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***Research paper***

# Investigation of exact traveling wave solution for the (2+1) dimensional nonlinear evolution equations via modified extended tanh-function method

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## Abstract

In this study, we have implemented the modified extended tanh-function method to obtain the exact travelling wave solutions for the general (2+1)-dimensional nonlinear evolution equations. By using this method, some travelling wave solutions are successfully obtained and which have been expressed by the trigonometric, hyperbolic and rational functions. These obtained solutions are an appropriate and desirable for instructive specific nonlinear physical phenomena in genuinely nonlinear dynamical systems. The method is an efficient and reliable mathematical tool for solving many nonlinear evolution equations arising in science and engineering problems.

***Keywords***: *Modified Extended Tanh-Function Method; Riccati Equation; The General (2+1) Dimensional Nonlinear Evolution Equations; Traveling Wave Solutions.*

## 1. Introduction

In recent years, nonlinear partial differential equations (NPDEs) is widely used to describe many important phenomena and dynamic processes in various fields of science and engineering, especially in fluid mechanics, hydrodynamics, mathematical biology, diffusion process, solid state physics, plasma physics, neural physics, chemical kinetics and geo-optical fibers. It's prominent that finding exact solutions of nonlinear evolution equations (NLEEs), by using different abundant method plays an important role in the proper understanding of mechanisms of the numerous physical phenomena in mathematical physics and become one of the furthermost exciting and awfully active areas of research investigation for mathematicians, physicist, and engineers.
On the basis of the finding new exact solutions of nonlinear evolution equations, many researchers [1-34] have devoted significant effort to study of exact explicit traveling and solitary wave solutions and several effective techniques have been proposed and developed such as the sine-cosine method [1-3], homogeneous balance method [4,5], auxiliary equation method [6,7], the tanh-function method [8], the extended tanh function method [9,10], the modified extended tanh-function method [11-13], the modified simple equation method [14-18], the $(G'/G)$-expansion method [19-23], the Exp-function method [24,25], the $\exp(-\varphi(\xi))$ expansion method [26-28], the F-expansion method [29-31], ansatz method [32-33] , the first integral method [ 34] and so on.
The extended tanh function method, which was developed by Wazwaz [9,10] is a direct and effective algebraic method for handling nonlinear equations and authors [11-12] have been applied the modified extended tanh-function method solving nonlinear partial differential equations.
The objective of this study is to apply the modified extended tanh-function method to find the exact traveling waves solutions of the generalized (2+1)-dimensional nonlinear evolution equation [35–37] in the form,

$$u_{xt}+au_xu_{xy}+bu_{xx}u_y+u_{xxxy}=0 \qquad (1)$$

where, a and b are arbitrary constants.
Recently, some special cases of Eq. (1) have been studied by several authors [18, 38-40]. When setting $a=4$ and $b=2$, Eq. (1) becomes the (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation:

$$u_{xt}+4u_xu_{xy}+2u_{xx}u_y+u_{xxxy}=0 \qquad (2)$$

When setting $a=-4$ and $b=-2$, Eq. (1) becomes the (2+1)-dimensional breaking soliton equation:

$$u_{xt}-4u_xu_{xy}-2u_{xx}u_y+u_{xxxy}=0 \qquad (3)$$

When setting $a=4$ and $b=4$, Eq. (1) becomes the (2+1)-dimensional Bogoyavlenskii's breaking soliton equation:

$$u_{xt}+4u_xu_{xy}+4u_{xx}u_y+u_{xxxy}=0 \qquad (4)$$

The rest of this paper is organized as follows: In section 2, the modified extended tanh-function method is discussed in details. In section 3, presents the application of this method to construct the exact traveling wave solutions of the nonlinear evolution equations and the section 4, we briefly make a conclusion to the results that have been obtained.

## 2. Methodology

In this section, we will describe the algorithm of the modified extended tanh-function method for finding traveling wave solutions of nonlinear evolution equations. Let us consider a general nonlinear PDE in the form

$$P(u,u_t,u_{tt},u_x,u_{xx},u_y,u_{yy},u_{xt},\ldots\ldots..), \tag{5}$$

Where, $u=u(x,y,t)$ is an unknown function, $P$ is a polynomial in $u(x,y,t)$ and its derivative in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. The main steps of this method are as follows:
Step 1: Combine the real variables $x$, $y$ and $t$ by a compound variable $\xi$

$$u(x,y,t)=u(\xi),\ \xi=x+y\pm Vt \tag{6}$$

where, $V$ is the speed of the traveling wave. The traveling wave transformation (6), converts Eq. (6) into an ordinary differential equation (ODE) for $u=u(\xi)$:

$$Q(u,u',u'',u''',\ldots\ldots\ldots\ldots..), \tag{7}$$

Where, $Q$ is a polynomial of $u$ and its derivatives and the superscripts indicate the ordinary derivatives with respect to $\xi$.
Step 2: Suppose the traveling wave solution of Eq. (7) can be expressed as follows:

$$u(\xi)=a_0+\sum_{i=1}^{i=n}\left(a_i\varphi^i+b_i\varphi^{-i}\right), \tag{8}$$

Where, the coefficients $a_i,b_i(1\le i\le n, n\in N)$ are constants to be determined and either $a_n$ or $b_n$ may be zero but both $a_n$ and $b_n$ cannot be zero simultaneously. The positive integer n can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Eq. (8). Moreover, we define the degree of $u(\xi)$ as $D(u(\xi))=n$, which gives rise to degree of another expression as follows:

$$D\left(\frac{d^q u}{d\xi^q}\right)=n+q\,,\ D\left(u^p\left(\frac{d^q u}{d\xi^q}\right)^s\right)=np+s(n+q)\,.$$

Therefore, we can find the value of n in Eq. (8), where $\varphi=\varphi(\xi)$ satisfies the following Riccati equation:

$$\varphi'(\xi)=\sigma+\varphi^2(\xi)\,, \tag{9}$$

where, $\sigma$ is a constant. Equation (9) admits several types of solutions according to the following:

Type-I (Hyperbolic function solution): If $\sigma<0$, then

$$\varphi(\xi)=-\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\xi\right)$$

or,

$$\varphi(\xi)=-\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\xi\right) \tag{10}$$

Type-II (Trigonometric function solution): If $\sigma>0$, then

$$\varphi(\xi)=\sqrt{\sigma}\tan\left(\sqrt{\sigma}\xi\right)$$

or,

$$\varphi(\xi)=-\sqrt{\sigma}\cot\left(\sqrt{\sigma}\xi\right) \tag{11}$$

Type-III (Rational function solution): If $\sigma=0$, then

$$\varphi(\xi)=-\frac{1}{\xi}\,, \tag{12}$$

Step 3: After we determine the index parameter n, we substitute Eq.(8) along Eq.(9) into Eq.(7) and collecting all the terms of the same power $\varphi^i$, $i=0,\pm1,\pm2,\ldots$ and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of $a_i$, $b_i$ and $V$. Substituting the values of $a_i$, $b_i$ and other values into Eq. (8) along with general solutions of Eq. (9) completes the determination of the solution of Eq. (7).

## 3. Application of the method

In this section, we implement the method described in Section 2 to find the exact traveling wave solutions of the (2+1) dimensional nonlinear evolution equation, Eq. (1).

### 3.1. The general (2+1) dimensional nonlinear evolution equation

We seek the exact traveling wave solution of the Eq. (1) using extended tanh-function method.
The traveling waves transformation

$$u(x,y,t)=u(\xi),\ \xi=x+y-Vt \tag{13}$$

Reduces Eq. (1) to the ODE of the form

$$-Vu''+(a+b)u'u''+u^{iv}=0 \tag{14}$$

Integrating once w.r.t. $\xi$ and setting the constant of integration to zero, yields

$$-Vu'+\left(\frac{a+b}{2}\right)(u')^2+u'''=0 \tag{15}$$

Where, primes denote differentiation with respect to $\xi$. By balancing the highest order derivative term $u'''$ with the nonlinear term $(u')^2$ in (15), gives $n=1$. Therefore, modified extended tanh-function method allows us to use the solution in the following form:

$$u(\xi)=a_0+a_1\varphi(\xi)+\frac{b_1}{\varphi(\xi)} \tag{16}$$

where, $a_0$ , $a_1$ and $b_1$ are constants that need to be determined such that $a_1 \neq 0$ or $b_1 \neq 0$ .

Now substituting Eq.(9), Eq.(16) and its derivative into Eq.(15), and collecting coefficients of $\varphi^i$ and equating them to zero, we obtain a system of algebraic equations for $a_0$ , $a_1$ , $b_1$ and respectively:

$$\frac{1}{2}a\sigma^2 b_1^2 + \frac{1}{2}b\sigma^2 b_1^2 - 6\sigma^3 b_1 = 0 \tag{17a}$$

$$-a\sigma^2 a_1 b_1 - b\sigma^2 a_1 b_1 + a\sigma b_1^2 + b\sigma b_1^2 + V\sigma b_1 - 8\sigma^2 b_1 = 0 \tag{17b}$$

$$\frac{1}{2}(a+b)\sigma^2 a_1^2 - 2(a+b)\sigma a_1 b_1 - V\sigma a_1 + \frac{1}{2}(a+b)b_1^2 + 2\sigma^2 a_1 + Vb_1 - 2\sigma b_1 = 0 \tag{17c}$$

$$a\sigma a_1^2 + b\sigma a_1^2 - aa_1 b_1 - ba_1 b_1 - Va_1 + 8\sigma a_1 = 0 \tag{17d}$$

$$\frac{1}{2}a a_1^2 + \frac{1}{2}ba_1^2 + 6a_1 = 0 \tag{17e}$$

Solving the obtained system of equations (17a-17e) by using Maple, the following sets of solutions are obtained:

Case-I: $V = -4\sigma,\ a_0 = a_0, a_1 = -\frac{12}{a+b}$ and $b_1 = 0$

Case-II: $V = -4\sigma,\ a_0 = a_0, a_1 = 0$ and $b_1 = \frac{12\sigma}{a+b}$

Case-III: $V = -16\sigma,\ a_0 = a_0, a_1 = -\frac{12}{a+b}$ and $b_1 = \frac{12\sigma}{a+b}$

Now substituting the values of $V$ , $a_0$ , $a_1$ and $b_1$ in the Eq. (16), then the general solution of the above cases is as follows:

For case-I: $u(\xi) = a_0 - \frac{12}{a+b}\varphi(\xi)$ , (18)

where $\xi = x + y + 4\sigma t$

For case-II: $u(\xi) = a_0 + \frac{12\sigma}{(a+b)\varphi(\xi)}$ , (19)

where $\xi = x + y + 4\sigma t$

For case-III: $u(\xi) = a_0 - \frac{12}{a+b}\varphi(\xi) + \frac{12\sigma}{(a+b)\varphi(\xi)}$ , (20)

where $\xi = x + y + 16\sigma t$

In case-I, we deduce the traveling wave solutions of Eq. (1) with the help of Eq. (10-12) and Eq. (18) is as follows.

$$u_1(x,y,t) = a_0 + \frac{12\sqrt{-\sigma}}{a+b}\tanh\left(\sqrt{-\sigma}\xi\right), \tag{21a}$$

or,

$$u_2(x,y,t) = a_0 + \frac{12\sqrt{-\sigma}}{a+b}\coth\left(\sqrt{-\sigma}\xi\right), \tag{21b}$$

$$u_3(x,y,t) = a_0 - \frac{12\sqrt{\sigma}}{a+b}\tan\left(\sqrt{\sigma}\xi\right), \tag{22a}$$

or,

$$u_4(x,y,t) = a_0 + \frac{12\sqrt{\sigma}}{a+b}\cot\left(\sqrt{\sigma}\xi\right), \tag{22b}$$

where $\xi = x + y + 4\sigma t$

$$u_5(x,y,t) = a_0 + \frac{12}{(a+b)\xi}, \tag{23}$$

where $\xi = x + y + 4\sigma t$

In case-II, we deduce the traveling wave solutions of Eq. (1) with the help of Eq. (10-12) and Eq. (19) is as follows.

$$u_6(x,y,t) = a_0 - \frac{12\sigma}{(a+b)\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\xi\right)}, \tag{24a}$$

or,

$$u_7(x,y,t) = a_0 - \frac{12\sigma}{(a+b)\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\xi\right)}, \tag{24b}$$

$$u_8(x,y,t) = a_0 + \frac{12\sqrt{\sigma}}{(a+b)\tan\left(\sqrt{\sigma}\xi\right)}, \tag{25a}$$

or,

$$u_9(x,y,t) = a_0 - \frac{12\sqrt{\sigma}}{(a+b)\cot\left(\sqrt{\sigma}\xi\right)}, \tag{25b}$$

where $\xi = x + y + 4\sigma t$

$$u_{10}(x,y,t) = a_0, \tag{26}$$

where $\xi = x + y + 4\sigma t$

In case-III, we deduce the traveling wave solutions of Eq. (1) with the help of Eq. (10-12) and Eq. (20) is as follows.

$$u_{11}(x,y,t) = a_0 + \frac{12\sqrt{-\sigma}}{a+b}\tanh\left(\sqrt{-\sigma}\xi\right) - \frac{12\sigma}{(a+b)\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\xi\right)} \tag{27a}$$

or,

$$u_{12}(x,y,t) = a_0 + \frac{12\sqrt{-\sigma}}{a+b}\coth\left(\sqrt{-\sigma}\xi\right) - \frac{12\sigma}{(a+b)\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\xi\right)} \quad (27b)$$

$$u_{13}(x,y,t) = a_0 - \frac{12\sqrt{\sigma}}{a+b}\tan\left(\sqrt{\sigma}\xi\right) + \frac{12\sqrt{\sigma}}{(a+b)\tan\left(\sqrt{\sigma}\xi\right)} \quad (28a)$$

or,

$$u_{14}(x,y,t) = a_0 + \frac{12\sqrt{\sigma}}{a+b}\cot\left(\sqrt{\sigma}\xi\right) - \frac{12\sqrt{\sigma}}{(a+b)\cot\left(\sqrt{\sigma}\xi\right)} \quad (28b)$$

where $\xi = x + y + 16\sigma t$

$$u_{15}(x,y,t) = a_0 + \frac{12}{(a+b)\xi}, \quad (29)$$

where $\xi = x + y + 16\sigma t$

### 3.1.1. (2+1) dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation

By using the section 3.1, setting $a = 4$ and $b = 2$ in the above Eqs. (21-29), then we explore the fifteen exact solutions of Eq. (2) which is indicated the symbol $u_1(x,y,t)$ - $u_{15}(x,y,t)$ . For special values of parameters, the shapes of traveling wave solutions are originated from the obtained exact solutions (see Figs. 1–4).

### 3.1.2. (2+1)-dimensional breaking soliton equation

By using the section 3.1, setting $a = -4$ and $b = -2$ in the above Eqs. (21-29), then we explore the fifteen exact solutions of Eq. (3) which is indicated the symbol $u_{16}(x,y,t)$ - $u_{30}(x,y,t)$ . For special values of parameters, the shapes of traveling wave solutions are originated from the obtained exact solutions (see Figs. 5–8).

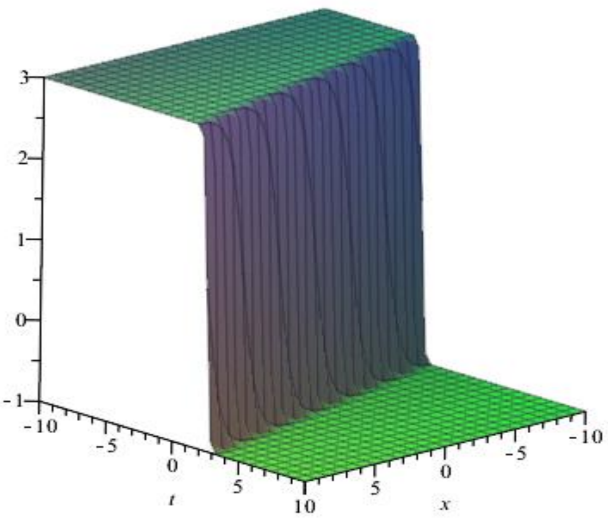


**Fig. 1:** 3D graphics (Kink profile) of $u_1(x,y,t)$ when $a_0 = 1, \sigma = -1$ , $y = 0$ and $-10 \le x,t \le 10$ .

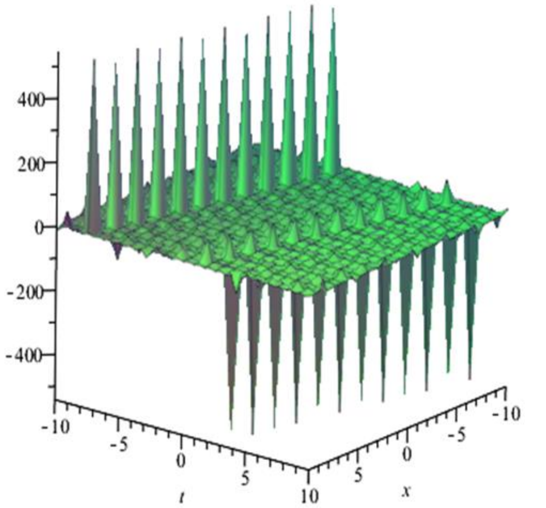


**Fig. 2:** 3D graphics (Periodic profile) of $u_3(x,y,t)$ when $a_0 = 1, \sigma = 1$ , $y = 0$ and $-10 \le x,t \le 10$ .

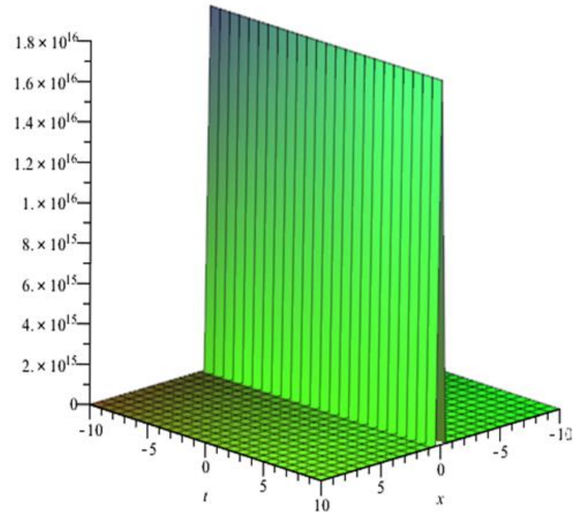


**Fig. 3:** 3D graphics (Singular cuspon profile) of $u_5(x,y,t)$ when $a_0 = 1, \sigma = 0$ , $y = 0$ and $-10 \le x,t \le 10$ .

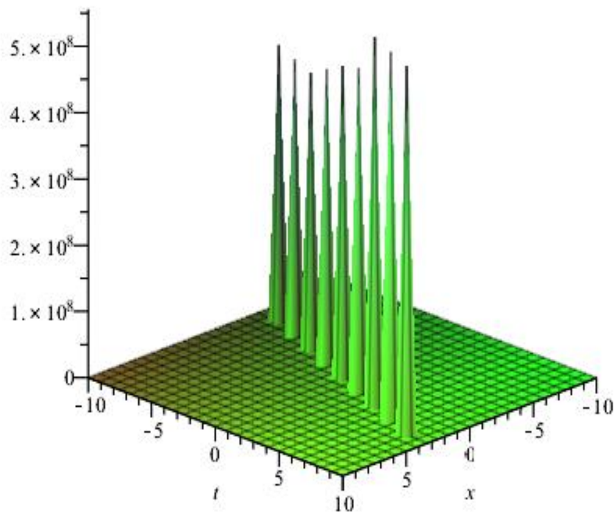


**Fig. 4:** 3D graphics (Periodic profile) of $u_{11}(x,y,t)$ when $a_0 = 1, \sigma = -0.1$ , $y = 0$ and $-10 \le x,t \le 10$ .

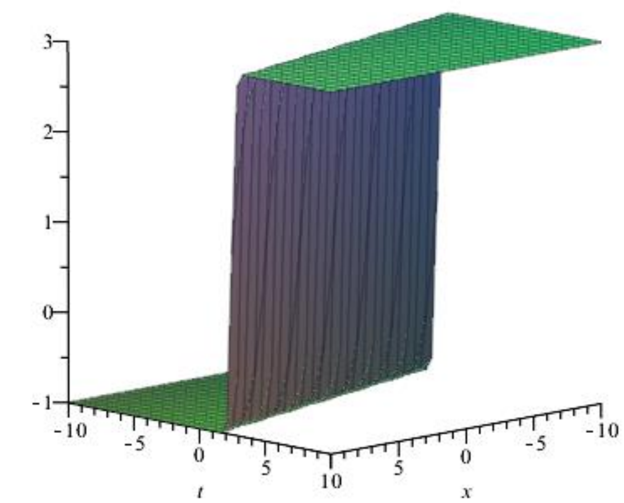


**Fig. 5:** 3D graphics (Kink profile) of $u_{16}(x,y,t)$ when $a_0 = 1, \sigma = -1$, $y = 0$ and $-10 \le x,t \le 10$.

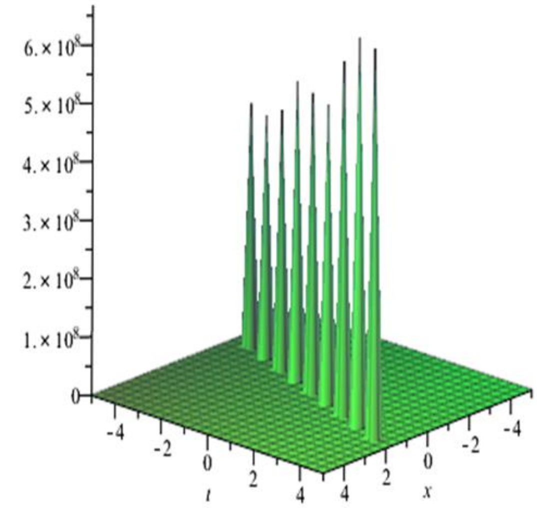


**Fig. 8:** 3D graphics (Periodic profile) of $u_{26}(x,y,t)$ when $a_0 = 1, \sigma = -0.1$, $y = 0$ and $-5 \le x,t \le 5$.

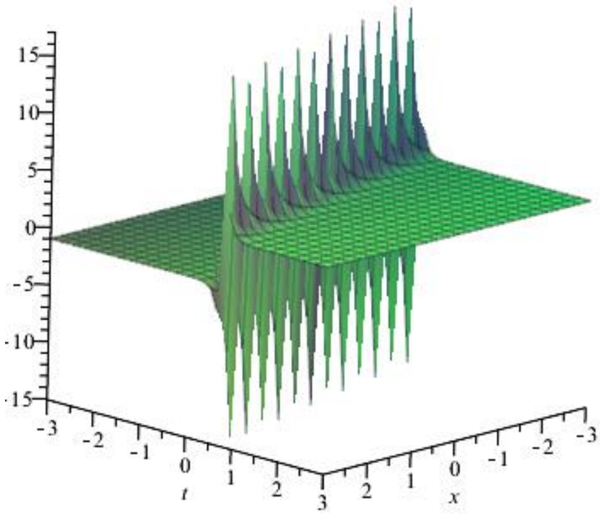


**Fig. 6:** 3D graphics (Singular kink profile) of $u_{17}(x,y,t)$ when $a_0 = 1, \sigma = -1$, $y = 0$ and $-3 \le x,t \le 3$.

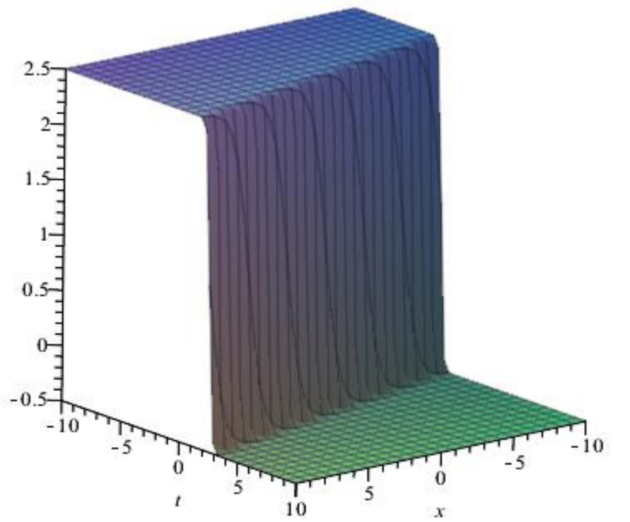


**Fig. 9:** 3D graphics (Kink profile ) of $u_{31}(x,y,t)$ when $a_0 = 1, \sigma = -1$, $y = 0$ and $-10 \le x,t \le 10$.

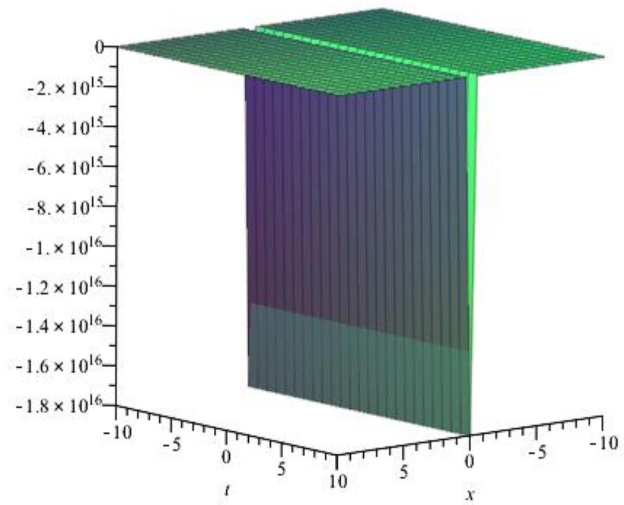


**Fig. 7:** 3D graphics (Singular cuspon profile) of $u_{20}(x,y,t)$ when $a_0 = 1, \sigma = 0$, $y = 0$ and $-10 \le x,t \le 10$.

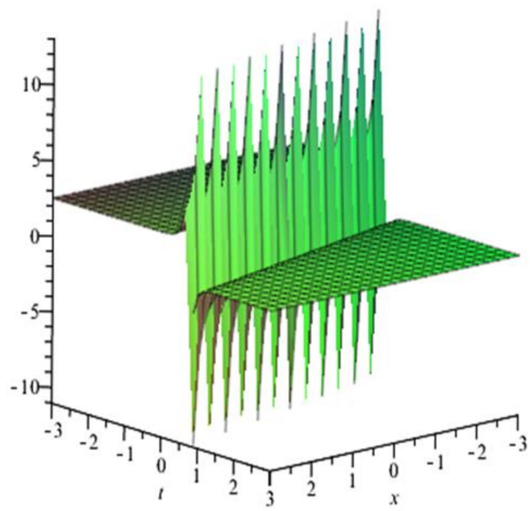


**Fig. 10:** 3D graphics (Singular kink profile) of $u_{32}(x,y,t)$ when $a_0 = 1, \sigma = -1$, $y = 0$ and $-3 \le x,t \le 3$.

#### 3.1.3. (2 + 1)-dimensional Bogoyavlenskii's breaking soliton equation

By using the section 3.1, setting $a = 4$ and $b = 4$ in the above Eqs. (21-29), then we explore the fifteen exact solutions of Eq. (4) which is indicated the symbol $u_{31}(x, y, t)$ - $u_{45}(x, y, t)$ . For special values of parameters, the shapes of traveling wave solutions are originated from the obtained exact solutions (see Figs. 8–12).

## 4. Conclusion

The modified extended tanh-function method has been successfully used to seek exact solutions of the general (2+1)-dimensional nonlinear evolution equations such as the (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation, the (2+1)-dimensional breaking soliton equation and the (2 + 1)-dimensional Bogoyavlenskii's breaking soliton equation. The performance of this method is reliable, simple and gives some new exact traveling wave solutions as well as solitons, kinks, and periodic solutions. We assure that the gained results will be helpful for further studies in mathematical physics and engineering.

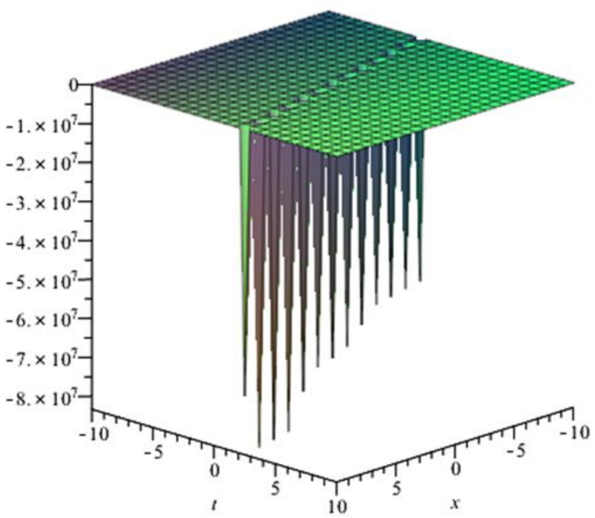


**Fig. 11:** 3D graphics (Kink profile) of $u_{41}(x, y, t)$ when $a_0 = 1, \sigma = -1$, $y = 0$ and $-10 \le x, t \le 10$ .

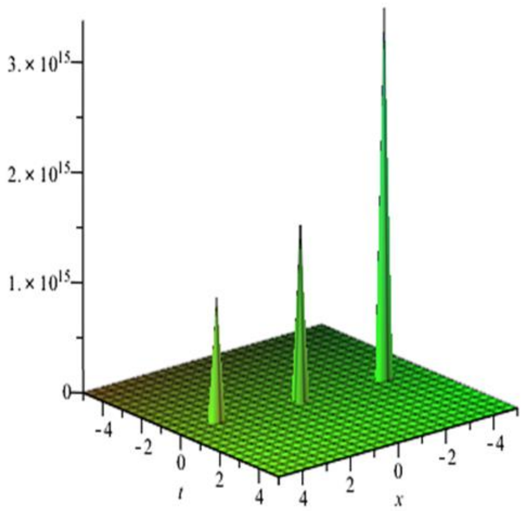


**Fig. 12:** 3D graphics (Periodic profile) of $u_{43}(x, y, t)$ when $a_0 = 1, \sigma = 1$ , $y = 0$ and $-5 \le x, t \le 5$ .

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