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Short communication

To unite nuclear and sub-nuclear strong interactions

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Abstract

With reference to 'reciprocal' of the strong coupling constant and 'reduced Compton's wavelength' of the nucleon, we make an attempt to understand the background of nuclear charge radius, binding energy and stability.

Keywords: *Strong Coupling Constant; Reduced Compton Wavelength of Nucleon; Beta Stability Line; Semi-Empirical Mass Formula (SEMF).*

1. Introduction

The modern theory of strong interaction is quantum chromo dynamics (QCD) [1]. It explores baryons and mesons in broad view with 6 quarks and 8 gluons. According to QCD, the four important properties of strong interaction are: 1) color charge; 2) confinement; 3) asymptotic freedom [2]; 4) short-range nature ($<10^{-15}$ m). Color charge is assumed to be responsible for the strong force to act on quarks via the force carrying agent, gluon. Experimentally it is well established that, strength of strong force depends on the energy through the interaction or the distance between particles. At lower energies or longer distances: a) color charge strength increase; b) strong force becomes 'stronger'; c) nucleons can be considered as fundamental nuclear particles and quarks seem to be strongly bound within the nucleons leading to 'Quark confinement'. At high energies or short distances: a) color charge strength decreases; b) strong force gets 'weaker';3) colliding protons generate 'scattered free quarks leading to 'Quark Asymptotic freedom'. Based on these points, low energy nuclear scientists assume 'strong interaction' as a strange nuclear interaction associated with binding of nucleons and implications and its implications were not considered. High-energy nuclear scientists consider nucleons as composite states of quarks and try to understand the nature and strength of strong interaction at sub nuclear level.

At this juncture, one important question to be answered and reviewed at the basic level is: How to understand nuclear interactions in terms of sub nuclear interactions? Unfortunately, the famous nuclear models like, Liquid drop model and Fermi's gas model [3-6] are lagging in answering this question. To find a way, we would like to suggest that, by implementing the 'strong coupling constant' $(\alpha_s \cong 0.1186)$, in low energy nuclear physics, nuclear binding energy and stability can be understood. In this new direction, we have developed interesting concepts and produced many semi empirical relations [7-9]. Even though it is in its budding stage, our model seems to be simple and realistic compared to the new integrated model proposed by N. Ghahramany et al [10-12]. It needs further study at a fundamental level.

2. Role of the strong coupling constant in low energy nuclear physics

We propose the following four assumptions.

1) Nuclear binding energy can be understood with a single energy coefficient associated with 'reciprocal' of the strong coupling constant.
2) Characteristic nuclear radius can be expressed as,
$$R_0 \cong \left(\frac{1}{\sqrt{\alpha_s}}\right)\left\{\frac{\hbar}{m_p c}+\frac{\hbar}{m_n c}\right\} \cong \left(\frac{1}{\sqrt{\alpha_s}}\right)\left(\frac{2\hbar}{m_p c}\right) \cong 1.22 \text{ fm.}$$
3) Characteristic nuclear binding energy potential can be,
$$B_0 \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0} \cong \left(\frac{\alpha}{2\sqrt{\alpha_s}}\right)m_p c^2 \cong 10.0 \text{ MeV} \quad \text{where}$$
$R_0 \approx 1.22$ fm.
4) (α_s) and $\exp\left(\frac{m_n - m_p}{m_e}\right) \cong 4\pi$ Seem to play a crucial role in understanding beta stability line and nuclear binding energy.

3. About the semi empirical mass formula

Let A be the total number of nucleons, Z the number of protons and N the number of neutrons. According to the semi-empirical mass formula (SEMF), nuclear binding energy:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{1}$$

Here a_v = volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus.

By maximizing $B(A,Z)$ with respect to Z, one can find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \approx \frac{A}{2+(a_c/2a_a)A^{2/3}} \text{ and } A-2Z \approx \frac{0.4A^2}{A+200} \quad (2)$$

By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound or most stable.

4. Beta stability line with respect to strong coupling constant

If $\alpha_s \cong 0.1186$, for Z >8, close to the line of beta stability, for Z >8,

$$A_s \cong \left(Z+\sqrt{\frac{1}{\alpha_s}}\right)^{6/5} \cong (Z+2.904)^{1.2} \quad (3)$$

$$\left.\begin{aligned} &\text{for } Z>16, \\ &(A_s-2Z) \cong (Z\beta+1)^2-4 \\ &\text{where } \beta = \left(\frac{3}{5}\right)\alpha_s. \end{aligned}\right\} \quad (4)$$

5. Beta stability line with respect to nucleon mass difference

With reference to nucleon and electron rest masses, we noticed that,

$$\exp\left(\frac{(m_n-m_p)c^2}{m_e c^2}\right) \cong 12.5659102 \cong 4\pi \quad (5)$$

$$\left\{\begin{aligned} &\text{where, } m_n c^2 \cong 939.565413 \text{ MeV}, \\ &m_p c^2 \cong 938.272081 \text{ MeV}; \; m_e c^2 \cong 0.5109989461 \text{ MeV} \end{aligned}\right.$$

Based on this observation, beta stability line can be understood with the following empirical relations.

$$\text{Let, } k \cong (1/4\pi)^2 \cong 0.006333 \quad (6)$$

$$\left.\begin{aligned} &\text{For } Z \geq 2, \\ &A_s \cong 2Z+(1/4\pi)^2 \cong 2Z+0.006333Z^2 \cong 2Z+kZ^2 \\ &N_s \cong Z+(1/4\pi)^2 \cong Z+0.006333Z^2 \cong Z+kZ^2 \\ &A_s-2Z \cong (1/4\pi)^2 \cong kZ^2 \text{ and } \frac{Z}{\sqrt{A_s-2Z}} \cong 4\pi \end{aligned}\right\} \quad (7)$$

Based on these relations,

$$\left.\begin{aligned} &\text{A) } \frac{(A_s-2Z)^2}{A_s} \cong k^2 A_s (A_s-Z)\sqrt{Z} \\ &\text{B) } \frac{A_s^{1/2}(A_s-Z)^{1/4}Z^{1/8}}{\sqrt{A_s-2Z}} \approx \frac{1}{\sqrt{k}} \approx 4\pi \end{aligned}\right\} \quad (8)$$

6. Relations for understanding nuclear binding energy

Based on the new integrated model proposed by N. Ghahramany et al [10-12] and with reference to relation (7), it is possible to show that, for $Z \cong (40 \text{ to } 83)$, close to the beta stability line,

$$\left[\frac{N_s^2-Z^2}{Z}\right] \cong kZA_s \quad (9)$$

Based on this strange and simple relation and with reference to our recent publications [8], [9], close to the beta stability line, numerically it is possible to show that, for $(Z \geq 24)$, if $\alpha_s \approx 0.1186$ and $R_0 \approx 1.22$fm,

$$\left.\begin{aligned} (B)_A &\cong \left[A - A^{1/3} - \frac{kA\sqrt{NZ}}{3.4}\right] \times 10.0 \text{ MeV} \\ &\cong \left\{A - A^{1/3} - \left[(\alpha_s)\left(\frac{a_c}{2a_a}\right)\left(A\sqrt{NZ}\right)\right]\right\} \times 10.0 \text{ MeV} \end{aligned}\right\} \quad (10)$$

where, $\alpha_s\left(\frac{a_c}{2a_a}\right) \cong \frac{k}{3.4}$. It is for further study.

For $(Z \geq 24)$, binding energy per nucleon can be expressed as:

$$\frac{(B)_A}{A} \cong \left[1 - A^{-2/3} - \frac{k\sqrt{NZ}}{3.4}\right] \times 10.0 \text{ MeV} \quad (11)$$

a) See table 1 for nuclear binding energy of stable and unstable isotopes of Z=23 to 100 estimated from relations (7) and (10).
b) See table 2 for nuclear binding energy of natural isotopes of Z=30, 40, 50, 60, 70, 80 and 92 estimated from relation (10).
c) See table 3 for nuclear binding energy of natural isotopes of Z=25 to 83 estimated from relation (10).

Table 1: Nuclear Binding Energy of Z = 23 To 100 Estimated From Relations (7) and (10)

Proton number	Mass number	Neutron number	Estimated binding energy(MeV)	Actual[10] or reference [4] binding energy (MeV)	Error (MeV)
23	49	26	431.1	426.34	-4.7
24	52	28	457.6	456.349	-1.2
25	54	29	475.1	472.33	-2.8
26	56	30	492.6	492.258	-0.4
27	59	32	518.8	517.313	-1.5
28	61	33	536.1	534.666	-1.4
29	63	34	553.4	551.385	-2.0
30	66	36	579.2	578.136	-1.1
31	68	37	596.3	590.61	-5.7
32	70	38	613.3	610.521	-2.8
33	73	40	638.8	634.34	-4.5
34	75	41	655.7	651.02	-4.7
35	78	43	680.9	676.11	-4.8
36	80	44	697.6	695.434	-2.2

37	83	46	722.6	720.46	-2.1
38	85	47	739.1	737.85	-1.3
39	88	49	763.9	763.88	0.0
40	90	50	780.2	783.893	3.7
41	93	52	804.7	805.765	1.1
42	95	53	820.9	821.625	0.7
43	98	55	845.1	844.4	-0.7
44	100	56	861.1	861.928	0.8
45	103	58	885.1	884.163	-0.9
46	105	59	900.9	899.914	-1.0
47	108	61	924.7	922.2	-2.5
48	111	63	948.2	947.622	-0.6
49	113	64	963.8	963.094	-0.7
50	116	66	987.1	988.684	1.6
51	118	67	1002.5	1000.48	-2.0
52	121	69	1025.5	1024.43	-1.1
53	124	71	1048.5	1046.32	-2.1
54	126	72	1063.5	1063.909	0.4
55	129	74	1086.2	1085.08	-1.1
56	132	76	1108.7	1110.038	1.4
57	135	78	1131.0	1131.00	0.0
58	137	79	1145.7	1145.7	0.0
59	140	81	1167.8	1168.67	0.9
60	143	83	1189.7	1191.266	1.5
61	146	85	1211.5	1209.52	-2.0
62	148	86	1225.8	1225.392	-0.4
63	151	88	1247.3	1244.141	-3.2
64	154	90	1268.7	1266.627	-2.1
65	157	92	1289.9	1287.38	-2.5
66	160	94	1311.0	1309.455	-1.5
67	162	95	1324.7	1321.18	-3.6
68	165	97	1345.5	1343.08	-2.5
69	168	99	1366.2	1363.31	-2.9
70	171	101	1386.7	1384.744	-1.9
71	174	103	1407.0	1404.44	-2.6
72	177	105	1427.2	1425.185	-2.0
73	180	107	1447.2	1444.663	-2.6
74	183	109	1467.1	1465.525	-1.6
75	186	111	1486.8	1484.63	-2.2
76	189	113	1506.4	1505.007	-1.4
77	192	115	1525.8	1523.81	-2.0
78	195	117	1545.0	1545.682	0.7
79	198	119	1564.1	1564.94	0.8
80	201	121	1583.1	1587.411	4.3
81	204	123	1601.9	1606.87	5.0
82	207	125	1620.5	1629.063	8.6
83	210	127	1639.0	1643.94	5.0
84	213	129	1657.3	1659.72	2.4
85	216	131	1675.4	1673.42	-2.0
86	219	133	1693.5	1690.59	-2.9
87	222	135	1711.3	1706.49	-4.8
88	225	137	1729.0	1724.18	-4.8
89	228	139	1746.6	1740.67	-5.9
90	231	141	1763.9	1759.14	-4.8
91	234	143	1781.2	1776.08	-5.1
92	238	146	1804.2	1801.69	-2.6
93	241	148	1821.1	1817.31	-3.8
94	244	150	1837.8	1835.45	-2.4
95	247	152	1854.4	1851.73	-2.7
96	250	154	1870.8	1868.97	-1.8
97	254	157	1892.8	1888.79	-4.0
98	257	159	1908.9	1906.19	-2.7
99	260	161	1924.8	1922.2	-2.6
100	263	163	1940.5	1939.52	-1.0

Table 2: Nuclear Binding Energy of Natural Isotopes of Z =30, 40, 50, 60, 70, 80, 92 Estimated from Relation (10)

Proton number	Mass number	Neutron number	Estimated binding energy (MeV)	Actual [10] binding energy (MeV)	Error (MeV)
30	64	34	561.9	559.098	-2.83
30	66	36	579.2	578.136	-1.05
30	67	37	587.8	585.189	-2.62
30	68	38	596.4	595.387	-1.03
30	70	40	613.6	611.087	-2.53
40	90	50	780.2	783.893	3.68
40	91	51	788.5	791.087	2.62
40	92	52	796.7	799.722	3.02
40	94	54	813.2	814.677	1.52
40	96	56	829.6	828.996	-0.59
50	112	62	955.6	953.532	-2.11
50	114	64	971.4	971.574	0.18

50	115	65	979.3	979.121	-0.13
50	116	66	987.1	988.684	1.58
50	117	67	995.0	995.627	0.67
50	118	68	1002.8	1004.955	2.16
50	119	69	1010.6	1011.438	0.82
50	120	70	1018.4	1020.546	2.11
50	122	72	1034.1	1035.53	1.47
50	124	74	1049.6	1049.963	0.32
60	142	82	1182.3	1185.142	2.84
60	143	83	1189.7	1191.266	1.53
60	144	84	1197.2	1199.083	1.92
60	145	85	1204.6	1204.838	0.25
60	146	86	1212.0	1212.403	0.41
60	148	88	1226.8	1225.028	-1.76
60	150	90	1241.6	1237.448	-4.11
70	168	98	1365.6	1362.793	-2.85
70	170	100	1379.7	1378.13	-1.55
70	171	101	1386.7	1384.744	-1.94
70	172	102	1393.7	1392.764	-0.91
70	173	103	1400.7	1399.131	-1.53
70	174	104	1407.6	1406.595	-1.05
70	176	106	1421.6	1419.283	-2.29
80	196	116	1550.2	1551.218	1.00
80	198	118	1563.4	1566.489	3.10
80	199	119	1570.0	1573.153	3.20
80	200	120	1576.5	1581.181	4.66
80	201	121	1583.1	1587.411	4.34
80	202	122	1589.6	1595.165	5.55
80	204	124	1602.7	1608.652	5.98
92	234	142	1780.2	1778.567	-1.63
92	235	143	1786.2	1783.864	-2.36
92	238	146	1804.2	1801.69	-2.56

Table 3: Nuclear Binding Energy of Natural Isotopes of Z = 25 To83 Estimated from Relation (10)

Proton number	Mass number	Neutron number	Estimated binding energy (MeV)	Actual [10] binding energy (MeV)	Error (MeV)
25	55	30	483.9	482.075	-1.84
27	59	32	518.8	517.313	-1.45
29	65	36	570.7	569.212	-1.46
31	69	38	604.9	601.996	-2.88
33	75	42	655.8	652.564	-3.26
35	79	44	689.3	686.321	-3.03
37	87	50	756.0	757.856	1.87
39	89	50	772.1	775.538	3.39
41	93	52	804.7	805.765	1.06
45	103	58	885.1	884.163	-0.95
47	109	62	932.6	931.727	-0.91
49	115	66	979.6	979.404	-0.15
51	123	72	1041.4	1042.097	0.66
53	127	74	1071.6	1072.577	0.99
55	133	78	1116.7	1118.528	1.83
57	139	82	1161.2	1164.551	3.36
59	141	82	1175.3	1177.919	2.64
63	153	90	1261.9	1258.998	-2.93
65	159	94	1304.3	1302.027	-2.30
67	165	98	1346.1	1344.256	-1.86
69	169	100	1373.2	1371.352	-1.88
71	175	104	1414.0	1412.106	-1.86
73	181	108	1454.1	1452.24	-1.84
75	187	112	1493.6	1491.877	-1.70
77	193	116	1532.5	1532.058	-0.40
79	197	118	1557.5	1559.386	1.86
81	203	122	1595.3	1600.87	5.52
83	209	126	1632.5	1640.23	7.68

With an error bar of 5 MeV, to some extent, relation (10) can be applied to light atomic nuclides and for Z = 3 to 100, modified relation can be expressed as,

$$(B)_A \cong \left[A - A^{1/3} - \frac{kA\sqrt{NZ}}{3.35} - 1\right] \times 10.09 \text{ MeV} \tag{12}$$

Estimated data can be compared with first four terms of the semi empirical mass formula.

7. Discussion

a) By considering coulombic repulsions and with reference to our earlier publications [7], [8], close to the beta stability line, binding energy can also be expressed with: for $(Z \geq 24)$,

$$(B)_A \cong \left[A - kN\left(\frac{kAA_s}{\sqrt{Z}} + 1\right)\right] \times 8.9 \text{ MeV} \tag{13}$$

where $\begin{cases} R_0 \cong 1.22 \text{ fm and} \\ \left\{\left[\left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right]-\left[\frac{e^2}{4\pi\varepsilon_0 R_0}\right]\right\} \cong 8.9 \text{ MeV} \end{cases}$

b) By fine tuning the values of $(\alpha_s \text{ and } R_0)$ and by considering even-odd corrections and shell corrections, accuracy can be improved.

c) With a suitable mathematical relation and with a single energy coefficient lying in between (8.9 to 10) MeV, it is possible to estimate nuclear binding energy.

d) Close to the beta stability line, ignoring the pairing energy term, semi empirical mass formula energy coefficients can be expressed as: $a_v \approx a_s \approx a_a \approx \frac{3}{2}\times 10.0 \text{ MeV} \approx 15.0 \text{ MeV}$ and $a_c \approx 0.73 \text{ MeV}$.

8. Conclusion

Understanding and estimating nuclear binding energy and stability with 'sub-nuclear strong interaction' seems to be quite interesting and needs a serious consideration at a fundamental level. We believe that, results obtained from above relations are simple to understand and seem to be more physical and relatively closer to the experimental data. With further research, current nuclear models and strong interaction concepts, pertaining to high-energy physics can be studied in a unified manner, and a realistic nuclear model can be developed.

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