (34)$$

$$f_1'(\lambda) = -0.0286 * \lambda + 0.05358 * \lambda^2 - 0.02500001 * \lambda^6 \quad (35)$$

$$f_1''(\lambda) = -0.0286 + 0.10716 * \lambda - 0.15000006 * \lambda^5 \quad (36)$$

$$f_2'(\lambda) = 0.00078784 * \lambda - 0.00573432 * \lambda^2 + 0.0071152 * \lambda^3 - 0.0014298 * \lambda^5 + 0.001785 * \lambda^6 - 0.002682 * \lambda^8 + 0.00011902 * \lambda^{10} \quad (37)$$

$$f_2''(\lambda) = 0.00078784 - 0.01146864 * \lambda + 0.0213456 * \lambda^2 - 0.0071490 * \lambda^4 + 0.010710 * \lambda^5 - 0.021456 * \lambda^7 + 0.00119020 * \lambda^9 \quad (38)$$

$$f_3'(\lambda) = -0.0045484 * \lambda + 0.0046503 * \lambda^2 + 0.01449700 * \lambda^3 - 0.014915 * \lambda^4 - 0.002712 * \lambda^5 + 0.0057085 * \lambda^6 - 0.0031768 * \lambda^7 + 0.0033489 * \lambda^8 + 0.00087530 * \lambda^9 - 0.0048587 * \lambda^{10} + 0.0008944 * \lambda^{12} - 3.9390 * 10^{-6} * \lambda^{14} \quad (39)$$

$$f_3''(\lambda) = -0.0045484 + 0.0093006 * \lambda + 0.04349100 * \lambda^2 - 0.059660 * \lambda^3 - 0.013560 * \lambda^4 + 0.0342510 * \lambda^5 - 0.0222376 * \lambda^6 + 0.0267912 * \lambda^7 + 0.00787770 * \lambda^8 - 0.0485870 * \lambda^9 + 0.0107328 * \lambda^{11} - 0.0000551460 * \lambda^{13} \quad (40)$$

$$f_4'(\lambda) = 0.00328450 * \lambda - 0.00753 * \lambda^2 + 0.0011376 * \lambda^3 + 0.001165 * \lambda^4 + 0.003036 * \lambda^5 - 0.00132797 * \lambda^6 + 2.784 * 10^{-6} * \lambda^7 + 0.00050769 * \lambda^8 - 0.0003240 * \lambda^9 + 0.0003113 * \lambda^{10} + 0.0001224 * \lambda^{11} - 0.0004849 * \lambda^{12} - 0.0000259616 * \lambda^{13} + 0.00013485 * \lambda^{14} - 0.00001292 * \lambda^{16} + .00000005454 * \lambda^{18} \quad (41)$$

$$f_4''(\lambda) = 0.00328450 - 0.01506 * \lambda + 0.0034128 * \lambda^2 + 0.004660 * \lambda^3 + 0.015180 * \lambda^4 - 0.00796782 * \lambda^5 + 0.000019488 * \lambda^6 + 0.00406152 * \lambda^7 - 0.0029160 * \lambda^8 + 0.0031130 * \lambda^9 + 0.0013464 * \lambda^{10} - 0.0058188 * \lambda^{11} - 0.0003375008 * \lambda^{12} + 0.00188790 * \lambda^{13} - 0.00020672 * \lambda^{15} + 9.81882 * 10^{-7} * \lambda^{17} \quad (42)$$

Using equation 34 to 42 we get the following form

$$f_5'''(\lambda) = A_5 - 2(1.3 - 1.5 * \lambda^2) * (0.00328450 * \lambda - 0.00753 * \lambda^2 + 0.0011376 * \lambda^3 + 0.001165 * \lambda^4 + 0.003036 * \lambda^5 - 0.00132797 * \lambda^6 + 2.784 * 10^{-6} * \lambda^7 + 0.00050769 * \lambda^8 - 0.0003240 * \lambda^9 + 0.0003113 * \lambda^{10} + 0.0001224 * \lambda^{11} - 0.0004849 * \lambda^{12} - 0.0000259616 * \lambda^{13} + 0.00013485 * \lambda^{14} - 0.00001292 * \lambda^{16} + 5.4549 * 10^{-8} * \lambda^{18}) - 2(-0.0286 * \lambda + 0.05358 * \lambda^2 - 0.02500001 * \lambda^6) * (-0.0045484 * \lambda + 0.0046503 * \lambda^2 + 0.01449700 * \lambda^3 - 0.014915 * \lambda^4 - 0.002712 * \lambda^5 + 0.0057085 * \lambda^6 - 0.0031768 * \lambda^7 + 0.0033489 * \lambda^8 + 0.00087530 * \lambda^9 - 0.0048587 * \lambda^{10} +$$

$0.0008944 * \lambda^{12} - 3.9390 * 10^{-6} * \lambda^{14}) - (0.00078784 * \lambda - 0.00573432 * \lambda^2 + 0.0071152 * \lambda^3 - 0.0014298 * \lambda^5 + 0.001785 * \lambda^6 - 0.002682 * \lambda^8 + 0.00011902 * \lambda^{10}) * (0.00078784 * \lambda - 0.00573432 * \lambda^2 + 0.0071152 * \lambda^3 - 0.0014298 * \lambda^5 + 0.001785 * \lambda^6 - 0.002682 * \lambda^8 + 0.00011902 * \lambda^{10}) + (1.3\lambda - .5\lambda^3)(0.00328450 - 0.01506 * \lambda + 0.0034128 * \lambda^2 + 0.004660 * \lambda^3 + 0.015180 * \lambda^4 - 0.00796782 * \lambda^5 + 0.000019488 * \lambda^6 + 0.00406152 * \lambda^7 - 0.0029160 * \lambda^8 + 0.0031130 * \lambda^9 + 0.0013464 * \lambda^{10} - 0.0058188 * \lambda^{11} - 0.0003375008 * \lambda^{12} + 0.00188790 * \lambda^{13} - 0.00020672 * \lambda^{15} + 9.81882 * 10^{-7} * \lambda^{17}) + (-.0143\lambda^2 + .01786\lambda^3 - 0.00357143\lambda^7)(-0.0045484 + 0.0093006 * \lambda + 0.04349100 * \lambda^2 - 0.059660 * \lambda^3 - 0.013560 * \lambda^4 + 0.0342510 * \lambda^5 - 0.0222376 * \lambda^6 + 0.0267912 * \lambda^7 + 0.00787770 * \lambda^8 - 0.0485870 * \lambda^9 + 0.0107328 * \lambda^{11} - 0.0000551460 * \lambda^{13}) + (.00039392\lambda^2 - .00191144\lambda^3 + .0017788\lambda^4 - .0002383\lambda^6 + .000255\lambda^7 - .000298\lambda^9 + .00001082\lambda^{11})(0.00078784 - 0.01146864 * \lambda + 0.0213456 * \lambda^2 - 0.0071490 * \lambda^4 + 0.010710 * \lambda^5 - 0.021456 * \lambda^7 + 0.00119020 * \lambda^9) + (-.0022742\lambda^2 + .0015501\lambda^3 + .00362425\lambda^4 - .002983\lambda^5 - .000452\lambda^6 + .0008155\lambda^7 - .0003971\lambda^8 + .0003721\lambda^9 + .00008753\lambda^{10} - .0004417\lambda^{11} + .0000688\lambda^{13} - .0000002626\lambda^{15})(-0.0286 + 0.10716 * \lambda - 0.15000006 * \lambda^5) + (.00164225\lambda^2 - .00251\lambda^3 + .0002844\lambda^4 + .000233\lambda^5 + .000506\lambda^6 - .00018971\lambda^7 + .000000348\lambda^8 + .00005641\lambda^9 - .0000324\lambda^{10} + .0000283\lambda^{11} + .0000102\lambda^{12} - .0000373\lambda^{13} - .0000018544\lambda^{14} + .00000899\lambda^{15} - .00000076\lambda^{17} + .000000002871\lambda^{19})(-3.0 * \lambda) \quad (43)$

Simplifying equation 43

$$f_5'''(\lambda) = A_5 - 0.014123350 * \lambda + 0.04504960542 * \lambda^2 - 0.000294123612 * \lambda^3 - 0.02608581737 * \lambda^4 - 0.03227586041 * \lambda^5 + 0.01841442283 * \lambda^6 + 0.001422607132 * \lambda^7 - 0.008023144857 * \lambda^8 + 0.005517237548 * \lambda^9 - 0.006586659071 * \lambda^{10} - 0.001660046028 * \lambda^{11} + 0.01017494765 * \lambda^{12} + 0.0002113377767 * \lambda^{13} - 0.001985264458 * \lambda^{14} + 0.00009391661831 * \lambda^{15} - 0.0001589346997 * \lambda^{16} + 0.00005890913285 * \lambda^{18} - 2.891917806 * 10^{-7} * \lambda^{20} \quad (44)$$

Integrating equation 44 with respect to λ two times and using constant c and d

$$f_5''(\lambda) = A_5 * \lambda - 0.007061675000 * \lambda^2 + 0.01501653514 * \lambda^3 - 0.00007353090300 * \lambda^4 - 0.005217163474 * \lambda^5 - 0.005379310068 * \lambda^6 + 0.002630631833 * \lambda^7 + 0.0001778258915 * \lambda^8 - 0.0008914605397 * \lambda^9 + 0.0005517237548 * \lambda^{10} - 0.0005987871883 * \lambda^{11} - 0.0001383371690 * \lambda^{12} + 0.0007826882808 * \lambda^{13} + 0.00001509555548 * \lambda^{14} - 0.0001323509639 * \lambda^{15} + 5.869788644 * 10^{-6} * \lambda^{16} - 9.349099982 * 10^{-6} * \lambda^{17} + 3.100480676 * 10^{-6} * \lambda^{19} - 1.377103717 * 10^{-8} * \lambda^{21} + c \quad (45)$$

$$f_5'(\lambda) = 0.5000000000 * A_5 * \lambda^2 - 0.002353891667 * \lambda^3 + 0.003754133785 * \lambda^4 - 0.00001470618060 * \lambda^5 - 0.0008695272457 * \lambda^6 - 0.0007684728669 * \lambda^7 + 0.0003288289791 * \lambda^8 + 0.00001975843239 * \lambda^9 - 0.00008914605397 * \lambda^{10} + 0.00005015670498 * \lambda^{11} - 0.00004989893236 * \lambda^{12} - 0.00001064132069 * \lambda^{13} + 0.00005590630577 * \lambda^{14} + 1.006370365 * 10^{-6} * \lambda^{15} - 8.271935244 * 10^{-6} * \lambda^{16} + 3.452816849 * 10^{-7} * \lambda^{17} - 5.193944434 * 10^{-7} * \lambda^{18} + 1.550240338 * 10^{-7} * \lambda^{20} - 6.259562350 * 10^{-10} * \lambda^{22} + c * \lambda + d \quad (46)$$

Using boundary condition to find the value of c and d

$f_5'(0) = 0 = d$;

$f_5'(1) = 0 = 0.5000000000 * A_5 + 0.00004521466005 + c + d$;

$d = 0;\ c = -0.5000000000 * A_5 - 0.00004521466005$

Putting values of c and d in equation 46 we get

$$f_5'(\lambda) = 0.5000000000 * A_5 * \lambda^2 - 0.002353891667 * \lambda^3 + 0.003754133785 * \lambda^4 - 0.00001470618060 * \lambda^5 - 0.0008695272457 * \lambda^6 - 0.0007684728669 * \lambda^7 + 0.0003288289791 * \lambda^8 + 0.00001975843239 * \lambda^9 - 0.00008914605397 * \lambda^{10} + 0.00005015670498 * \lambda^{11} - 0.00004989893236 * \lambda^{12} - 0.00001064132069 * \lambda^{13} + 0.00005590630577 * \lambda^{14} + 1.006370365 * 10^{-6} * \lambda^{15} - 8.271935244 * 10^{-6} * \lambda^{16} + 3.452816849 * 10^{-7} * \lambda^{17} - 5.193944434 * 10^{-7} * \lambda^{18} + 1.550240338 * 10^{-7} * \lambda^{20} - 6.259562350 * 10^{-10} * \lambda^{22} + \lambda(-0.5000000000 * A_5 - 0.00004521466005) \quad (47)$$

Integrating equation 47 and using constant c

$$f_5(\lambda) = -0.0005884729168 * \lambda^4 + 0.0007508267570 * \lambda^5 - 2.451030100 * 10^{-6} * \lambda^6 - 0.0001242181780 * \lambda^7 - 0.00009605910836 * \lambda^8 + 0.00003653655323 * \lambda^9 + 1.975843239 * 10^{-6} * \lambda^{10} - 8.104186725 * 10^{-6} * \lambda^{11} + 4.179725415 * 10^{-6} * \lambda^{12} - 3.838379412 * 10^{-6} * \lambda^{13} - 7.600943350 * 10^{-7} * \lambda^{14} + 3.727087051 * 10^{-6} * \lambda^{15} + 6.289814781 * 10^{-8} * \lambda^{16} - 4.865844261 * 10^{-7} * \lambda^{17} + 1.918231583 * 10^{-8} * \lambda^{18} - 2.733654965 * 10^{-8} * \lambda^{19} + 7.382096848 * 10^{-9} * \lambda^{21} - 2.721548848 * 10^{-11} * \lambda^{23} + 0.1666666667 * A_5 * \lambda^3 (-A_5/2 - 904293201/20000000000000) + c \quad (48)$$

Using boundary condition, we find the value of c

$f_5(0) = 0 = c$;

$f_5(1) = 0 = 0.9999729176 + 0.1666666667 * A_5 + c$;

$A_5 = -5.999837504, c = 0$

Putting values of c and A_5 in equation 48 we get

$$f_5(\lambda) = \lambda^2(2.999873537) - 0.9999729175 * \lambda^3 - 0.0005884729168 * \lambda^4 + 0.0007508267570 * \lambda^5 - 2.451030100 * 10^{\wedge}(-6) * \lambda^6 - 0.0001242181780 * \lambda^7 - 0.00009605910836 * \lambda^8 + 0.00003653655323 * \lambda^9 + 1.975843239 * 10^{\wedge}(-6) * \lambda^{10} - 8.104186725 * 10^{-6} * \lambda^{11} + 4.179725415 * 10^{-6} * \lambda^{12} - 3.838379412 * 10^{-6} * \lambda^{13} - 7.600943350 * 10^{-7} * \lambda^{14} + 3.727087051 * 10^{-6} * \lambda^{15} + 6.289814781 * 10^{-8} * \lambda^{16} - 4.865844261 * 10^{-7} * \lambda^{17} + 1.918231583 * 10^{-8} * \lambda^{18} - 2.733654965 * 10^{-8} * \lambda^{19} + 7.382096848 * 10^{-9} * \lambda^{21} - 2.721548848 * 10^{-11} * \lambda^{23} \quad (49)$$

Hence the solution of above non-linear differential equation can be obtained by putting the value of

$f_0(\lambda), f_1(\lambda), f_2(\lambda), f_3(\lambda), f_4(\lambda), f_5(\lambda)$

$$f(\lambda) = f_0(\lambda) + Ref_1(\lambda) + Re^2 f_2(\lambda) + Re^3 f_3(\lambda) + Re^4 f_4(\lambda) + Re^5 f_5(\lambda)$$

$$f(\lambda) = 1.3\lambda - .5\lambda^3 + Re(-.0143\lambda^2 + .01786\lambda^3 - 0.00357143\lambda^7) + Re^2(.00039392\lambda^2 - .00191144\lambda^3 + .0017788\lambda^4 - .0002383\lambda^6 + .000255\lambda^7 - .000298\lambda^9 + .00001082\lambda^{11}) + Re^3(-.0022742\lambda^2 + .0015501\lambda^3 + .00362425\lambda^4 - .002983\lambda^5 - .000452\lambda^6 + .0008155\lambda^7 - .0003971\lambda^8 + .0003721\lambda^9 + .00008753\lambda^{10} - .0004417\lambda^{11} + .0000688\lambda^{13} - .0000002626\lambda^{15}) + Re^4(.00164225\lambda^2 - .00251\lambda^3 + .0002844\lambda^4 + .000233\lambda^5 + .000506\lambda^6 - .00018971\lambda^7 + .000000348\lambda^8 + .00005641\lambda^9 - .0000324\lambda^{10} + .0000283\lambda^{11} + .0000102\lambda^{12} - .0000373\lambda^{13} - .0000018544\lambda^{14} + .00000899\lambda^{15} - .00000076\lambda^{17} + .000000002871\lambda^{19}) + Re^5(\lambda^2(2.999873537) - 0.9999729175 * \lambda^3 - 0.0005884729168 * \lambda^4 + 0.0007508267570 * \lambda^5 - 2.451030100 * 10^{\wedge}(-6) * \lambda^6 - 0.0001242181780 * \lambda^7 - 0.00009605910836 * \lambda^8 + 0.00003653655323 * \lambda^9 + 1.975843239 * 10^{\wedge}(-6) * \lambda^{10} - 8.104186725 * 10^{-6} * \lambda^{11} + 4.179725415 * 10^{-6} * \lambda^{12} - 3.838379412 * 10^{-6} * \lambda^{13} - 7.600943350 * 10^{-7} * \lambda^{14} + 3.727087051 * 10^{-6} * \lambda^{15} + 6.289814781 * 10^{-8} * \lambda^{16} - 4.865844261 * 10^{-7} * \lambda^{17} + 1.918231583 * 10^{-8} * \lambda^{18} - 2.733654965 * 10^{-8} * \lambda^{19} + 7.382096848 * 10^{-9} * \lambda^{21} - 2.721548848 * 10^{-11} * \lambda^{23})$$

4. Graphical analysis

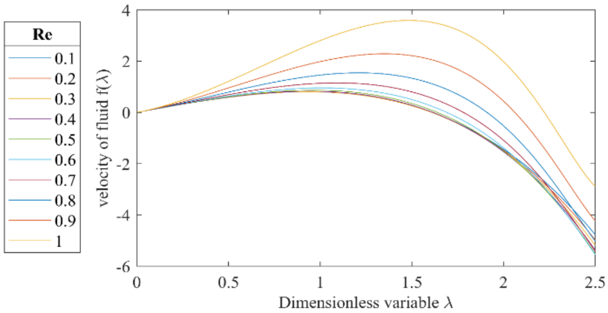

Fig. 3: Fluid Velocity Against Dimensionless Variable for Re Numbers .1, .2, .3 … 1.

Figure 3 shows the relationship between fluid velocity and dimensionless variable if we take the value of Re .1, .2, .3 … 1. From the figure we can observe that when the value of Re is .1 the fluid velocity is almost linear with regard to the dimensionless variable range 0 to 1. Furthermore, if the value of Re is increased to 1 the velocity is also increased and if we compare it with all other values in the range .1 to 1 the graph bends upward, i.e. the velocity is increase gradually.

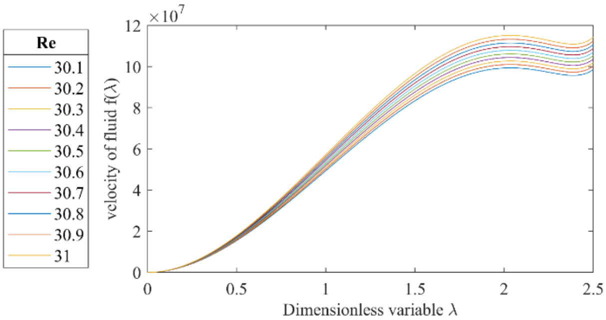

Fig. 4: Fluid Velocity Against Dimensionless Variable for Re Numbers 30.1, 30.2, 30.3 …. 31.

In figure 4 we have analyzed, if we take the value of Re 30.1, 30.2, 30.3 …. 31 then what will be the value of f(λ). From the figure we can see that the velocity of the fluid when value of Re is 30.1, 30.2, 30.3 … 31 which is much different than .1, .2, .3 … 1. However, if we compare this two, we can say that the velocity is much higher than previous one here velocity is 10^7 compare to previous one. Furthermore, if we look carefully, we can see that the velocity is also increasing, i.e., the velocity is increase gradually with respect to the dimensionless variable.

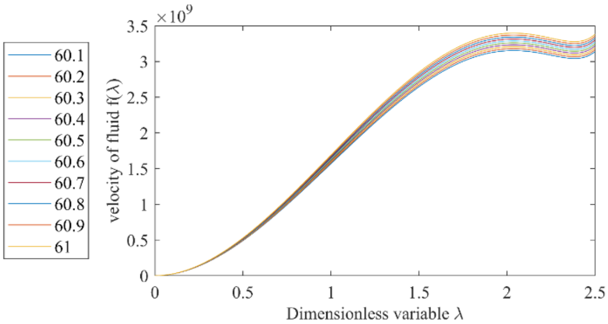

Fig. 5: Fluid Velocity Against Dimensionless Variable for Re Numbers 60.1, 60.2, 60.3 …. 61.

Figure 5 illustrates that what will be the velocity of the fluid if we take the value of Re 60.1, 60.2, 60.3 …. 61. We can see from the graph that the velocity of the fluid increases as the value of Re increases, however if we look closely, we can notice that the plot line of the graph is denser than the preceding one. Because as Re increases, so does the velocity; in this case, the velocity is expressed by the terms 10^9, whereas the preceding one is given by 10^7. Furthermore, if we look meticulously, we can observe that the velocity progressively increases as Re increases.

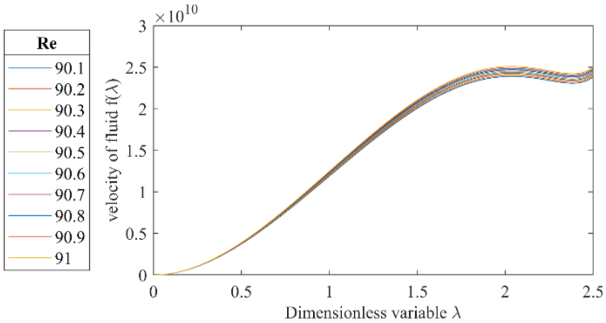

Fig. 6: Fluid Velocity Against Dimensionless Variable for Re Numbers 90.1, 90.2, 90.3 … 91.

In Figure 6, we investigated what the fluid velocity would be if Re 90.1, 90.2, 90.3... 91 were used. The figure shows that the velocity of the fluid increases as the value of Re increases, yet when we look closely, we are able to observe that the plot line of the graph is thicker than the preceding one. It is because as Re increases, also rises the velocity. Here, the velocity is expressed by terms of 10^{10}, whereas the previous one is denoted by 10^9. If we look closely, we can see that the velocity increases steadily as Re increases. When we compare the four figures, we can observe that if we increase the value by 30, the velocity increases compare to the previous one.

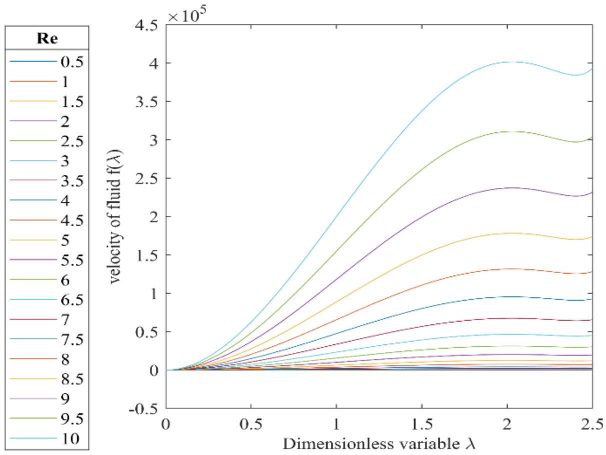

Fig. 7: Fluid Velocity Against Dimensionless Variable for Re Numbers 0.5, 1, 1.5,2 …. 10.

Figure 7 depicts the value of the Re number ranging from.5 to 10. We can observe that the plot displays the fluid velocity for different Re values ranging from.5 to 10 with respect to the value ofλ. If we look thoroughly, we can see that the velocity gradually increases as the Re number increases. However, when we compare the ranges.5 to 4 and 5 to 10, we can observe that the velocity increases significantly higher in the range of 5 to 10 than in the range.5 to 4.

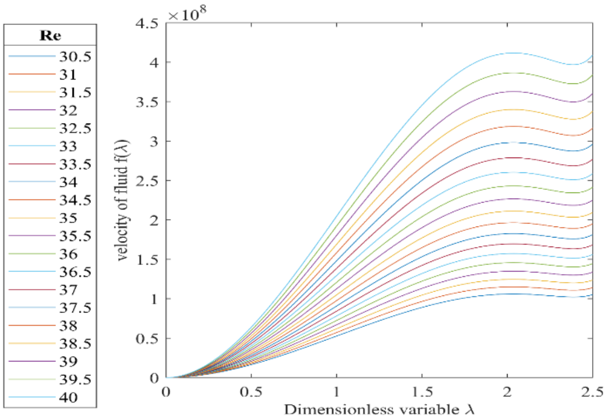

Fig. 8: Fluid Velocity Against Dimensionless Variable for Re Numbers 30.5, 31, 31.5 …. 40.

The figure 8 exhibits the fluid velocity for various Re values ranging from 30.5 to 40 in relation to the value ofλ. If we look carefully at the graph, we can observe that the velocity increases steadily as the Re number increases. However, when we compare the ranges 30.5 to 32.5 and 35 to 40, we can see that the velocity increases significantly more in the range 35 to 40 than in the range 30.5 to 32.5.

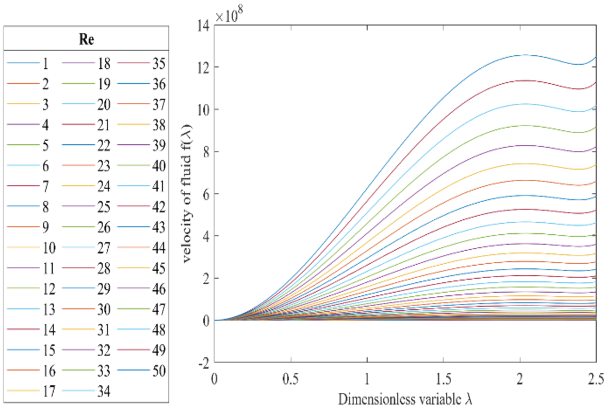

Fig. 9: Fluid Velocity Against Dimensionless Variable for Re Numbers 1, 2 3… 48, 49, 50.

Figure 9 shows the value of Re 1, 2, 3 ...48, 49, 50. We examine that the velocity of the fluid is virtually identical to the previous one as the value of Re increases by 1 at the initial state. It appears that the fluid is in steady state, but it is not because the velocity in the upper range is significantly higher than the velocity in the lower range. If we look closely, we can see that the velocity increases as the value of Re increases in relation to the dimensionless variableλ. In addition, when we compare 1 to 10 and 40 to 50, we can see that there is a significant variation in velocity.

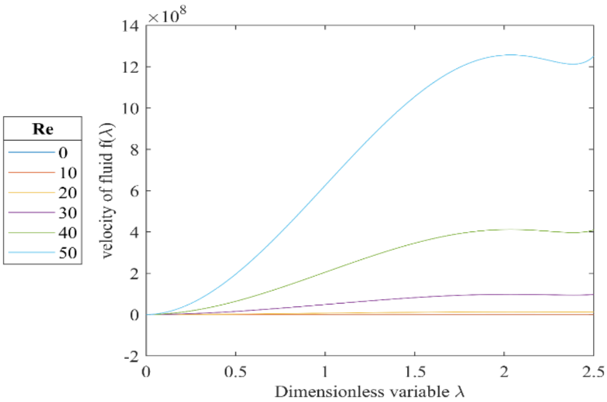

Fig. 10: Fluid Velocity Against Dimensionless Variable for Re Numbers 0, 10, 20,30,40,50.

We attempted to examine the graph or velocity in this graph by selecting a number between 0 and 50. In this case, we used the numbers 0, 10, 20, 30, 40, and 50. When we compare 0 and 10, it appears that the velocity does not rise significantly, yet it does. For example, we can look at figure 7 where we take .5 to 10 and see that the velocity is progressively increasing. However, it appears differently here. It appears to be almost identical. As we increase the amount by 10, we can observe that as the value of e increases, so does the velocity.
The graphs above attempt to predict fluid velocity when Reynolds number is altered while dimensionless variable λ remains constant. In the following section, we evaluate the influence of fluid velocity when dimensionless variable λ changes and Re remains constant. In this stage, the value of λ is increased by 0.5, extending the range of Reynold number from 0 to 10.

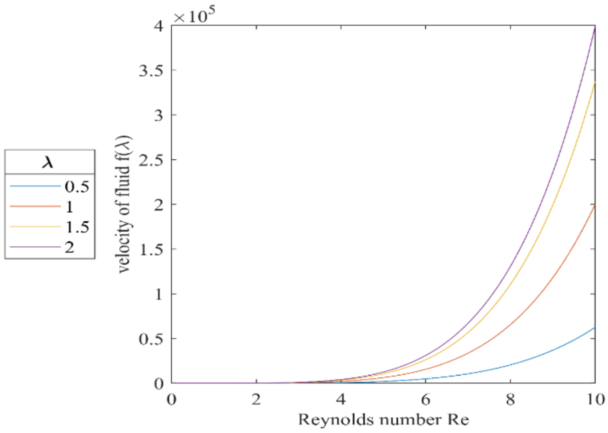

Fig. 11: Fluid Velocity Against Reynolds Numbers for Λ .5,1, 1.5,2.

Figure 11 shows the relationship between fluid velocity and Reynold number if we take the value of λ .5 ,1 1.5 and 2. According to the definition of the Reynold number, $\mathrm{Re} = \frac{\text{inertial force}}{\text{viscous force}}$, as the value of Reynold number increases, the viscous force decreases, resulting in an increase in velocity. However, when we compare it to R. Yadav (2016), we observe that it was not following the basic principle of Reynold number. The present graph shows that increasing the value of λ from.5 to 2 by.5 leads to an increase in velocity.

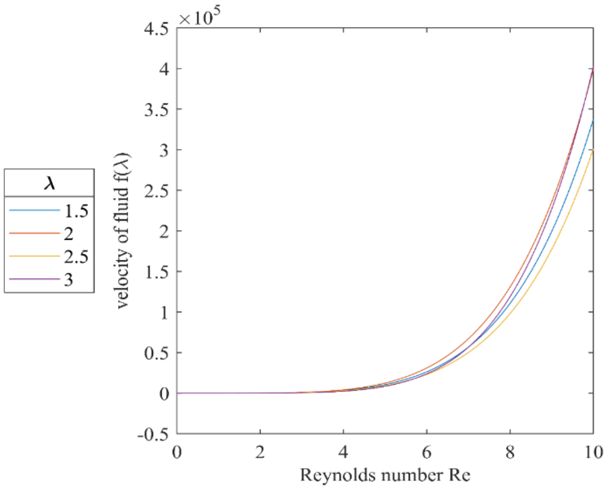

Fig. 12: Fluid Velocity Against Reynolds Numbers for Λ 1.5,2, 2.5,3.

In figure 12 we have analyzed, if we take the value λ is 1.5,2,2.5 and 3 then what will be the value of f(λ). The graph shows that the velocity increases for 1.5, 2, and 2.5, but the velocity for 3 is between 2.5 and 2. Furthermore, it intersects with a velocity of 1.5. In this portion, velocity was stable at 2 to 2.5 for λ 3.

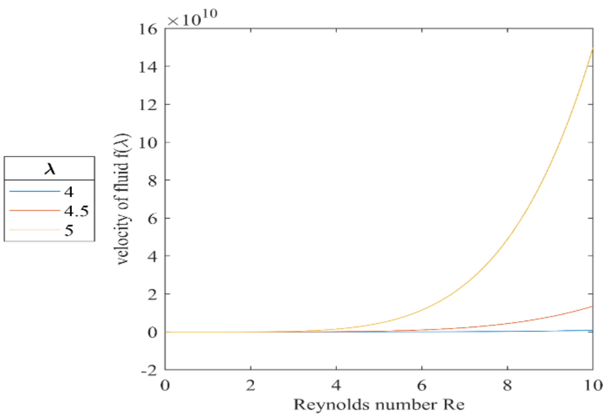

Fig. 13: Fluid Velocity Against Reynolds Numbers for Λ 4,4.5, 5.

The figure 13 exhibits the fluid velocity for various λ values 4, 4.5 and 5 in relation to the value of Re. For the value of 4 it seems that the velocity of fluid is almost zero compare to the value of 4.5 and 5. However, if we take a look at the value of $\lambda = 5$, it can be seen that the velocity increases significantly compare to the other values.

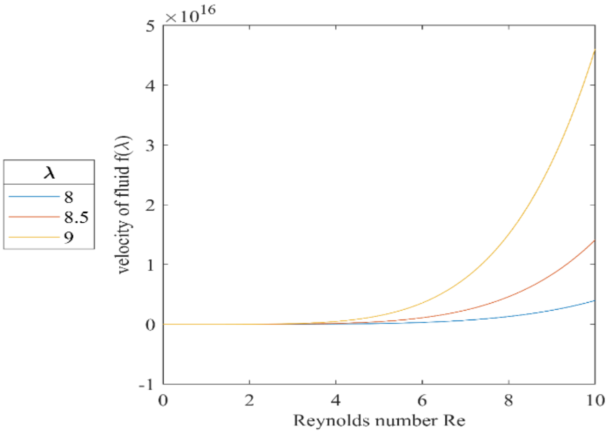

Fig. 14: Fluid Velocity Against Reynolds Numbers for Λ 8,8.5,9.

Figure 14 displays the velocity distribution for λ = 8, 8.5 and 9. We examine that the velocity of the fluid is almost identical to the graph of figure13. However, this portion has a far higher velocity than the prior one. i.e., the graph shows a comparable result, but the velocity is significantly higher than the prior one. If we look thoroughly, we can see that the velocity increases mostly for Reynolds numbers greater than 4.

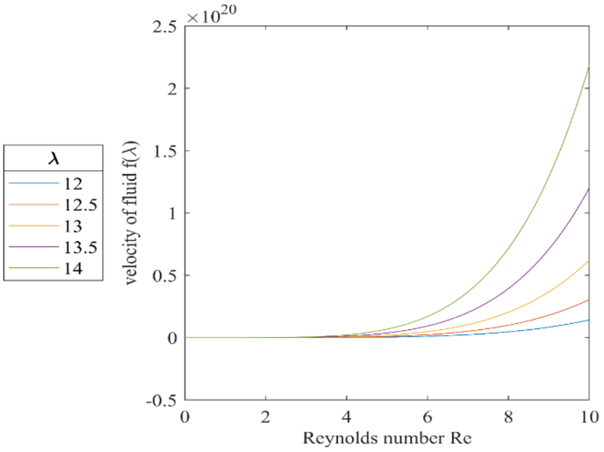

Fig. 15: Fluid Velocity Against Reynolds Numbers for Λ 12,12.5,13,13.5,14.

Figure 15 demonstrates that the velocity increases when the value of λ increases from 12 to 14 by 0.5. We can observe a uniform increase in velocity at the beginning. The velocity increases with rising Reynold number. Initially, the velocity for all λ increased with a same phase and steadily increased as Re increased.

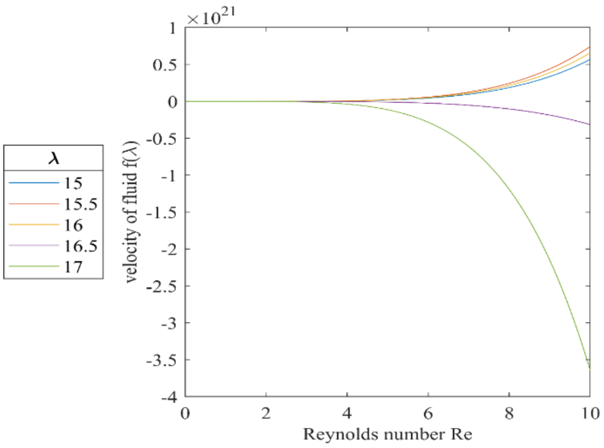

Fig. 16: Fluid Velocity Against Reynolds Numbers for Λ 15,15.5,16,16.5,17.

Figure 16 illustrates the velocity change when λ is increased from 15 to 17. The velocity was growing as λ increased, but as it reached 16, the velocity began to decrease. There is no velocity at 16.5 and 17, because the velocity cannot be negative. Thus, we can state that velocity will be positive or there will be velocity with the range of λ from.5 to 16. Further we shall investigate whether there is any velocity for λ values higher than 17.

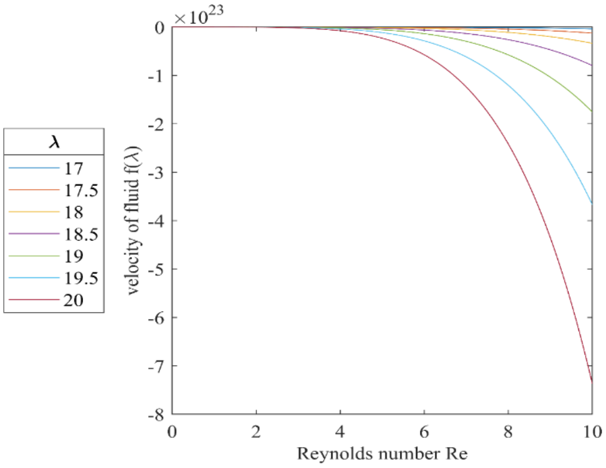

Fig. 17: Fluid Velocity Against Reynolds Numbers for Λ 17,17.5,18,18.5,19,19.5,20.

For our analysis, we used λ values greater than 17, and as expected, the results demonstrate a negative velocity as λ increases. The velocity decreases as the λ value increases. As a result, no additional graphical analysis is required because the velocity is going negative.

5. Conclusion

In this research we have used 5th term of perturbation method to solve an ordinary differential equation and the main aim is to analysis the laminar flow with viscous fluid between two parallel porous walls, with Reynolds numbers as the primary goals. We have used maple to obtain the required solution and used matlab for finding the graph. Various Reynold number is used ranging from .1 to 50 for the graphical representation, discussion and find the result. The result revealed that the velocity of the fluid is gradually increases as the Reynold number is increased. We also try to comprehend what will happen if we keep the Reynolds number constant and alter the dimensionless variable λ, i.e. vary the diameter of the hole in the wall. Figures 16 and 17 show that when λ is changed rather than Reynold number, the basic rule of viscosity is followed until a certain point. However, as the diameter of the hole decreases, the velocity of the fluid decreases, indicating that it does not follow the basic rule of viscosity. These will assist us in determining the dimension of the hole and the number of Reynolds numbers that will pass through the porous boundary wall, as well as their velocity. Applications for viscous fluid flow include geothermal energy extraction, plasma research, the petroleum industry, electrostatic precipitation, and technical and biological challenges involving accelerators and electrostatics.

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