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He's variational method for a (2 + 1)-dimensional soliton equation

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Abstract

In this Letter, a (2 + 1)-dimensional soliton equation is studied by He's variational approach. The solitary solutions are obtained using the Ritz method.

Keywords: *Variational method, Solitary solution, Soliton equation, Ritz method, exact solution.*

1 Introduction

In this letter, we consider the following (2 + 1) dimensions soliton equation to reveal new exact traveling wave solutions using He's variational method

$$\begin{aligned} &i\,u_t + u_{xx} + u\,v = 0, \\ &v_t + v_y + (v\,u^*)_x = 0, \end{aligned} \tag{1}$$

where $i = \sqrt{-1}, u(x, y, t)$ is a complex function and $v(x, y, t)$ is a real function which has studied in [1] by using the bifurcation theory.

Soliton is an important feature of nonlinearity and can be found in many applications of science. Many effective and reliable methods are used in the literature to investigate solitons and in particular multiple soliton solutions of completely integrable equations. Nonlinear evolution equations have been noticed in plasma physics, fluid dynamics, optical fibers, biological systems and other applications. In the past decades, there has been an increased interest on studying the nonlinear evolution equations. Exact solutions play a vital role in understanding various qualitative and quantitative features of nonlinear phenomena. There are diverse classes of interesting exact solutions, such as traveling wave solutions, but it often needs specific mathematical techniques to construct exact solutions due to the nonlinearity present in dynamics [2, 3]. It has recently become more interesting to obtain exact solutions of nonlinear partial differential equations (NPDEs) using symbolic computation softwares such as Maple, Mathematica and Matlab that facilitate complex and tedious algebraical computations. In recent years, various effective methods have been developed to find the exact solutions of NPDEs, such as tanh-function method [4, 5, 6, 7, 8, 9, 10, 11], generalized hyperbolic function method [12], homogeneous balance method [13, 14], Jacobi-elliptic function method [15, 16, 17], exp-function method [18, 19], auxiliary equation method [20, 21, 22, 23, 24] and so on, e.g. see [25, 26, 27].

2 He's variational method

In order to seek exact solutions of Eq. (1), we suppose that

$$\begin{aligned} &u(x,y,t)=\phi(\xi)\,\exp(i\,\eta), \quad v(x,y,t)=v(\xi),\\ &\eta=k\,x+l\,y+\lambda\,t, \quad \xi=K(x+L\,y-2k\,t), \end{aligned} \tag{2}$$

where $\phi(\xi)$ and $v(\xi)$ are real functions, k, l, λ, K and L are real constants to be determined later. Substituting Eq. (2) into Eq. (1), we have

$$K^2\,\phi^{''}(\xi)-(\lambda+k^2)\,\phi(\xi)+\phi(\xi)\,v(\xi)=0, \tag{3}$$

$$(L-2\,k)v^{'}(\xi)+(\phi^2(\xi))^{'}=0. \tag{4}$$

where prime denotes the differentiation with respect to ξ. Integrating Eq. (4) with respect to ξ and taking the integration constant as zero yields

$$v(\xi)=\frac{1}{2\,k-L}\,\phi^2(\xi), \qquad \text{if } L\neq 2k. \tag{5}$$

Substituting Eq. (5) into Eq. (3) yields

$$\phi^{''}(\xi)+\alpha\,\phi(\xi)-\beta\,\phi^3(\xi)=0, \tag{6}$$

and

$$\alpha=\frac{-\lambda-k^2}{K^2}\quad,\quad \beta=\frac{1}{K^2(L-2k)}, \qquad L\neq 2k.$$

According to Ref. [29], upon using He's semi-inverse method [30], we can arrive at the following variational formulation:

$$J(\phi) = \int_0^\infty \left[\frac{1}{2} (\phi')^2 - \frac{\alpha}{2} \phi^2 + \frac{\beta}{4} \phi^4 \right] d\xi. \tag{7}$$

We assume the soliton solution in the following form

$$\phi(\xi) = A\text{sech}(\xi) \tag{8}$$

where A is an unknown constant to be further determined.
By Substituting Eq. (8) into Eq. (7) we obtain

$$J = \frac{1}{6} \beta A^4 + \frac{1}{6} A^2 - \frac{1}{2} \alpha A^2. \tag{9}$$

For making J stationary with respect to A

$$\frac{\partial J}{\partial A} = \frac{2}{3} \beta A^3 + \frac{1}{3} A - \alpha A \tag{10}$$

from Eq. (10), we have

$$A = \pm \frac{\sqrt{2\beta\, (3\,\alpha - 1)}}{2\beta} \tag{11}$$

and

$$\alpha = \frac{-\lambda - k^2}{K^2} \quad , \quad \beta = \frac{1}{K^2(L - 2k)}, \qquad L \neq 2k.$$

The solitary solution is, therefore, obtained as follows:

$$\phi(\xi) = \pm \frac{\sqrt{2\beta\, (3\,\alpha - 1)}}{2\beta} \text{sech}(\xi). \tag{12}$$

By Eqs. (2) and (5), we have

$$\begin{aligned} u(x, y, t) &= A\text{sech}(\xi)\, e^{i\,\eta} = \pm \frac{\sqrt{2\beta\, (3\,\alpha - 1)}}{2\beta} \text{sech}(\xi)\, e^{i\,\eta}, \\ v(\xi) &= \frac{3\,\alpha - 1}{2\beta\, (2\,k - L)} \text{sech}^2(\xi), \qquad \text{if } L \neq 2k, \end{aligned} \tag{13}$$

where

$$\eta = k\,x + l\,y + \lambda\,t \quad , \quad \xi = K(x + L\,y - 2k\,t). \tag{14}$$

We search another soliton solution in the form

$$\phi(\xi) = D\text{sech}^2(\xi) \tag{15}$$

where D is an unknown constant to be further determined.
By Substituting Eq. (15) into Eq. (7) we obtain

$$J = \frac{4}{35}\,D^4\beta - \frac{1}{3}\,\alpha\,D^2 + \frac{4}{15}\,D^2. \tag{16}$$

For making J stationary with respect to D

$$\frac{\partial J}{\partial D} = \frac{16}{35}\,D^3\beta - \frac{2}{3}\,\alpha\,D + \frac{8}{15}\,D. \tag{17}$$

From Eq. (17), we have

$$D = \pm\frac{\sqrt{42\beta\;(5\,\alpha - 4)}}{12\beta} \tag{18}$$

and

$$\alpha = \frac{-\lambda - k^2}{K^2} \quad , \quad \beta = \frac{1}{K^2(L-2k)}, \quad L \neq 2k.$$

The solitary solution is, therefore, obtained as follows:

$$\phi(\xi) = \pm\frac{\sqrt{42\beta\;(5\,\alpha - 4)}}{12\beta}\,\mathrm{sech}^2(\xi). \tag{19}$$

By Eqs. (2) and (5), we have

$$\begin{aligned} u(x,y,t) &= D\mathrm{sech}^2(\xi)\,\mathrm{e}^{i\,\eta} = \frac{\sqrt{42\beta\;(5\,\alpha - 4)}}{12\beta}\,\mathrm{sech}^2(\xi)\,\mathrm{e}^{i\,\eta}, \\ v(\xi) &= \frac{73\,\alpha - 1}{24\beta\,(2\,k - L)}\,\mathrm{sech}^4(\xi), \qquad \text{if } L \neq 2k, \end{aligned} \tag{20}$$

where

$$\eta = k\,x + l\,y + \lambda\,t \quad , \quad \xi = K(x + L\,y - 2k\,t). \tag{21}$$

3 Conclusions

In this letter, we have used He's variational method to search for solitary solutions. He's variational principle is a very dominant instrument to find the solitary solutions for various nonlinear equations.

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