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Research paper

Geometric Approach to Optimal Path Problem with Uncertain Arc Lengths

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Abstract

In this paper, the problem of finding the shortest paths, one of the most important problems in science and technology has been geometrically studied. Shortest path algorithm has been generalized to the shortest cycles in each homotopy class on a surface with arbitrary topology, using the universal covering space notion in the algebraic topology. Then, a general algorithm has been presented to compute the shortest cycles (geometrically rather than combinatorial) in each homotopy class. The algorithm can handle surface meshes with the desired topology, with or without boundary. It also provides a fundamental framework for other algorithms based on universal coverage space due to the capacity and flexibility of the framework.

Keywords: *Shortest path problem; Homotopy; Covering space; Cycle*

1. Introduction

The shortest problem is finding a path with minimal distance, time, or cost between the source node and the destination node. Many programming problems, including transportation, routing, communications, and supply chain management, can be seen as special cases of the shortest path problem. In the 1950s and 1960s, effective algorithms were proposed or developed by Bellman [1], Dijkstra [2], Dreyfus [3] and Floyd [4], which made the short-haul problem a central element of a network.

These algorithms have been called classical algorithms. In classical algorithms, the network must have deterministic arc lengths. However, for reasons of failure, maintenance or other reasons, arc lengths are not deterministic in many situations. Therefore, it is inappropriate to use conventional algorithms in these situations. Some researchers have estimated that these non-deterministic phenomena are compatible with randomness and have introduced the theory of probabilities into the shortest path problem; see Frank [5], Hall [6], Loui [7], Mirchandani [8], for example. Since Dubois and Prade [9] proposed the Short Fuzzy Path (FSPP) problem in 1980, the fuzzy theory has begun to attract networked researchers. Some researchers, such as Klein [10], Ji and Iwamura [11] have worked extensively in this area.

In 2007, Liu [12] proposed the theory of uncertainty to describe non-deterministic phenomena, especially expert data and subjective estimates. Since then, the theory of uncertainty has provided a new approach to addressing non-deterministic factors in programming problems. Obviously, the length of each path in an uncertain network is uncertain and we cannot get the shortest path in the normal direction. What we are studying is the distribution of the length of the shortest path. Nowadays, the concepts of the time-dependent shortest paths in large graphs and the shortest path in an uncertain network is taken into consideration. These two concepts are duels of each other and constitute the best path under certain constraints of the level of confidence [13], [14-15]. However, the exact methods for finding their shortest path is not indicated.

Solving problems with routes with a geometric approach will lead to useful extensions of them [16]. In this paper, after a literature review about the shortest path problem algorithms, a new approach has been presented. In the sequel, algebraic concepts has been explained and then, the shortest path problem can be solved geometrically rather than combinatorial on a surface with arbitrary topology with or without boundary.

2. Literature Review

Shortest path problem is finding a path between two vertices (or nodes) of a graph so as to minimize the sum of the weights of its constituent edges.

The most important algorithms to solve this problem are [17-18]:

Dijkstra's algorithm solves the problem of the shortest single source path.

The Bellman-Ford algorithm solves the single source problem if the edge weights can be negative.

A *-search algorithm solves the shortest path of a pair using heuristic methods to try to speed up the search.

The Floyd-Warshall algorithm solves all the shortest paths in pairs.

Johnson's algorithm solves all the shortest path pairs and can be faster than Floyd-Warshall on sparse graphs.
The Viterbi algorithm solves the shortest stochastic path problem with additional probabilistic weight on each node.
The problem of finding the shortest paths in graphs is a fundamental optimization problem for many applications, such as routing in networks, image segmentation in vision, surface segmentation in graphics, motion and robot navigation, speech recognition and VLSI design, etc.

3. Methodology

The concepts presented in this section can be found in the standard books of geometry and algebraic topology, including [19].
A continuous map $f: I \to X$ where I is the unit interval [0,1] and X is a space is called a path in X. The idea of permanently deforming a path while keeping its determined ends is specified by the following definition. A homotopy of paths in X is a family $f_t: I \to X$, $0 \le t \le 1$, such that

(1) The extremities $f_t(0) = x_0$ and $f_t(1) = x_1$ are independent of t.

(2) The associated map $F: I \times I \to X$ defined by $F(s,t) = f_t(s)$ is continuous. When two paths f_0 and f_1 are connected in this way by a homotopy f_t, they are called homotopes. The notation for this is $f_0 \simeq f_1$.

Let $f, g: I \to X$ be such that $f(1) = g(0)$, there exists a path of composition or of product $f.g$ which crosses first f then g, defined by the formula

$$f.g(s) = \begin{cases} f(2s) & 0 \le s \le \frac{1}{2} \\ g(2s-1) & \frac{1}{2} \le s \le 1 \end{cases} \qquad (1)$$

Thus, f and g are traveled twice as fast so that $f.g$ is traveled in the unit time. This produces operation respects the homotopy classes because if $f_0 \simeq f_1$ and $g_0 \simeq g_1$ via homotopies f_t and g_t, and if $f_0(1) = g_0(0)$ so that $f_0.g_0$ is defined, then $f_t.g_t$ is defined and provides a homotopy $f_0.g_0 \simeq f_1.g_1$.

In particular, the main focus will be on the paths $f: I \to X$ with the same starting and ending points $f(0) = f(1) = x_0 \in X$. These paths are called loops and the point of common start and end x_0 is called the base point. The set of all the homotopy classes $[f]$ of the loops $f: I \to X$ at the base point x_0 is denoted $\pi_1(X, x_0)$.

$\pi_1(X, x_0)$ is a group with respect to the product $[f][g] = [f.g]$. This group is called the fundamental group of X at the base point x_0.

Thus, if X is path-connected, the group $\pi_1(X, x_0)$ is, up to isomorphism, independent of the choice of base point x_0. In this case, the notation $\pi_1(X, x_0)$ is often abbreviated to $\pi_1(X)$, or $\pi_1 X$.

In general, a space is called simply connected if it is path-connected and has a trivial fundamental group.

A space X is simply connected if and only if there is a unique homotopy class of paths connecting any two points of X.

Let X be a space, a covering space of X consists of a space $\tilde{X}$ and a map $p: \tilde{X} \to X$ satisfying the following condition:

For each point $x \in X$, there exists an open neighborhood U of x in X such that $p^{-1}(U)$ is a union of disjoint open sets, each of which is mapped homeomorphically onto U by p.

4. Results

Theorem. The shortest path problem can be solved geometrically rather than combinatorial on a surface with arbitrary topology with or without boundary.

Proof. As mentioned about homotopy paths, without losing the whole, it can be studied a restricted version of the above problem:

Restricted shortest cycle problem: With a surface mesh X with an arbitrary topology, it should be found in each homotopy class the shortest cycle passing through a given point $x_0 \in X$.

For this purpose, it has been proposed a general framework using universal covering space to turn the problem from the search of the shortest cycles into a problem of finding the shortest paths. In order to avoid the exponential blowing up of the space required by naïve when building the universal covering space, it suffices a space efficient data structure to manage the universal covering space will be developed.

On the other hand, there are three facts about coverage spaces $p: \tilde{X} \to X$.

(a) For each path $f: I \to X$ starting at a point $x_0 \in X$ and each $\tilde{x}_0 \in p^{-1}(x_0)$, there exists a unique lift $\tilde{f}: I \to \tilde{X}$ starting at $\tilde{x}_0$.

(b) For each homotopy $f_t: I \to X$ of paths beginning with x_0 and each $\tilde{x}_0 \in p^{-1}(x_0)$, there exists a unique high homotopy $\tilde{f}_t: I \to \tilde{X}$ of paths starting with $\tilde{x}_0$.

(c) Let map $F: Y \times I \to X$ and map $\tilde{F}: Y \times \{0\} \to \tilde{X}$ by raising $F|_{Y \times \{0\}}$, there exists then a unique map $\tilde{F}: Y \times I \to \tilde{X}$ raises F and limits to $\tilde{F}$ given on $Y \times \{0\}$.

In the sequel of the process, a set of homology bases as a section graph by cutting will be used, the section graph in the original mesh will become the cutting limit in the fundamental domain.

In this way, finding the shortest path is limited to calculate a fundamental domain $\tilde{X}$ by cutting the mesh X along a set of curves. Then by taking a central copy of $\tilde{X}$ and glue more copies of it along the cut segments, the optimal path is found.

5. Conclusions

A general algorithm had been presented to compute the shortest cycles (geometrically rather than combinatorial) in each homotopy class. It is proved that the shortest path problem can be solved geometrically rather than combinatorial on a surface with arbitrary topology with or without boundary.

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