International Journal of Scientific World, 2 (1) (2014) 1-9
©Science Publishing Corporation
www.sciencepubco.com/index.php/IJSW
doi: 10.14419/ijsw.v2i1.1780
Research Paper

Statistical inference for generalized Pareto distribution based on progressive Type-II censored data with random removals

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Abstract

In this article, We consider the estimation problems of the parameter and reliability function of the generalized Pareto distribution based on a progressively type-II censored sample with random (Binomial) removals. we use the method of maximum likelihood and Bayesian estimation to estimate parameter and reliability function. Bayesian estimates are derived under squared error and LINEX loss functions. we also construct the confidence interval for the parameter of generalized Pareto distribution based on a progressively type-II censored sample with random removals. The comparisons between different estimators are made based on simulation study.

Keywords: *Generalized Pareto distribution, progressive Type-II censored, random removals, Bayesian estimates, reliability function*

1. Introduction

In various life-testing and reliability studies, experiments must often terminate before all units on test have failed. In such cases, one has complete information only on part of the sample. On all units which have not failed, one has only partial information. Such data are called censored. There are several types of censored tests. One of the most common censored tests is progressive type II censoring. In progressive type II censoring, Suppose that n units are placed on a life test and the experimenter decides beforehand quantity m, the number of units, to be failed. Now at the time of the first failure, R_1 of the remaining $n-1$ surviving units are randomly removed from the experiment. Continuing on, at the time of the second failure, R_2 of the remaining $n-R_1-2$ units are randomly drawn from the experiment. Finally, at the time of the mth failure, all the remaining $R_m = n-m-R_1-R_2-...-R_{m-1}$ surviving units are removed from the experiment. Note that, in this scheme, $R_1, R_2, ..., R_m$ are all pre-fixed. However, in some practical situations, these numbers may occur at random. for example, in some reliability experiments, an experimenter may decide that it is inappropriate or too dangerous to carry on the testing on some of the tested units even though these units have not failed. In such cases, the pattern of removal at each failure is random (Zeinab [3], Yen and Tse [2]). This leads to progressive censoring with random removals (illustrated by "Table 1").

There have been several references about the statistical inference on lifetime distributions under progressive censoring with random removals; for example, we refer to Yuen and Tse [2], Tse and Yuen [5], Tse et al. [10], Shuo and Tao [12], Wu [11], Wu and Chang [4] and Wu et al. [9].

Based on a progressively type-II censored sample with random removals, we consider the problem of estimating

Table 1: A schematic representation of the progressive type-II censoring with binomial removals

Process	The number in life testing	Failures	Binomial Removals	Remains
1	n	1	$R_1 \sim B(n-m,p)$	$n-1-R_1$
2	$n-1-R_1$	1	$R_2 \sim B(n-m-R_1,p)$	$n-2-R_1-R_2$
...	...	...	...	...
$m-1$	$n-(m-2)-\sum_{j=1}^{m-2} R_j$	1	$R_{m-1} \sim B(n-m-\sum_{j=1}^{m-2} R_j,p)$	$n-(m-1)-\sum_{j=1}^{m-1} R_j$
m	$n-(m-1)-\sum_{j=1}^{m-1} R_j$	1	$R_m = n-m-\sum_{j=1}^{m-1} R_j$	0

parameter and reliability function of the two parameter generalized Pareto distribution with the shape parameter θ and the scale parameter σ, proposed by Castillo and Hadi [7]. under both classical and Bayesian (with different loss functions) contexts.

The cumulative distribution function, probability density and reliability functions of two parameter generalized Pareto distribution are respectively given by

$$F(x|\sigma,\theta) = 1 - \left(1-\frac{x}{\sigma}\right)^{\frac{1}{\theta}}; \theta > 0, 0 < x < \sigma \tag{1}$$

and

$$f(x|\sigma,\theta) = \frac{1}{\theta\sigma}\left(1-\frac{x}{\sigma}\right)^{\frac{1}{\theta}-1}; \theta > 0, 0 < x < \sigma \tag{2}$$

and

$$R(x|\sigma,\theta) = \left(1-\frac{x}{\sigma}\right)^{\frac{1}{\theta}}; \theta > 0, 0 < x < \sigma$$

for more detail about two parameter generalized Pareto distribution see Castillo and Hadi [7]. two parameter generalized Pareto distribution were widely used by several authors, Among others, we refer to Grimshaw [6], Castillo and Hadi [7], and Castillo et al. [8].

2. Maximum likelihood estimation

Let $X_1 < X_2 < ... < X_m$ be the ordered failure times out of n randomly selected times, where m is predetermined before the test. At the ith failure, R_i items are removed from the test. For progressive censoring with pre-determined number of removals $R = (R_1 = r_i, ..., R_{m-1}) = r_{m-1}$, the likelihood function can be defined as the following form

$$L(\sigma,\theta,x|R) = c\prod_{i=1}^{m} f(x_i|\sigma,\theta)[1-F(x_i|\sigma,\theta)]^{r_i} \tag{3}$$

where $c = n(n-1-R_1)\cdots\left(n-\sum_{i=1}^{m-1}(R_i+1)\right)$. Equation (3) is derived conditional on R_i. Each R_i can be of any integer value between 0 and $n-m-\sum_{j=1}^{i-1}(R_j)$. It is different from progressive censoring with fixed removal that R_i is a random number and is assumed to follow a binomial distribution with parameter p. It means that each unit leaves with equal probability p and the probability of R_i units leaving after the ith failure occurs is

$$P(R_1 = r_1) = p^{r_1}(1-p)^{n-m-r_1}$$

$$P(R_i = r_i|R_{i-1} = r_{i-1}, ..., R_1 = r_1) = (n-m-\sum_{j=1}^{i-1} r_j)p^{r_i}(1-p)^{n-m-\sum_{j=1}^{m-1} r_j}$$

where $0 \le r_i \le n - m - \sum_{j=1}^{i-1} r_j (i = 1, ..., m-1)$. Furthermore, we assume that R_i is independent of X_i for all i. The joint likelihood function of $X = (X_1, X_2, ..., X_m)$ and $R = (r_1, r_2, ..., r_m)$ can be found as

$$L(\sigma, \theta, x, p) = L(\sigma, \theta, x|R)P(R, p) \tag{4}$$

where $P(R, p)$ is the joint probability distribution of $R = (r_1, r_2, ..., r_m)$ and in particular

$$P(R, p) = P(R_m = r_m | R_{m-1} = r_{m-1}, ..., R_1 = r_1) \times ...$$
$$\times P(R_2 = r_2 | R_1 = r_1) P(R_1 = r_1)$$

Therefore

$$P(R, p) = \frac{(n-m)!}{(n - m - \sum_{j=1}^{m-1} r_j)! \prod_{j=1}^{m-1} r_j} p^{\sum_{j=1}^{m-1} r_j} (1-p)^{(m-1)(n-m) - \sum_{j=1}^{m-1}(m-j)r_j}$$

Substituting (1) and (2) into (4), the likelihood function takes the following form

$$L(\sigma, \theta, x, p) \propto \sigma^{-m} \theta^{-m} \exp\left\{ \frac{1}{\theta} \sum_{i=1}^{m} (r_i + 1) \log\left(1 - \frac{x_i}{\sigma}\right) \right\}$$

$$\times \exp\left\{ -\sum_{i=1}^{m} \log\left(1 - \frac{x_i}{\sigma}\right) \right\} p^{\sum_{j=1}^{m-1} r_j} (1-p)^{(m-1)(n-m) - \sum_{j=1}^{m-1}(m-j)r_j} I_{(0,\sigma)}(x_i) \tag{5}$$

The first partial derivatives of log-likelihood function with respect to θ is

$$\frac{\partial \log L(\sigma, \theta, x, p)}{\partial \theta} = -\frac{m}{\theta} - \frac{\sum_{i=1}^{m} (r_i + 1) \log\left(1 - \frac{x_i}{\sigma}\right)}{\theta^2} = 0$$

therefor we get the MLE of θ as in the following form

$$\hat{\theta} = -\frac{\sum_{i=1}^{m} (r_i + 1) \log\left(1 - \frac{x_i}{\sigma}\right)}{m} \tag{6}$$

By the invariant property of the MLE, the MLE of the reliability function, $S = R(t)$, with fixed $t > 0$. is given by

$$\hat{S}_{MLE} = \left(1 - \frac{t}{\sigma}\right)^{\frac{1}{\hat{\theta}_{MLE}}} \tag{7}$$

3. Exact interval estimation

Let $X_1 < X_2 < \cdots < X_m$ be a progressively type II censored sample from the generalized Pareto distribution . Furthermore, let $Y_i = -\frac{1}{\theta} \ln\left(1 - \frac{x_i}{\sigma}\right)$, $i = 1, ..., m$. It is easy to show that $Y_1, \cdots, Y_m$ is a progressively Type II censored sample from the standard exponential distribution. For a fixed set of $R = (r_1, ..., r_m)$, let us consider the following transformation:

$$Z_1 = nY_1$$

$$Z_2 = (n - r_1 - 1)(Y_2 - Y_1)$$

$$\vdots$$

$$Z_m = (n - r_1 - \cdots - r_m - m + 1)(Y_m - Y_{m-1}) \tag{8}$$

Balakrishnan and Aggarwala [1] showed that the progressively type II right censored spacings $Z_1, Z_2, ..., Z_m$ as defined in equation (8), are independent and identically distributed as a standard exponential distribution. Hence, $2Z_1$ has a chi-square distribution with 2 degrees of freedom. Now, let

$$W = 2\sum_{i=1}^{m} Z_i = -\frac{2}{\theta}\sum_{i=1}^{m}(r_i + 1)\log\left(1 - \frac{x_i}{\sigma}\right) = \frac{2m\hat{\theta}}{\theta}$$

It is easy to see that W has a χ^2 distribution with $2m$ degrees of freedom. Confidence interval for θ can be obtained through the pivotal quantity $\frac{2m\hat{\theta}}{\theta} \sim \chi^2_{(2m)}$. Since the pivotal quantity $\frac{2m\hat{\theta}}{\theta} \sim \chi^2_{(2m)}$, then we have

$$1 - \alpha = P\left(\chi^2_{1-\frac{\alpha}{2}}(2m) < \frac{2m\hat{\theta}}{\theta} < \chi^2_{\frac{\alpha}{2}}(2m)\right)$$

$$= P\left(\frac{2m\hat{\theta}}{\chi^2_{1-\frac{\alpha}{2}}(2m)} < \theta < \frac{2m\hat{\theta}}{\chi^2_{\frac{\alpha}{2}}(2m)}\right)$$

$$= P\left(\frac{-2\sum_{i=1}^{m}(r_i + 1)\log\left(1 - \frac{x_i}{\sigma}\right)}{\chi^2_{1-\frac{\alpha}{2}}(2m)} < \theta < \frac{-2\sum_{i=1}^{m}(r_i + 1)\log\left(1 - \frac{x_i}{\sigma}\right)}{\chi^2_{\frac{\alpha}{2}}(2m)}\right) \tag{9}$$

4. Bayesian estimation

In the following, we present Bayes estimators of the shape parameter and reliability function of generalized Pareto distribution when samples are drawn from progressively Type-II censoring data with binomial removals For this purpose we assume the parameters θ and p behave as independent random variables. In this paper, for parameter θ we consider inverted-gamma prior distribution of the form

$$\pi_1(\theta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\theta^{-(\alpha+1)}e^{-\left(\frac{\beta}{\theta}\right)}; (\alpha, \beta) > 0$$

Independently from parameter θ, p has a beta prior distribution with parameters a and b of the form

$$\pi_2(p) = \frac{1}{B(a, b)}p^{a-1}(1 - p)^{b-1}, 0 < p < 1; \quad (a, b) > 0$$

Based on the prior $\pi_1(\theta)$ and $\pi_2(p)$, the joint prior PDF of (θ, p) is

$$\pi(\theta, p) = \pi_1(\theta)\pi_2(p), \theta > 0, \quad 0 < p < 1$$

$$= \frac{\beta^{\alpha}}{\Gamma(\alpha)B(a, b)}p^{a-1}(1 - p)^{b-1}\theta^{-(\alpha+1)}e^{-\left(\frac{\beta}{\theta}\right)}; \theta > 0, \quad 0 < p < 1 \tag{10}$$

It follows, from (3) and(10), that the joint posterior distribution of (θ, p) is

$$\pi(\theta, p|\mathbf{x}, \mathbf{r}) = \frac{\beta^{*\alpha^*}}{\Gamma(\alpha^*)B(a^*, b^*)}\theta^{-(\alpha^*+1)}e^{-\frac{\beta^*}{\theta}}p^{a^*-1}(1 - p)^{b^*-1} \tag{11}$$

Where $\alpha^* = m + \alpha$, $\beta^* = \left(\beta - \sum_{i=1}^{m}(r_i + 1)\log\left(1 - \frac{x_i}{\sigma}\right)\right)$,$a^* = a + \sum_{j=1}^{m-1} r_j$ $b^* = b + (m-1)(n-m)\sum_{j=1}^{m-1}(m-j)r_j$ Therefore, the marginal posterior PDFs of θ and p are given respectively by

$$\pi(\theta|\mathbf{x}, \mathbf{r}) = \frac{\beta^{*\alpha^*}}{\Gamma(\alpha^*)}\theta^{-(\alpha^*+1)}e^{-\frac{\beta^*}{\theta}} \tag{12}$$

and

$$\pi(p|\mathbf{x}, \mathbf{r}) = \frac{1}{B(a^*, b^*)}p^{a^*-1}(1 - p)^{b^*-1}$$

4.1. Bayesian Estimation Under squared error loss function

Under SE loss function (symmetric), the estimator of a parameters is the posterior mean. Thus, Bayes estimators of the parameter θ is obtained by using the posterior density (12)

$$\hat{\theta}_S = E(\theta|\mathbf{x},\mathbf{r}) = \frac{\beta - \sum_{i=1}^{m}(r_i+1)\log\left(1-\frac{x_i}{\sigma}\right)}{m+\alpha-1} \tag{13}$$

4.2. Bayesian Estimation under LINEX loss function

The LINEX loss function for θ can be expressed as the following proportional(see Basu and Ebrahimi [13])

$$L(\Delta) \propto \exp(c\Delta) - c\Delta - 1; \qquad c \neq 0$$

where $\Delta = \frac{\hat{\theta}-\theta}{\theta}$ and $\hat{\theta}$ is an estimate of θ. The Bayes estimator of θ, denoted by $\hat{\theta_L}$ under the LINEX loss function is the solution of the following equation.

$$E\left[\frac{1}{\theta}\exp\left(\frac{c\hat{\theta}_L}{\theta}\right)|\mathbf{x},\mathbf{r}\right] = e^{c}E\left[\frac{1}{\theta}|\mathbf{x},\mathbf{r}\right]$$

Therefore we have

$$\hat{\theta}_L = \frac{\beta - \sum_{i=1}^{m}(r_i+1)\log\left(1-\frac{x_i}{\sigma}\right)}{c}\left(1-e^{-\frac{c}{\alpha+m+1}}\right) \tag{14}$$

5. Bayesian estimation of reliability function $S = R(t)$

Other problems of interest are those of estimating the reliability function $R(t)$, with fixed $t > 0$. Let the reliability $S = R(t)$ be a parameter itself. replacing $\theta = \frac{\ln\frac{\sigma}{\sigma-t}}{-\ln S}$ in terms of S by that of equation (12), we obtain the posterior density function S as

$$\pi\left(S|\mathbf{X}\right) = \frac{\nu(x_i,t)^{\alpha^*}}{\Gamma(\alpha^*)}\left(-\ln s\right)^{\alpha^*-1} s^{\nu(x_i,t)-1} \tag{15}$$

where,

$$\nu(x_i,t) = \frac{\beta^*}{\ln\frac{\sigma}{\sigma-t}}$$

By using posterior density function S (15), the Bayes estimate of the $S = R(t)$ relative to quadratic loss is

$$\hat{S}_S = \left(\frac{\nu(x_i,t)}{\nu(x_i,t)+1}\right)^{\alpha^*} \tag{16}$$

Under LINEX loss function, the Bayes estimate of $S = R(t)$ using equation (15) is

$$\hat{S}_L = -\frac{1}{c}\ln\left[\sum_{l=0}^{\infty}\frac{(-c)^l}{l!}\left(\frac{\nu(x_i,t)}{\nu(x_i,t)+l}\right)^{\alpha^*}\right] \tag{17}$$

6. Numerical study

In this section, a Monte Carlo simulation study is conducted with various choices of sample sizes.We firstly generate the numbers of progressive censoring with binomial removals $r_i (i = 1, 2, ..., m)$, and progressive censoring with binomial removals samples generated from generalized Pareto distribution by using the algorithm described in Balakrishnan and Aggarwala [1]. We used the following steps to generate a progressive censoring with binomial removals samples generated from generalized Pareto distribution

1. Generate a group values

$$r_i \sim Binomial(n - m - \sum_{j=1}^{i-1} r_j, p),$$

$$r_m = n - m - \sum_{i=1}^{m-1} r_i \quad , i = 1, 2, ..., m-1$$

according to the relevant value of p.

2. Simulate m independent exponential random variables $Z_1, Z_2, ..., Z_m$.
This can be done using inverse transformation $Z_i = -\ln(1 - U_i)$ where U_i are independent $uniform(0, 1)$ random variables.

3. Set

$$X_i = \frac{Z_1}{n} + \frac{Z_2}{n - R_1 - 1} + \frac{Z_3}{n - R_1 - R_2 - 2} + \cdots + \frac{Z_i}{n - R_1 - R_2 - \cdots - R_{i-1} - i + 1}$$

for $i = 1, 2, ..., m$. This is the required progressively type-II censored sample with binomial removals from the standard exponential distribution.

4. Finally, we set $Y_i = F^{-1}(1 - \exp(-X_i))$,for $i = 1, 2, ..., m$, where $F^{-1}(.)$ is the inverse cumulative distribution function of the generalized Pareto distribution. Then $Y_1, Y_2, \cdots, Y_m$ is the required progressively type-II censored sample with binomial removals from the generalized Pareto distribution.

5. We compute the MLE of θ and $R(t) = S$ by (6) and (7).

6. We compute the the Bayes estimates θ and $R(t) = S$ by respectively, using (13), (14), (16) and (17).

7. We compute the confidence interval of θ by using (9).

8. We repeat the above steps 2000 times. We then obtain the means and the MSEs (mean squared error), where

$$MSE = 2000^{-1} \sum_{i=1}^{2000} (\phi - \hat{\phi})^2$$

and $\hat{\phi}$ is the estimator of ϕ

In all above cases the prior parameters chosen as $(\alpha = 2, \beta = 1)$ which yield the generated value of $\theta = 2$ as the true value. The true values of $R(t)$ in $t = 0.5$ is obtained $R(0.5) = 0.9486833$. The results are summarized in Tables 2-5.

7. Conclusion

This paper presents different methods of estimation to estimate parameter and reliability function of two parameter generalized Pareto distribution based on a progressively type-II censored sample with random removals. Our observations about the results are stated in the follow:

- Table 2 and 4 shows that the Bayes estimates under squared error loss function has the smallest MSE's as compared with other estimates (Bayes estimates under the LINEX loss function and maximum likelihood estimator. However, maximum likelihood estimator method relatively more accurate estimators as compared with the Bayes estimation. It is immediate to note that the MSE's decrease as sample size n increases.

- Table 3 and 5 shows that the Bayes estimates of reliability function under squared error loss function has the smallest estimated MSE's as compared with Bayes estimates under the LINEX loss function and maximum likelihood estimator. However, Bayes estimates under the LINEX loss function relatively more accurate estimators as compared with the maximum likelihood estimator and Bayes estimates under squared error loss function. Also, the MSE's decrease as n increases.

- Table 2 and 4 shows that for different size n and m, the width of the Confidence interval for θ, decrease as n and m increases.

Table 2: Averaged values of MSEs for estimates of θ, $(P = 0.1)$

n	m	Generated R_i	$\hat{\theta}_{MLE}$ MSE	$\hat{\theta}_{SE}$ MSE	$\hat{\theta}_{LI}(c = 0.1)$ MSE	$\hat{\theta}_{LI}(c = -0.1)$ MSE	$\%95CI$ Width
10	5	(1, 1, 0, 0, 3)	2.0097	1.8414	1.3724	1.3897	(0.9811,6.1896)
			0.7815	0.56781	0.6952	0.6814	5.2089
	7	$(0 * 5,\ 2,\ 1)$	2.0138	1.8847	1.5002	1.5153	(1.0779,5.0021)
			0.5887	0.4639	0.5352	0.5261	
20	10	(0,1,2,2,0*5,5)	2.0144	1.9222	1.6202	1.6327	(1.1790,4.2008)
			0.4066	0.3419	0.3828	0.3772	
	15	(0,1,0*8,1,0,1,0,2)	1.9976	1.9353	1.7155	1.7250	(1.2756,3.5692)
			0.2795	0.2499	0.2740	0.2708	
30	20	(4,0,1,0*7	2.0150	1.9667	1.7918	1.7996	(1.3582,3.2989)
		,1,0,1,0*5,1,2)	0.2007	0.1829	0.1942	0.1924	
	25	(0*3,1,2,0,0	2.0226	1.9832	1.8383	1.8449	(1.4159,3.1254)
		,1,0*16,1)	0.1670	0.1542	0.1584	0.1572	
40	30	(1,2,0*5,1,0,2,0,1	2.0131	1.9805	1.8576	1.8632	(1.4501,2.9838)
		,0*4,1,0*4,1,0*7,1)	0.1285	0.1206	0.1260	0.1251	
	35	(0,0,1,0,1,0,	2.0063	1.9783	1.8717	1.8767	(1.4779,2.8804)
		1,0,0,2,0*25)	0.1132	0.1074	0.1122	0.1114	

Table 3: Averaged values of MSEs for estimates of the reliability function, $P = 0.1$

n	m	$\hat{S}_{MLE}$ MSE	$\hat{S}_S$ MSE	$\hat{S}_L(c = 0.1)$ MSE	$\hat{S}_L c = -0.1$ MSE
10	5	0.92483	0.93723	0.92896	0.92868
		0.001604	0.001219	0.001454	0.001457
	7	0.93153	0.94101	0.93576	0.93543
		0.000951	0.000702	0.000842	0.000847
20	10	0.93676	0.94367	0.94107	0.94071
		0.000544	0.000401	0.000471	0.000474
	15	0.94029	0.94505	0.94466	0.94428
		0.000305	0.000231	0.000258	0.000260
30	20	0.94298	0.94656	0.94738	0.94700
		0.000188	0.000151	0.000162	0.000162
	25	0.94433	0.94720	0.94875	0.94836
		0.000140	0.000117	0.000125	0.000125
40	30	0.94502	0.94743	0.94945	0.94906
		1.056e-04	8.948e-05	9.549e-05	9.466e-05
	35	0.94537	0.94744	0.94981	0.94941
		9.127e-05	7.855e-05	8.391e-05	8.286e-05

Table 4: Averaged values of MSEs for estimates of θ, $(P = 0.6)$

n	m	Generated R_i	$\hat{\theta}_{MLE}$ MSE	$\hat{\theta}_{SE}$ MSE	$\hat{\theta}_{LI}(c = 0.1)$ MSE	$\hat{\theta}_{LI}(c = -0.1)$ MSE	$\%95CI$ Width
10	5	(4,0,1,0,0)	2.0142	1.8452	1.3753	1.3926	(0.9833, 6.2035)
			0.8381	0.6058	0.7135	0.7003	
	7	(3,0*6)	2.0329	1.9038	1.5154	1.5307	(1.0896, 5.0564)
			0.5661	0.4419	0.5089	0.49991	
20	10	(7,2,1,0*7)	2.0113	1.9194	1.6179	1.6304	(1.1773, 4.1944)
			0.3999	0.3369	0.3807	0.3750	
	15	(2,1,2,0*12)	1.9959	1.9337	1.7141	1.7236	(1.2745, 3.5662)
			0.2752	0.2463	0.2718	0.2685	
30	20	(3,3,1,0,1,2,0*14)	1.9946	1.9472	1.7741	1.7818	(1.3445, 3.2655)
			0.1927	0.1776	0.1961	0.1939	
	25	(2,2,1,0*22)	2.0082	1.9694	1.8255	1.8320	(1.4059, 3.1031)
			0.1597	0.1485	0.1572	0.1559	
40	30	(4,2,2,2,0*26)	1.9999	1.9676	1.8456	1.8512	(1.4405, 2.9642)
			0.1344	0.1269	0.1345	0.1335	
	35	(3,0,1,1,0*31)	2.0059	1.9780	1.8714	1.8763	(1.4777, 2.8798)
			0.1204	0.1142	0.1183	0.1176	

Table 5: Averaged values of MSEs for estimates of the reliability function, $P = 0.6$

n	m	$\hat{S}_{MLE}$ MSE	$\hat{S}_S$ MSE	$\hat{S}_L c = 0.1$ MSE	$\hat{S}_L c = -0.1$ MSE
10	5	0.92450	0.93668	0.92862	0.92834
		0.001708	0.001433	0.001557	0.001559
	7	0.93224	0.94166	0.93648	0.93615
		0.000932	0.000706	0.000830	0.000833
20	10	0.93682	0.94376	0.94113	0.94077
		0.000515	0.000370	0.000442	0.000446
	15	0.94021	0.94496	0.94457	0.94420
		0.000316	0.000241	0.000268	0.000270
30	20	0.94246	0.94606	0.94685	0.94647
		0.000195	0.000154	0.000164	0.000165
	25	0.94402	0.94691	0.94844	0.94805
		0.000139	0.000114	0.000121	0.000121
40	30	0.94457	0.94699	0.94899	0.94860
		1.173e-04	9.869e-05	1.033e-04	1.028e-04
	35	0.94526	0.94733	0.94969	0.94930
		9.851e-05	8.506e-05	9.033e-05	8.934e-05

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