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Magnetohydrodynamic free convection flow of a heat generating fluid past a semi-infinite vertical porous plate with variable suction

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Abstract

In this paper, a magnetohydrodynamic convection flow of an electrically conducting heat generating fluid past a semi-infinite vertical porous plate with variable suction is considered. The fluid flow is unsteady and a variable magnetic field is transversely applied to the plate. Evaluation of velocity gradients, temperature gradients and concentration gradients across the plate is done. Observations and discussions of the effects of various parameters on flow variables are done. The non-dimensional parameters observed and discussed are Hall parameter, M; Magnetic number, M^2; Eckert number, Ec; Rotational parameter, Er; Suction parameter, S and Injection parameter, w. The velocity profiles, temperature profiles and concentration profiles are presented graphically for both convectional heating and free convectional cooling of the plate. The skin friction and rate of heat transfer values are obtained and presented in tables.
For free convectional heating and cooling of the plate, the Grashof number is taken as constants -5 and 5 respectively. Prandtl number is 0.71 which corresponds to air. The variation of the parameters mentioned above is noted to increase or decrease or had no effect on the skin friction, mass transfer, rate of heat transfer, the velocity profiles, concentration profiles and temperature profiles.

Keywords: *Finite difference method, Free convection, MHD, Semi infinite plate, Variable suction order.*

1 Introduction

The study of MHD rotating fluids has had considerable progress in the last few decades. MHD free convection flow past a semi-infinite vertical porous plate with variable suction is a study which has many applications such as in MHD pumps, MHD power generator, purification of crude oil in petroleum industries, polymer technology and aerodynamic heating and accelerators. The study of MHD free convection flow of a heating generating fluid past a semi-infinite vertical porous plate with variable suction finds very many applications in cooling of electronic devices (e.g. mobiles, computers etc.) and solar panels. Some other applications in this study are design of; flow meters, MHD generators, heat exchangers, space vehicle, propulsion and breaking, electromagnetic pumps and MHD electrical power generation. Fluid flow involving rotation is observed in earth's atmosphere and in oceans. Meteorologist can use this study to understand dynamics of meteorology and air pollution. It is in the light of this that this study will be useful to welfare of mankind.

MHD flow past an infinite porous plate with variables suction was studied by [1]. An investigatation of MHD free convection and Mass transfer flow through a porous medium with heat source was undertaken [2]. A study of Heat and Mass transfer of viscous heat generating fluid with Hall currents has been accomplished [7]. [11] investigated the hydromagnetic convective flow of a heat generating fluid past a vertical plate with Hall current and heat flux through a porous medium. MHD free convection heat and mass transfer of a heat generating fluid past an impulsively started infinite vertical porous plate with Hall current and radiation absorption was investigated [5]. Studies on MHD Stokes free convection flow past an infinite vertical porous plate subjected to constant heat flux with ion-slip current and radiation absorption was done [6]. Hydromagnetic flow past parallel porous plate was studied by [10] and [3] explained the effect of magnetic field on a rotating porous plate.

The flow of an incompressible viscous fluid past an impulsively started horizontal plate and Magnetohydrodynamics flow past an infinite plate with a constant and variable suction was performed [1]. Further studies on the finite difference of MHD stokes problem for a vertical infinite plate in a dissipative heat generating fluid with Hall and Ion-slip current were also executed. [3] studied the MHD stokes problem for a vertical infinite plate in a dissipative rotating

fluid with Hall current and they later investigated the effect of both Hall and Ion-slip currents on the flow of heat generating rotating fluid system. [4] studied the finite difference analysis of free convection effects on MHD problem for a vertical plate in a dissipative rotating fluid system with constant heat flux and Hall current. [5] also did a finite difference analysis of MHD Stokes problem for a vertical infinite plate in a dissipative fluid with constant heat and Hall current. A study on the viscous dissipation effects on heat transfer in boundary layer flow past a semi infinite horizontal flat plate was done [9]. Solution of the MHD stokes problem of a convective flow past a vertical infinite plate in a rotating fluid was attained [7]. They investigated the problem of hydro magnetic free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux and he solved the MHD stokes problem for a vertical infinite plate with Hall and ion-slip currents by explicit finite difference method.
In spite of all these investigations, much has not been done on MHD convection flow of an electrically conducting heat generating fluid past a semi-infinite vertical porous plate with variable suction in a rotating system in presence of a variable magnetic field. Our present investigation therefore seeks to study magnetohydrodynamic convection flow of an electrically conducting heat generating fluid past a semi-infinite vertical porous plate with variable suction in a rotating system in presence of a variable magnetic field. The aim of the present investigation is to obtain an approximate velocity profiles, temperature profiles, and concentration profiles of a heat generating fluid past a semi-infinite vertical porous plate with variable suction.

2 Mathematical analysis

In the present study MHD convection flow of an electrically conducting heat generating fluid past a semi-infinite vertical porous plate with variable suction in rotating system in presence of a variable magnetic field is considered. The magnetic field is applied transversely along the z-axis and perpendicular to the vertical plate. The plate is non-conducting and the fluid is electrically conducting.
Consider flow of a viscous incompressible MHD free convection heat generating fluid past an impulsively started semi-infinite vertical porous plate with variable suction in presence of variable magnetic field. The plate is suddenly set into motion in its own plane with constant velocity. It is assumed that the variable magnetic field is applied perpendicular to the direction of the flow as illustrated in the figure below (Fig 1).

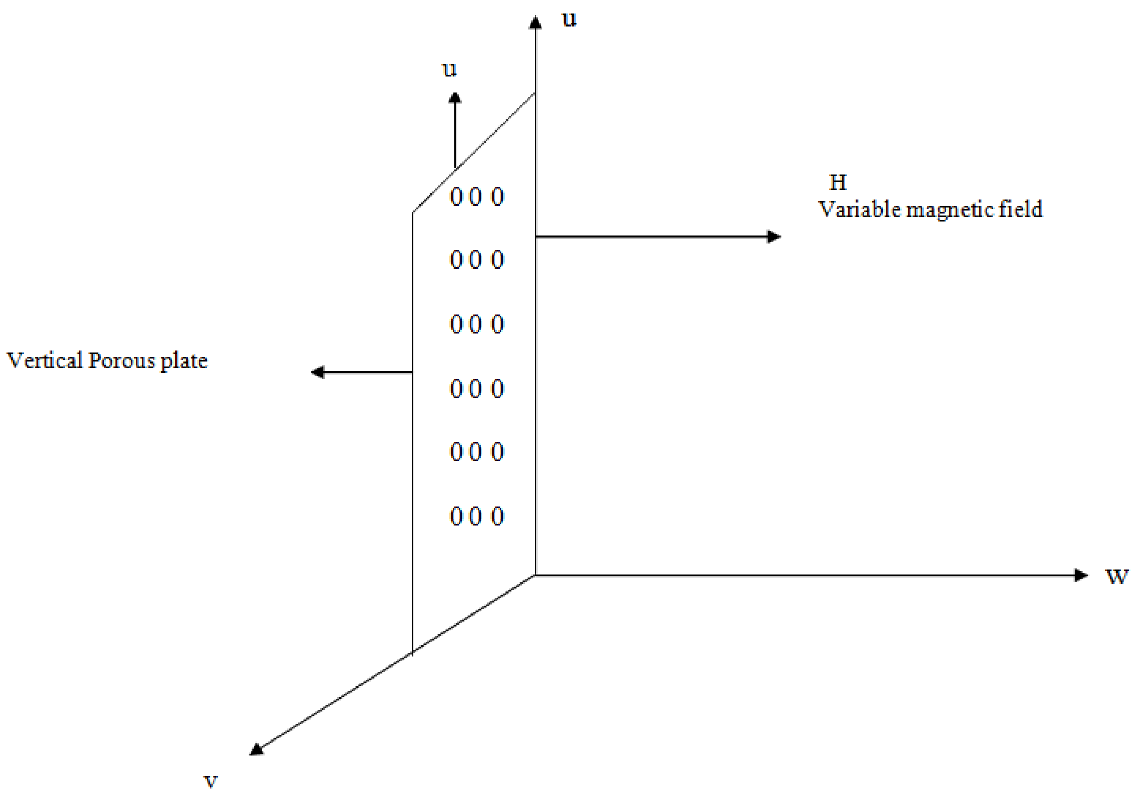


Fig 1: The flow configuration with the co-ordinate system of magnetohydrodynamic free convention flow of a heat generating fluid past a semi-infinite vertical porous plate with variable suction

An assumption is made that the induced magnetic field is negligible such that $H = (0, 0, H_z)$. This assumption is justified owing to the fact that the Magnetic Reynolds number of a partially ionized fluid is very small.
We consider the system to be rotating with uniform angular velocity Ω about the z-axis which is taken normal to the plate. Since the plate is semi-infinite in length, the variables are functions of y+ and t+ only. At time $t^+ > 0$, the plate start moving impulsively in its own plane with constant velocity v_o and its temperature instantaneously increased or decreased to T^+_w which is maintained constant later. Initially the temperature of the fluid and the plate are assumed to be the same.
The equation of conservation of electric charge $\nabla.\vec{J} = 0$ gives $\vec{Jz}^+ = Constant$ where $\vec{J} = (Jx^+, Jy^+, Jz^+)$. This constant is assumed to be equal to zero since $Jz^+ = 0$ at the plate which is electrically non-conducting. Thus $Jz^+ = 0$ everywhere in the flow. The generalized ohm's law must be modified to include the effects of Hall currents and variable magnetic field as follows:

$$\vec{J} + \frac{\omega_e \tau_e}{H_o} \vec{J} \times \vec{H_y} = \sigma\left[\vec{E} + \mu_e \vec{q} \times H_y + \frac{1}{e\eta_e} \nabla.P_e\right] \tag{1}$$

Where $\sigma, \mu_e, \tau_e, e, \eta_e$ and P_e is the electric conductivity, the magnetic permeability, the cyclotron frequency, the collision time, the electric charge, the number density of electron and the electron pressure respectively.
It is assumed that $\omega_e \tau_e \leq 1$. The induced magnetic field is assumed to be zero and the pressure gradient may be neglected. The ion-slip and thermoelectric effects are also neglected. Thus equation (1) yields

$$\left(J_x^+, J_y^+\right) + \frac{\omega_e \tau_e}{\vec{H_o}}\left(J_y^+ H_o, -J_x^+ H_o\right) = \sigma\mu_e\left(v^+ H_o, -u^+ H_o\right) \tag{2}$$

Equating the x^+ and the y^+ component in the above equation gives:

$$\left.\begin{aligned} J_x^+ + \omega_e \tau_e J_y^+ &= \sigma\mu_e v^+ \vec{H_o} \\ J_y^+ - \omega_e \tau_e J_x^+ &= -\sigma\mu_e u^+ \vec{H_o} \end{aligned}\right\} \tag{3}$$

Solving for J_x^+ and J_y^+ by first eliminating J_y^+ yields

$$Jx^+ = \frac{\sigma o \mu e Ho}{1+m^2}\left(v^+ + mu^+\right) \tag{4}$$

And consequently eliminating J_x^+

$$J_y^+ = \frac{\sigma_o \mu_e H_o}{1+m^2}\left(mv^+ - u^+\right) \tag{5}$$

When the effect of rotation is considered, the Coriolis force has to be included in the momentum equation. Considering a rotating frame of reference with a uniform angular velocity, Ω, the equations of motion become:

$$\frac{\partial u^+}{\partial t^+} - wo^+ \frac{\partial u^+}{\partial z^+} - 2\Omega v^+ = \nu \frac{\partial^2 u^+}{\partial z^{+2}} + g\beta^*\left(T^+ - T^+_\infty\right) + \frac{\mu_e J_y^+ H_o}{\rho} + g\beta^*\left(C^+ - C^+_\infty\right) \tag{6}$$

$$\frac{\partial v^+}{\partial t^+} - w_o \frac{\partial v^+}{\partial z^+} + 2\Omega u^* = \nu \frac{\partial^2 u^+}{\partial z^{+2}} - \frac{\mu_e J_x^+ H_o}{\rho} \tag{7}$$

$$\frac{\partial T^+}{\partial t^+} - w_o \frac{\partial T^+}{\partial z^+} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^+}{\partial z^{+2}} + \frac{\nu}{Cp}\left[\left(\frac{\partial v^+}{\partial z^+}\right)^2 + \left(\frac{\partial u^+}{\partial z^+}\right)^2\right] \tag{8}$$

$$\frac{\partial C^+}{\partial t^+} - w_o \frac{\partial C^+}{\partial z^+} = D \frac{\partial^2 C^+}{\partial z^{+2}} \tag{9}$$

$$\frac{\partial B_y^+}{\partial t^+} + u^+ \frac{\partial B_y^+}{\partial z^+} - B_y^+ \frac{\partial u^+}{\partial y^+} = 0 \tag{10}$$

In the dimensionalization process the following set of general scaling variables are utilized in our study

$$\left.\begin{aligned} & t=\frac{t^{+}U^{2}{}_{0}}{\nu}, z=\frac{z^{+}U_{0}}{\nu}, u=\frac{u^{+}}{U_{0}}, w_{o}=\frac{wo^{+}}{uo} \\ & v=\frac{v^{+}}{u_{0}}, x=\frac{x^{+}U^{2}{}_{0}}{\nu}, \theta=\frac{T^{+}-T^{+}{}_{\infty}}{T^{+}{}_{w}-T^{+}{}_{\infty}}, y=\frac{y^{+}u_{o}{}^{2}}{\nu} \\ & G_{r}=\nu g\beta'\frac{\left(T^{*}{}_{w}-T^{*}{}_{\infty}\right)}{u_{o}{}^{3}}, P_{r}=\frac{\mu C_{p}}{\kappa}, Sc=\frac{D}{\nu} \\ & M^{2}=\frac{\sigma H_{o}{}^{2}\mu_{e}{}^{2}\nu}{\rho u_{o}{}^{2}}, E_{r}=\frac{\Omega\nu}{u_{o}{}^{2}}, P_{r}=\frac{\mu C_{p}}{\kappa}, C=\frac{C^{+}-C^{+}{}_{\infty}}{C^{+}{}_{w}-C^{+}{}_{\infty}} \end{aligned}\right\} \quad (11)$$

The initial boundary conditions are

For $t<0$:

$$u^{+}\left(z^{+},t\right)=0, v^{+}\left(z^{+},t\right)=0, T^{+}\left(z^{+},t\right)=T^{+}{}_{\infty}, C^{+}\left(z^{+},t\right)=C^{+}{}_{\infty} \quad (12)$$

For $t>0$:

$$u^{+}\left(0,t\right)=U, v^{+}\left(0,t\right)=0, T^{+}\left(0,t\right)=T^{+}{}_{w}, C^{+}\left(0,t\right)=C^{+}{}_{w} \quad (13)$$

$$u^{+}\left(\infty,t\right)=0, v^{+}\left(\infty,t\right)=0, T^{+}\left(\infty,t\right)=T^{+}{}_{\infty}, C^{+}\left(\infty,t\right)=C^{+}{}_{\infty} \quad (14)$$

Let Q^{+} be the internal heat generation. The internal heat generation is assumed to be of the form

$$Q^{+}=-\left(T^{+}-T^{+}{}_{\infty}\right)Q \quad (15)$$

On substituting equations (4) and (5) into equations (6) and (7), the system of equations governing the problem becomes:

$$\frac{\partial u^{+}}{\partial t^{+}}-wo^{+}\frac{\partial u^{+}}{\partial z^{+}}-2\Omega v^{+}=g\beta^{*}\left(T^{+}-T^{+}{}_{\infty}\right)+g\beta^{*}\left(C^{+}-C^{+}{}_{\infty}\right)+\nu\frac{\partial^{2}u^{+}}{\partial z^{+2}}+\frac{\mu_{e}{}^{2}J_{y}{}^{+}{}_{H}{}^{2}\sigma}{\rho\left(1+m^{2}\right)}\left(mv^{+}-u^{+}\right) \quad (16)$$

$$\frac{\partial v^{+}}{\partial t^{+}}-wo\frac{\partial v^{+}}{\partial z^{+}}-2\Omega v^{+}=\nu\frac{\partial^{2}u^{+}}{\partial z^{+2}}-\frac{\mu_{e}J_{x}{}^{+}H_{o}{}^{2}\sigma}{\rho\left(1+m^{2}\right)}\left(mu^{+}+v^{+}\right) \quad (17)$$

$$\frac{\partial T^{+}}{\partial t^{+}}-wo\frac{\partial T^{+}}{\partial z^{+}}=\frac{\kappa}{\rho C_{p}}\frac{\partial^{2}T^{+}}{\partial z^{+2}}+\frac{Q^{+}}{\rho C_{p}} \quad (18)$$

$$\frac{\partial C^{+}}{\partial t^{+}}-w_{o}\frac{\partial C^{+}}{\partial Z^{+}}=D\frac{\partial^{2}C^{+}}{\partial z^{+2}} \quad (19)$$

$$\frac{\partial B_{y}^{+}}{\partial t^{+}}+u^{+}\frac{\partial B_{y}^{+}}{\partial z^{+}}-B_{y}^{+}\frac{\partial u^{+}}{\partial y^{+}}=0 \quad (20)$$

The non-dimensional forms of the initial and boundary conditions are:

$t<0$:	$q(z,0)=0$,	$\theta(z,0)=0$,	$C(z,0)=0$	(21a)
$t>0$:	$q(0,t)=1$,	$\theta(0,t)=1$,	$C(0,t)=1$	(21b)
$q(\infty,t)=0$,	$\theta(\infty,t)=0$,	$C(\infty,t)=0$		(21c)

3 Method of solution

As the exact solution of equations (16) to (20) together with initial and boundary conditions (21 a, b, c) is not possible, we solve them by finite difference method. Equations (16) to (20) in finite difference form becomes

$$\begin{aligned} \frac{q\left(i,j+1\right)-q\left(i,j\right)}{\Delta t}= & w_{\circ}\frac{\left[q\left(i,j\right)-q\left(i-1,j\right)\right]}{\Delta z}+Gr\theta\left(i,j\right)+GcC\left(i,j\right)+m^{2}q\left(i,j\right) \\ & +\frac{\left[q\left(i-1,j\right)-2q\left(i,j\right)+q\left(i+1,j\right)\right]}{\left(\Delta z\right)^{2}} \end{aligned} \quad (22)$$

$$\Pr\frac{\theta(i,j)-\theta(i,j)}{\Delta t}=\frac{w_{\circ}\left[\theta(i,j)-\theta(i-1,j)\right]}{\Delta z}+\frac{\left[\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)\right]}{(\Delta z)^{2}}$$
$$+\Pr Ec\left(\frac{q(i+1,j)-q(i,j)}{\Delta z}\right)\left(\frac{\vec{q}(i+1,j)-\vec{q}(i,j)}{\Delta z}\right) \quad (23)$$

$$\frac{C(i,j+1)-C(i,j)}{\Delta t}=\frac{w_{\circ}\left[C(i,j)-C(i-1,j)\right]}{\Delta z}+Sc\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta z)^{2}}\right] \quad (24)$$

$$\left[\frac{H^{i+1}(i,j)-H_{y}^{i}(i,j)}{\Delta t}+u^{i+1}\frac{H^{i}(i+1,j)-H^{i}(i-1,j)}{2\Delta x}\right]R_{m}=\frac{H^{i+1}(i+1,j)-2H^{i+1}(i,j)+H^{i+1}(i-1,j)}{(\Delta x)^{2}} \quad (25)$$

We can compute consecutive terms of temperature, concentration and velocity profiles computed using the following finite difference form of equations;

$$q(i,j+1)=\Delta t\left\{\left[\frac{q(i+1,j)-2q(i,j)+q(i-1,j)}{(\Delta z)^{2}}\right]+w_{\circ}\left[\frac{q(i,j)-q(i-1,j)}{\Delta z}\right]+Gr\theta+GcC(i,j)-m_{2}q(i,j)\right\}+q(i,j) \quad (26)$$

$$\theta(i,j+1)=\frac{\Delta t}{\Pr}\left\{\begin{array}{l}\left[\frac{\Pr w_{\circ}\left[\theta(i,j)-\theta(i-1,j)\right]}{\Delta z}\right]+\left[\frac{\theta(i+1,j)-2\theta(i,j)+\theta(i-1,j)}{(\Delta z)^{2}}\right]\\+\Pr Ec\left[\left(\frac{q(i+1,j)-q(i,j)}{\Delta z}\right)\left(\frac{\vec{q}(i+1,j)-q(i,j)}{\Delta z}\right)\right]\end{array}\right\} \quad (27)$$

and

$$C(i,j+1)=\Delta t\left\{\left[\frac{w_{\circ}\left[C(i,j)-C(i-1,j)\right]}{\Delta z}\right]+Sc\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta z)^{2}}\right]\right\} \quad (28)$$

$$\left[\frac{H^{i+1}(i,j)-H_{y}^{i}(i,j)}{\Delta t}+u^{i+1}\frac{H^{i}(i+1,j)-H^{i}(i-1,j)}{2\Delta x}\right]R_{m}=\frac{H^{i+1}(i+1,j)-2H^{i+1}(i,j)+H^{i+1}(i-1,j)}{(\Delta x)^{2}} \quad (29)$$

In equations (28) and (29) the index i' refers to z and j' to time. The mesh system in this case is divided by taking $\Delta z=0.1$ and $\Delta t=0.00125$. From equation (21) the initial conditions at $z=0$ takes the form

$q(0,0)=1,\ \theta(0,0)=1,\ q(i',0)=\theta(i',0)=0$, For all except $i'=0$ (30)

In finite difference the boundary condition (21b) takes the form

$q(0,j')=1,\ \theta(0,j')=1$ For all j' (31)

Though the boundary condition (34c) applies at $z=\infty$, we take $z=4.1$ as corresponding to $z=8$, since both the values of q and θ tend to zero as $z\to 4$. Therefore in this section we set $q(41,j')=\theta(41,j')=0$ for all j'. From (26) we note that the velocity at the end of time step $q(i',j'+1), i'=1,2,\ldots\ldots\ldots\ldots,40$ is computed in terms of velocities and temperatures at points on earlier time step. Similarly, $\theta(i',j'+1)$ is computed from equation (27). The procedure is repeated till $j'=400$ i.e. up to time $t=0.5$. During the computation, to test the convergence and stability of the finite difference scheme, computations were made with smaller values of t, viz $t=0.0009, 0.001$ and 0.0002. In our analysis we noted that increasing the number of mesh points by using smaller values of Δt does not have a significant effect on the result, thus the finite difference scheme used is stable and convergent.

In order to get the physical understanding of this problem and for the purpose of discussing the results, the numerical calculations have been carried out as explained above for both velocity and temperature. In our calculations the Prandtl number is taken to be equal to $0\cdot 7$ which corresponds to air and magnetic parameter $M_{1}^{2}=5.0$ which signifies strong magnetic field. The calculations were carried out for both $Gr>0(=5.0)$ in the presence of cooling of the plate by free convection currents) and $Gr<0(=-0.5)$ in the presence of heating of the plate by free-convection currents).Now the results obtained for the unsteady flow for various parameters are displayed in Figures (to be sated after the results).In the next section, a presentation of the numerical method employed in computing the skin friction and the rate of heat transfer at the plate is done.

3.1 Calculation of the skin friction and rate of heat transfer

After obtaining the velocity and temperature distributions of the flow as explained in the previous section we now compute the skin friction given by

$$\tau = -\left.\frac{\partial q}{\partial z}\right|_{z=0} \tag{32}$$

Where $\tau = \frac{\tau^*}{\rho u_o^2}$ .On the other hand the heat flux $\dot{q}$ at the wall is given by

$$\dot{q} = -\left.\frac{\partial \theta}{\partial z}\right|_{z=0} \tag{33}$$

In order to solve equations (32) and (33) we apply a second-order least squares correlation used over the gradients of the first ten points.

4 Discussion of results

A program was written and run for different values of non-dimensional parameters to determine velocity profiles, temperature profiles and concentration profiles when the plate was conventionally heated or conventionally cooled. The Non-dimensional parameters which were used are rotational parameter, Hall parameter, Eckert number, suction parameter, magnetic parameter and injection parameter.

The velocities were classified as primary and secondary velocity along x and y-axes. Numerical computations for the velocities (both primary and secondary) profiles, temperature profiles and concentration profiles were obtained and the unsteady flow results obtained were presented in form of graphs as in Figures 4.1 to 4.8. Prandtl number Pr and magnetic parameter M used were 0.71 and 5.0 respectively. The magnetic parameter M^2 =5.0 signifies strong Magnetic field. Gr>0 (=5) corresponds to cooling of the plate by free convection currents since the plate is at a higher temperature than the surrounding and Gr<0(= -5) to heating of the plate by free convection currents since the plate is at lower temperature than the surrounding.

Case 1: Heating at the Plate

In the presence of heating of the plate by free convection current for the case of Gr<0 (=-5).

From figure 2 and 3 we observe that:

- For Ec= 0.02, a decrease in the rotational parameter leads to an increase in both the primary and secondary velocity profilers. Since the wall moves in opposite direction to that of the free stream, it tends to retard the flow. Similarly, the convectional currents due to rotation cause the fluid to retard in motion.
- An increase in the Magnetic parameter leads to an increase in both the primary velocity and the secondary velocity profiles. Inclusion of Hall parameter decreases the resistive force imposed by the magnetic field due to its effect in reducing the effective conductivity $\frac{\sigma}{1+m^2}$.
- We also observe that an increase in the suction parameter causes no effect in the primary velocity profile but decreases secondary velocity profiles. Introduction of suction retards fluid flow due to increase of convection currents of the fluid across the plates.
- Removal of injection increases both the primary and secondary velocity profiles. This is because the convectional currents of the fluid are reduced.
- An increase in Eckert number results to an increase in the primary velocity and secondary velocity profiles. This is due to the fact that an increase in Ec increases convectional currents which cause a slight decrease in the primary velocity.

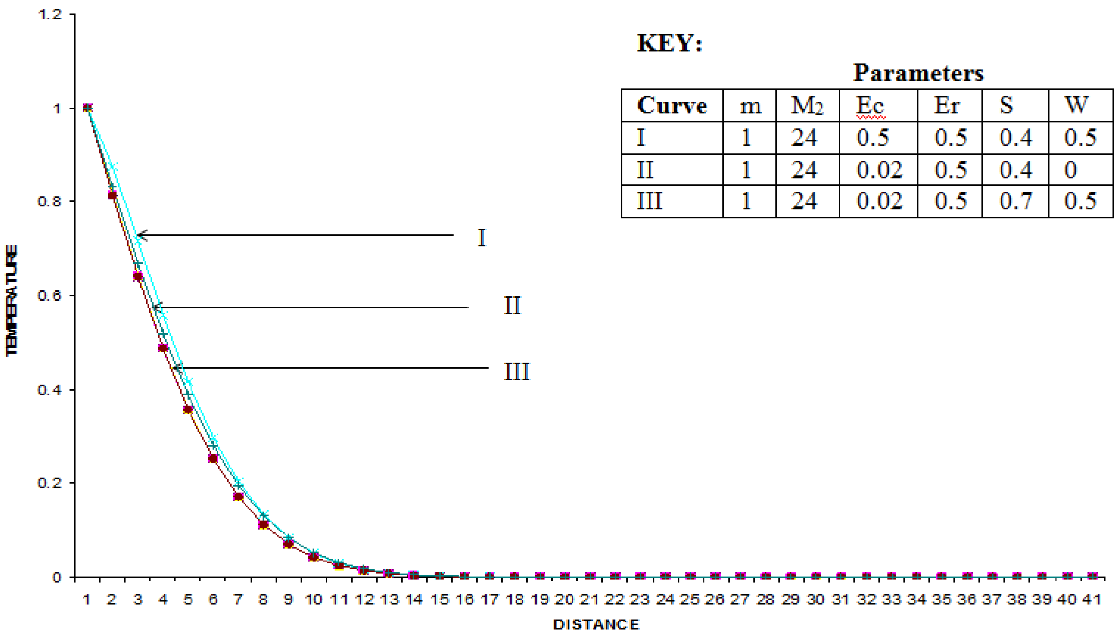


Fig 2: Primary velocity profiles in the presence of heating of the plate by free convection current for the case of Gr<0 (= - 5)

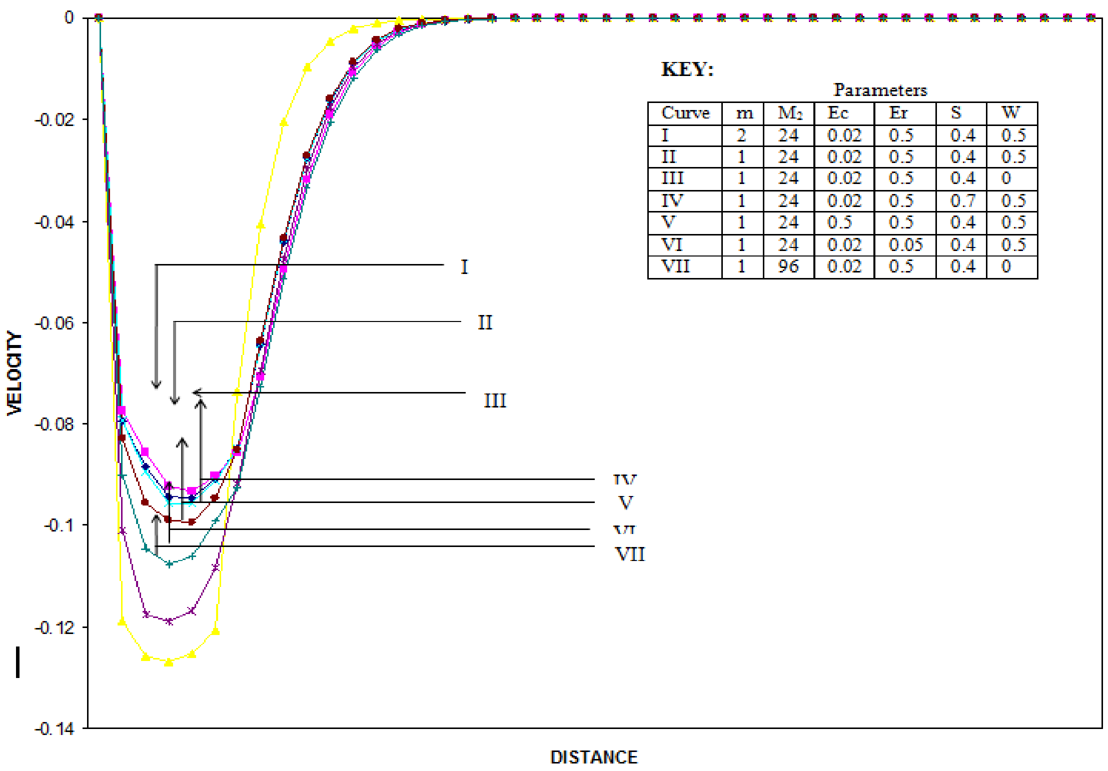


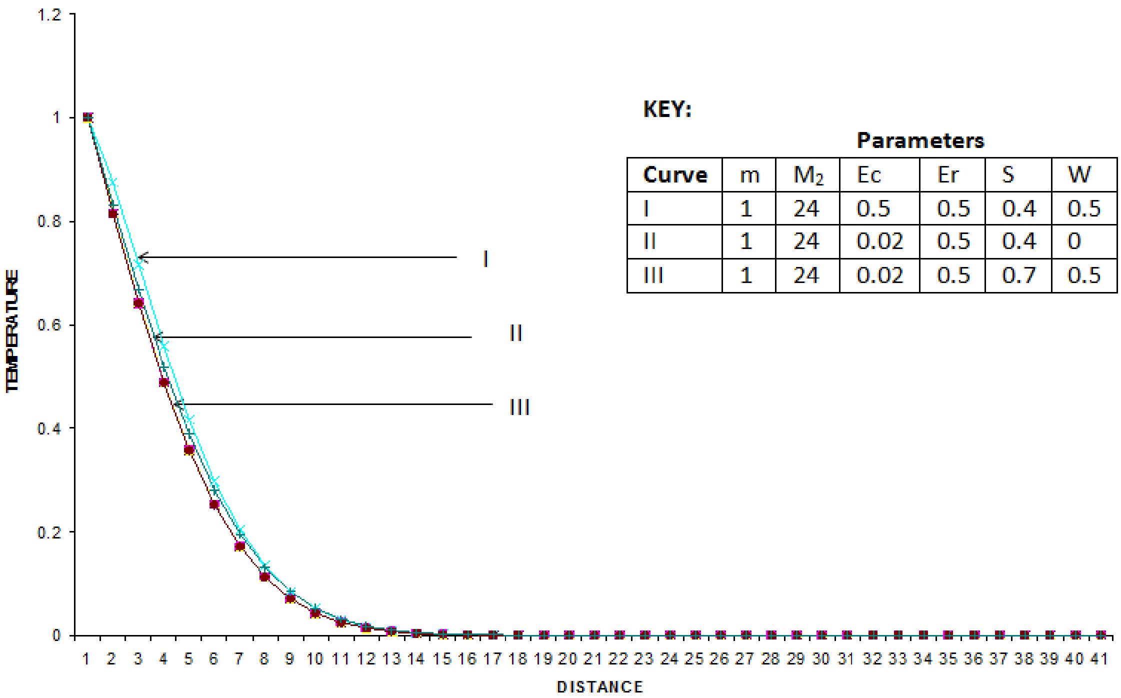
Fig 3: Secondary velocity profiles in the presence of heating of the plate by free convection current for the case of Gr<0 (= -5)

From Fig 4: (Temperature profile for Gr< 0) we observe that;

- A decrease in the Rotational parameter Er gives rise to an increase in the temperature profile. The rotation causes the circulation of induced currents at the surface of the fluid, that is, the increase of the temperature affects the current distribution. Rotation leads up to additional transport; this contribution is a consequence of the decrease of the ion rotation. Viscous dissipation would immediately lead to an increase of ion-temperature, increasing ion momentum and thermal transport.
- An increase in Hall parameter causes a decrease in the temperature profile. As the distance from the plate increases, these profiles increase. However, as the distance from the plate increases these profiles remain

constant. Further, an increase of Hall parameter increases cyclotron frequency and hence the rotation and collision of electrons increases. An increase in Hall parameter leads to a decrease in the effective conductivity ($\frac{\sigma}{1+m^2}$) which reduces magnetic damping force on the velocity and thus the velocity increases.

- Increase in Eckert number leads to an increase in the temperature profiles. Increasing the Eckert number causes the fluid to become warmer and therefore increase its temperature. This is attributed to the viscous dissipation.
- Increase in the Magnetic parameter leads to an increase in the temperature profiles. The increase in the fluid temperature induces more flow in the boundary layer causing the velocity of the fluid there to increase. The magnetic field produces a huge increment in the magnitude of the temperature. This can be explained physically as follows: it is well known that a magnetic field imparts some rigidity to the conducting fluid. Thus, with increase in the magnetic field, greater effort will be necessary to maintain the rotation of the plate and this implies an increase in temperature with an increase of the parameter M.



Curve	m	M_2	Ec	Er	S	W
I	1	24	0.5	0.5	0.4	0.5
II	1	24	0.02	0.5	0.4	0
III	1	24	0.02	0.5	0.7	0.5

Fig 4: Temperature profiles in the presence of heating of the plate by free convection current for the case of Gr <0 (= -5)

From Fig 5: (Concentration profile for Gr< 0) we observe that:

- A decrease in the Rotational parameter Er has no effect to the concentration profiles. Rotation has been achieved by a transfer of angular momentum. Once this is drastically reduced, the rate at which the particles move and collide is too small such that the change is insignificant.
- An increase in Hall parameter has no effect to the concentration profiles. An increase in Hall parameter which is due to the increase of collisions has no effect to the concentration profile. This is because there is no change in the charge carriers hence the effect is neutralized. Since no polarization voltage is imposed on the fluid, the concentration profile is not affected.
- Removal of injection causes a rise in concentration profile. Removal of injection means an increase in the molecular diffusivity which consequently results in the rise of the concentration.
- Increase in Eckert number has no effect to the concentration profiles. The increase in an Eckert number increases thermal energy which consequently increases temperature and this does not affect the concentration of the fluid but increases the mass diffusion.
- Increase in suction parameter leads to a decrease in the concentration profiles. Suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth. Sucking decelerates the fluid particles through the porous wall hence reducing the concentration.

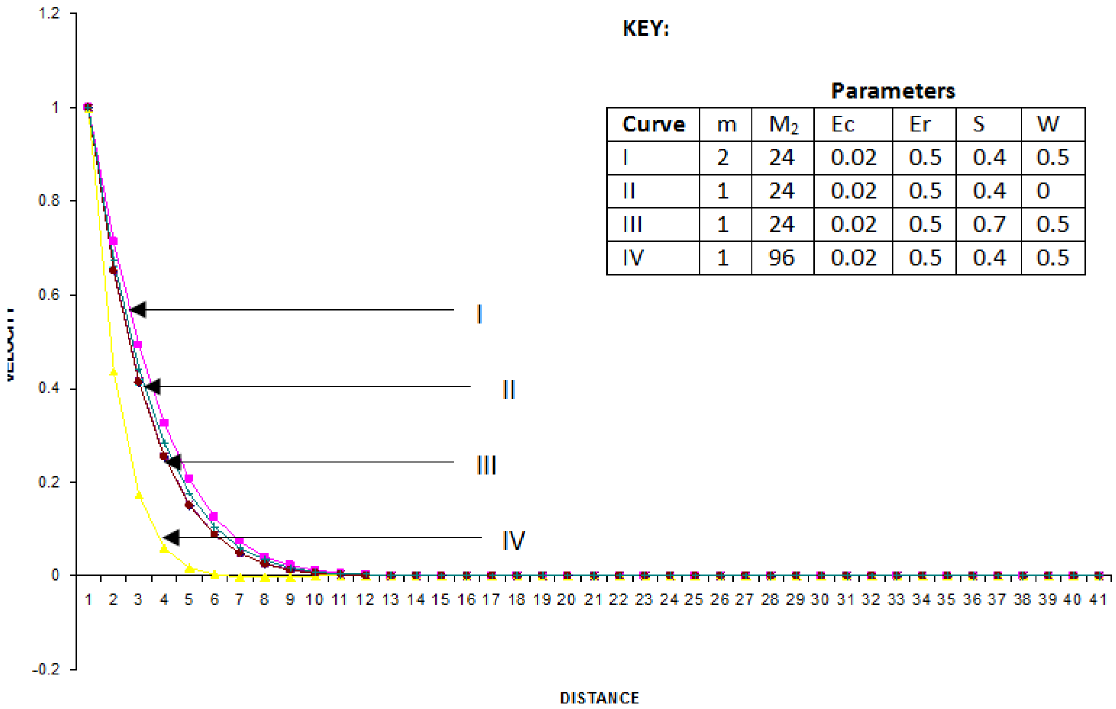


Fig 5**:** Concentration profiles in the presence of heating of the plate by free convection current for the case of Gr<0 (= -5)

Case 2. Cooling at the Plate

In the presence of cooling of the plate by free convectional currents i.e. when the Grashof number is greater than zero (equal to five) from Figure 6 and Figure 7 we note that:

- For Ec = 0.02, a decrease in the rotational parameter leads to an increase in both the primary and secondary velocity profiles. This is because the presence of the transverse magnetic field creates a resistive force similar to the drag force that acts in the opposite direction of the fluid; thus causing the velocity of the fluid to decrease.
- An increase in the Magnetic parameter leads to a decrease in the primary velocity and an increase in the secondary velocity profiles. Due the Lorentz force, there is a resistive force along the x-axis and this reduces the primary velocity but the secondary velocity profile increases since it is in the direction of the induced force.
- An increase in Hall parameter leads to an increase in the primary velocity and a decrease in the secondary velocity profiles. When the Hall parameter is increased the induced current along x-axis increases and this translates to an increase in the primary velocity while the induced current along the y-axis decreases slightly and thus a reduction in the secondary velocity profiles.
- We also observe that an increase in the suction parameter causes no effect in the primary velocity profiles but a decrease in secondary velocity profiles. Increasing suction parameter means pumping more fluid to the surface of the plate. This does not affect the primary velocity profiles but decreases secondary velocity profiles since action and reaction forces which are in play are equal and opposite in nature.
- Removal of injection increases both the primary and secondary velocity profiles. This is because convectional currents which are interfering with the fluid flow are reduced and thereby increasing both primary and secondary velocity profiles.
- An increase in Eckert number results to an increase in both the primary and secondary velocity profiles. An increase in Eckert number means an increase in kinetic energy of the fluid particles and for this reason both primary and secondary velocity profiles.

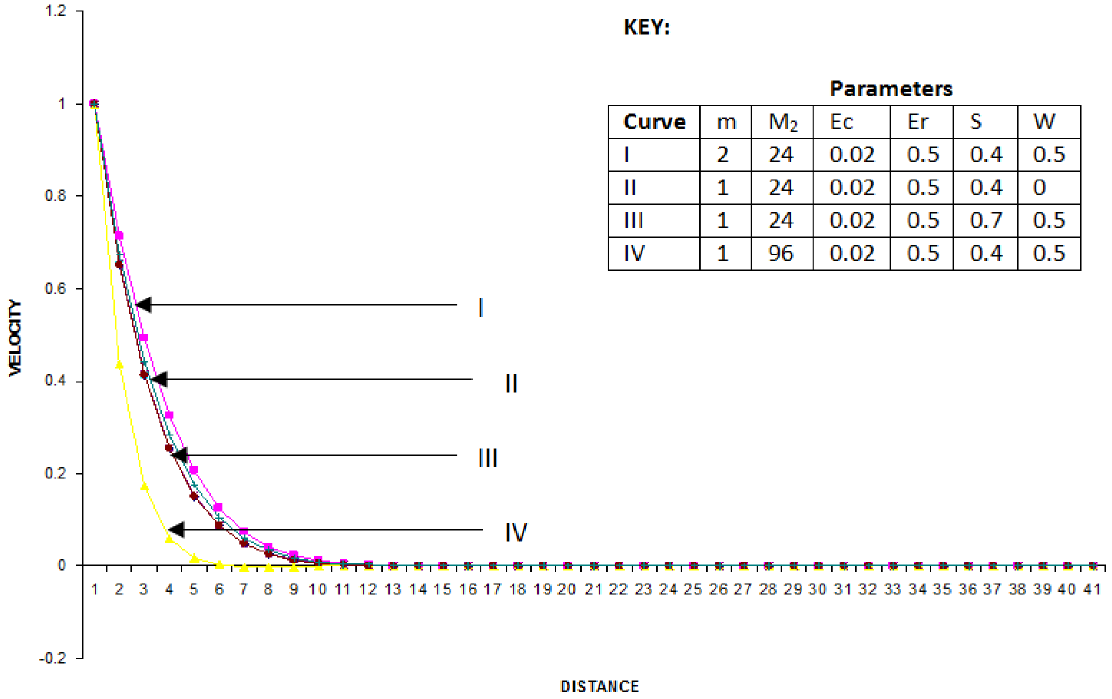


Fig 6: Primary Velocity profiles in the presence of cooling of the plate by free convection current for the case of Gr>0 (= 5)

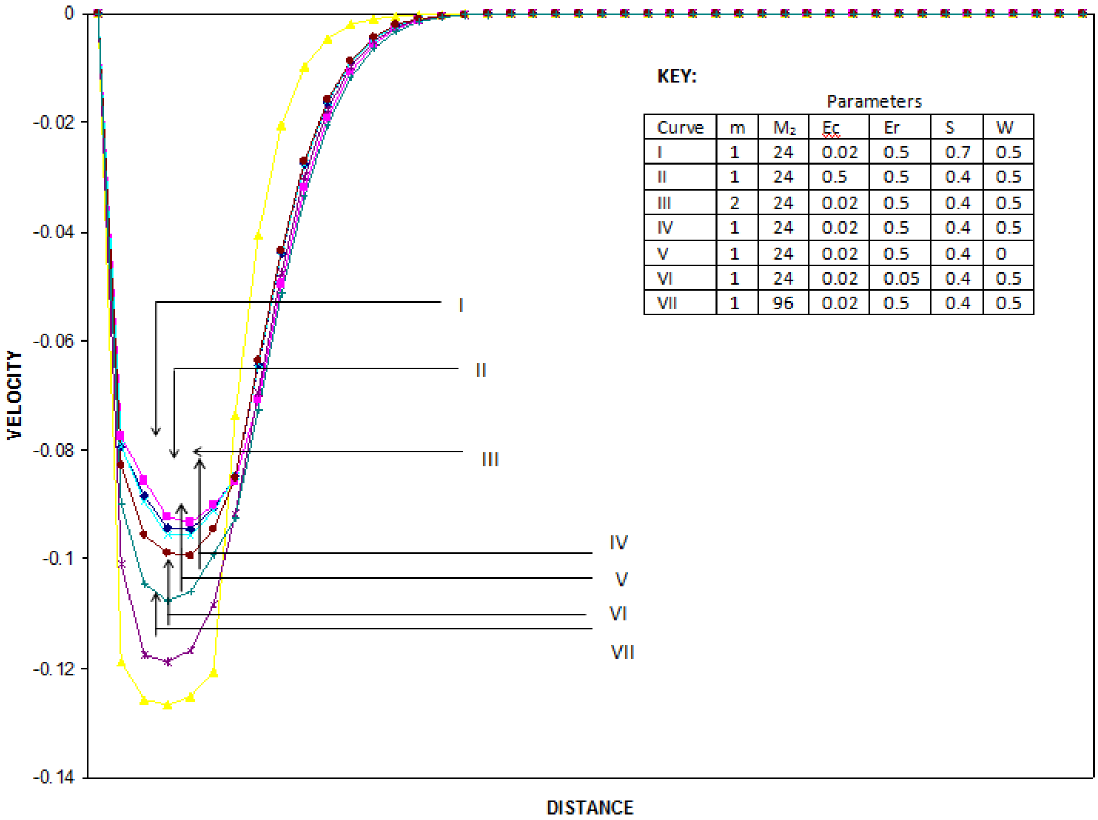


Fig 7: Secondary Velocity Profiles in the presence of cooling of the plate by free convection current for the case of Gr>0 (= 5)

From Fig 8: Temperature profiles for Gr>0 we observe that:

- A decrease in the Rotational parameter Er, leads to an increase in the temperature profiles. The rotation causes the circulation of induced currents at the surface of the fluid, that is, the increase of the temperature affects the current distribution. Rotation leads up to additional transport; this contribution is a consequence of the decrease of the ion rotation. Viscous dissipation would immediately lead to an increase of ion-temperature, thus increasing ion momentum and thermal transport.
- An increase in Hall parameter causes an increase in the temperature profile. This is because in Hall parameter means an increase of ion collisions which translates to more thermal generation hence increasing the temperature profiles.
- Removal of injection causes a rise in temperature profiles. The high injection current causes a strong self-heating effect which reduces the quantum of the particles.
- Increase in Eckert number leads to an increase in the temperature profiles. Increasing the Eckert number causes the fluid to become warmer and therefore increase its temperature. This is attributed to the viscous dissipation. Increasing Ec can lead to a situation that the viscous dissipation becomes significant hence increasing the temperature.
- Increase in suction parameter has no effect on the temperature profiles. Increase in suction means a decrease in molecular diffusivity (D) and this means that the temperature profiles are not altered.

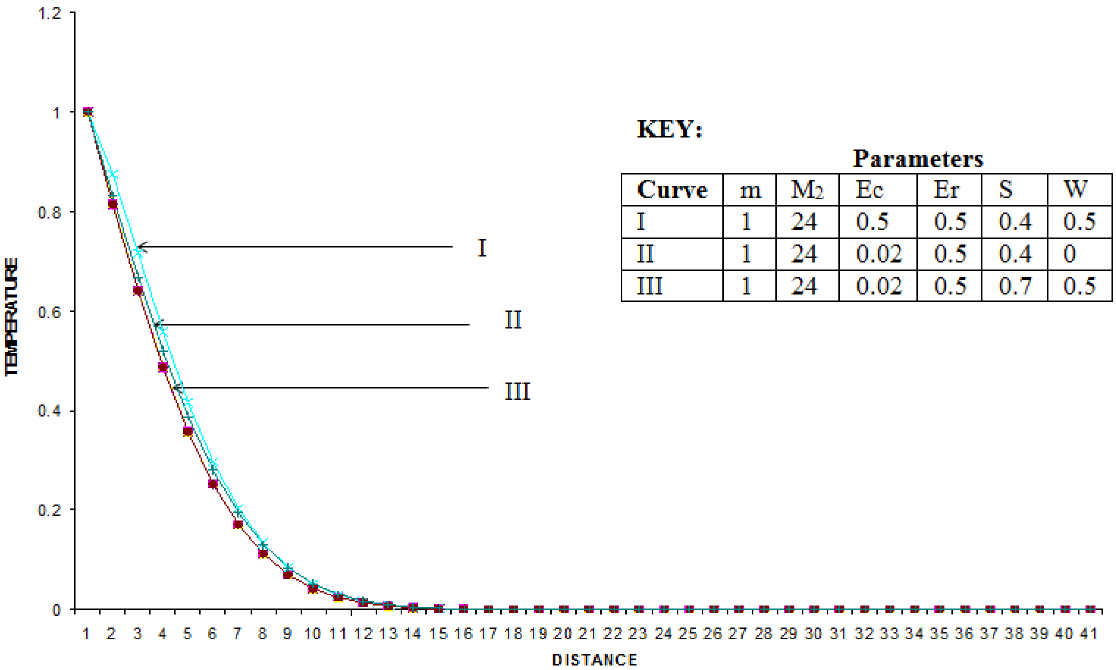


Fig 8: Temperature profiles in the presence of cooling of the plate by free convection current for the case of Gr>0 (= 5)

From fig 9: Concentration profile for Gr > 0 we observe that;

- A decrease in the Rotational parameter Er has no effect to the concentration profiles. Rotation has been achieved by a transfer of angular momentum. Once this is drastically reduced, the rate at which the particles move and collide is too small such that the change is insignificant.
- An increase in Hall parameter has no effect to the concentration profile. An increase in Hall parameter which is due to the increase of collisions has no effect to the concentration profile. This is because there is no change in the charge carriers hence the effect is neutralized. Since no polarization voltage is imposed on the fluid, the concentration profile is not affected.
- Removal of injection causes a rise in concentration profiles. Removal of injection means an increase in the molecular diffusivity which consequently results in the rise of the concentration.
- Increase in the Magnetic parameter has no effect to concentration profiles. As M increases, the Lorentz force which tends to oppose the flow also increases. The effect is enhanced deceleration which when combined with the momentum diffusivity has no effect to concentration. Again there is no increment in the buoyancy ratio hence no change in the concentration.

- Increase in suction parameter leads to a decrease in the concentration profiles. Suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth. Sucking decelerates the fluid particles through the porous wall hence reducing the concentration.

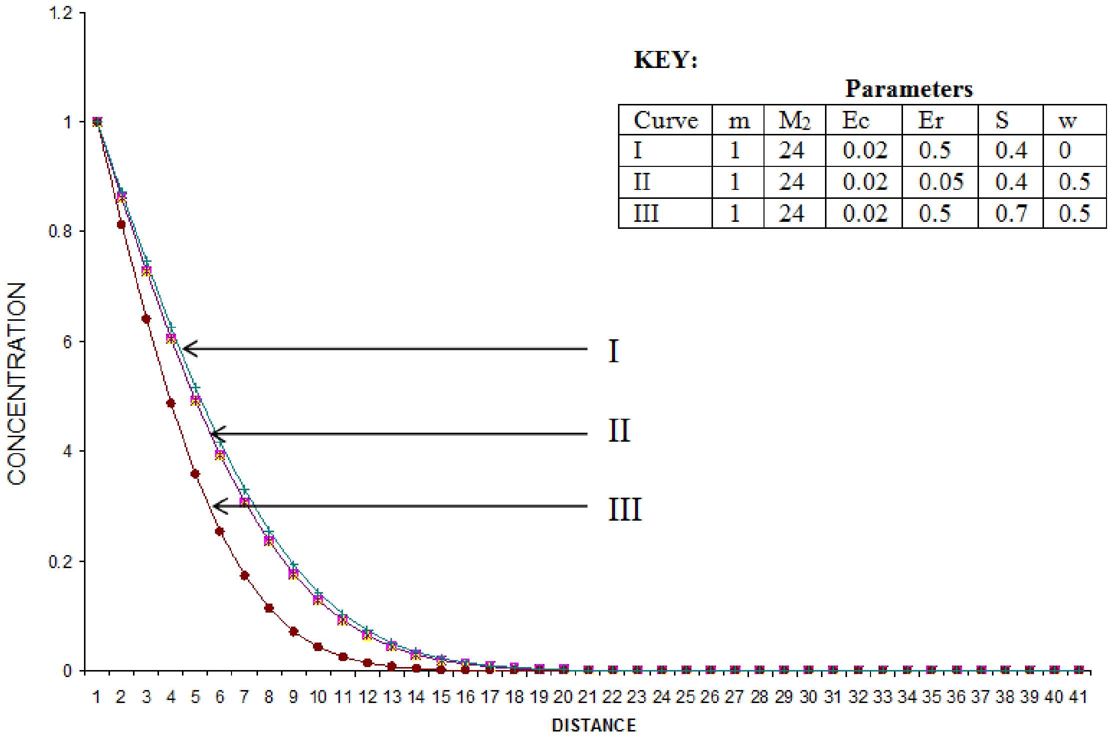


Fig 9: Concentration profiles of cooling of the plate by free convection current for the case of Gr>0 (= 5)

From Table 1 we observe that with Gr=5.0

- Removal of injection leads to a decrease in the rate of heat transfer.
  Removal of injection means the reduction of the particles which were causing collisions resulting in heat changes. This means that the collision times, allows the removal of air and the consequence is reduced heat transfer.
- An increase in suction parameter or magnetic parameter leads to an increase in the rate of heat transfer. The increase of S means a decrease of molecular diffusivity (D) that result in increment of the rate of heat transfer. The application of the externally variable magnetic field reduces the velocity vectors and leads to an increase in the rate of heat transfer.
- A decrease in rotational parameter Er leads to an increase in the rate of heat transfer. Due the presence of the Lorentz force and the gravitational force rotating at very low speeds, a friction factor is realized that results in thermal dispersion thereby increasing the rate of heat transfer.
- Removal of w decreases the rate of heat transfer. Injection causes convectional currents on the plate and this reduces the rate of heat transfer.
- Increase in the rotational parameter m leads to a decrease in the heat transfer. This reduction is due to the increase in the momentum, thermal and magnetic boundary layer thickness which in turn are caused by the deceleration of the magnetic field.

Table 1: Rate of heat transfer Nu with cooling Pr=0.71, Gr=5.0

m	M_2	Ec	Er	S	w	Nu
1	24	0.02	0.5	0.4	0.5	2.661606153
2	24	0.02	0.5	0.4	0.5	2.659182396
1	96	0.02	0.5	0.4	0.5	2.67362258
1	24	0.5	0.5	0.4	0.5	2.478347258
1	24	0.02	0.05	0.4	0.5	2.661558481
1	24	0.02	0.5	0.7	0.5	2.661558481
1	24	0.02	0.5	0.4	0	2.531911988

From Table 2 it is observed that with Gr=5.0

- Removal of injection parameter leads to an increase in (τ_x) but a decrease in (τ_y). The removal of injection parameter means reduction of the particles which were causing collisions this increases skin friction along x-axis and a decrease in skin friction along y-axis.
- An increase in Hall parameter leads to a decrease in (τ_x) but an increase in (τ_y). The skin friction in the y-direction tends to be negative since it is in the opposite direction to that of gravitational force.
- A decrease in rotational parameter Er lead to an increase in both (τ_x) and (τ_y). Due the presence of the Lorentz force and the gravitational force rotating at very low speeds, a friction factor is realized and hence an increase in both (τ_x) and (τ_y).
- Increase in Magnetic parameter leads to a rise in both (τ_x) and (τ_y). The application of the externally variable magnetic field reduces the velocity vectors and since velocity is inversely proportional to frictional force and this means that both (τ_x) and (τ_y) increases.
- Increase in S leads to decrease in both (τ_x) and (τ_y). Basically suction stabilizes the hydrodynamic, thermal as well as concentration boundary layers growth. Sucking decelerates the fluid particles through the porous wall, consequently this leads to decrease in both (τ_x) and (τ_y).

Table 2: Skin frictions with cooling Pr=0.71, Gr=5.0

m	M_2	Ec	Er	S	w	τ_x	τ_y
1	24	0.02	0.5	0.4	0.5	3.303558315	0.07671002
2	24	0.02	0.5	0.4	0.5	3.224548046	0.104524876
1	96	0.02	0.5	0.4	0.5	3.388281036	0.322132187
1	24	0.5	0.5	0.4	0.5	3.300116591	0.077290887
1	24	0.02	0.05	0.4	0.5	3.303796334	0.082941388
1	24	0.02	0.5	0.7	0.5	3.311764194	0.07528562
1	24	0.02	0.5	0.4	0	3.223194216	0.121241577

Table 3: Rate of heat transfer Nu with heating Pr=0.71, Gr=-5.0

m	M_2	Ec	Er	S	w	Nu
1	24	0.02	0.5	0.4	0.5	2.661961166
2	24	0.02	0.5	0.4	0.5	2.659542075
1	96	0.02	0.5	0.4	0.5	2.674019269
1	24	0.5	0.5	0.4	0.5	2.48783689
1	24	0.02	0.05	0.4	0.5	2.661914493
1	24	0.02	0.5	0.7	0.5	2.661914493
1	24	0.02	0.5	0.4	0	2.532264669

Skin Friction along x-axis and along the y-axis

From table 4, we note that

i. An increase in the rotation parameter Er leads to an increase in (τ_x) and a decrease in (τ_y).

ii. An increase in the Eckert number Ec leads to an increase in both (τ_x) and (τ_y)

iii. An increase in the Hall parameter m leads to a decrease in both (τ_x) and (τ_y). The skin friction in the y-direction is negative since it is in the opposite direction to that of gravitational force.

iv. An increase in magnetic parameter M_2 leads to a decrease in (τ_x) and an increase in (τ_y).

Table 4: Skin frictions with heating Pr=0.71, Gr=-5.0

m	M_2	Ec	Er	S	w	τ_x	τ_y
1	24	0.02	0.5	0.4	0.5	3.324194444	0.071301668
2	24	0.02	0.5	0.4	0.5	3.249333774	0.099589728
1	96	0.02	0.5	0.4	0.5	3.393047856	-0.3369896
1	24	0.5	0.5	0.4	0.5	3.327540829	0.070710633
1	24	0.02	0.05	0.4	0.5	3.3245373	0.077084675
1	24	0.02	0.5	0.7	0.5	3.332400323	0.069874435
1	24	0.02	0.5	0.4	0	3.24851838	0.115209036

5 Conclusion

The following conclusions on the effects of each non-dimensional parameter were made;
Increase of suction parameter: There was no effect on primary velocity profiles, temperature profiles in both free convectional heating and cooling of the plate, though there was a decrease in; secondary velocity profiles, concentration profiles, rate of heat transfer and skin friction along y-axis. Free convectional heating of the plate increased skin friction along x-axis.
Removal of injection: Both free convectional heating and cooling of the plate increased primary and secondary velocity profiles, concentration profiles, temperature profiles and skin friction along x-axis, but there was a decrease in the rate of heat transfer and skin friction along y-axis.
Increase in Eckert number: In both free convectional heating and cooling of the plate, there was an increase in primary and secondary velocity profiles and temperature profiles, but a decrease in heat transfer on heating of the plate. In both cases, there was no effect on concentration profiles.
Increase of magnetic parameter: An increase of magnetic parameter free convectional heating and cooling of the plate increased secondary velocity profiles, rate of heat transfer and skin friction both along x-axis and y-axis, but there was no effect on concentration profiles. The free convectional heating of the plate increased primary velocity profiles but decreased primary velocity profiles on free convectional cooling of the plate.
Decrease of rotational parameter: A decrease of rotational parameter in both free convectional heating and cooling of the plate increased primary and secondary velocity profiles, temperature profiles and both skin friction along x-axis and y-axis, but there was no effect on concentration profiles. The rate of heat transfer decreased on convectional heating of the plate while it increased on convectional cooling of the plate.
Increase of hall parameter: Increase of hall parameter both on free convectional cooling and heating increased skin friction along y-axis and decreased skin friction along x-axis but there was no effect on concentration. It was noted that there was a decrease in temperature profiles and the rate of heat transfer in free convectional heating on the plate and an increase of primary velocity profiles and temperature profiles in free convectional cooling on the plate.

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NOMENCLATURE	**ROMAN SYMBOLS QUANTITY**
$\vec{E}$	Electric intensity vector (V/m)
$\vec{F}$	Body force vector (N)
e	Unit charge (C)
L	Characteristic length (m)
$\vec{J}$	Current density vector (Am^{-2})
P	Pressure force vector (Nm^{-2})
U	Characteristic velocity (ms^{-1})
t^*	Dimensional Time (S)
κ	Thermal conductivity (Wm^{-1} k^{-1})
$\vec{q}$	Velocity vector (ms^{-1})
$\vec{B}$	Magnetic field vector (Wbm^{-2})
$\vec{D}$	Electric displacement vector (cm^{-2})
$\vec{H}$	Magnetic field intensity vector (Wbm^{-2})
$\vec{i}, \vec{j}, \vec{k}$	Unit vectors in the x, y and z directions respectively
u, v, w	Components of velocity vector q
$\vec{F}_e$	Electromagnetic force (kgm^{-2})
g	Acceleration due to gravity (ms^{-2})
$\frac{D}{Dt}\left(=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}\right)$	Material derivative
Q	Amount of heat added to the system (Nm)
h	Dimensional distance between vertical Plates
T	General fluid temperature
C_p	Specific heat at constant pressure (Jkg^{-1} K^{-1})
U_∞	Free stream fluid velocity (ms^{-1})
T_∞	Characteristic free stream temperature (K)
m	Hall parameter

GREEK SYMBOL	**QUANTITY**
μ	Coefficient of Viscosity, Kg/ms.
ν	Kinematic Viscosity m2s-1
ρ	Fluid density, kg/m3.
ρ_e	Electrical charge density (cm-2)
σ	Electrical conductivity (Ω-1m-1)
μ_e	Magnetic permeability (Hm-1)
$\Delta t, \Delta y, \Delta z$	Time and distance intervals respectively(s, m)
ΔT	Temperature change (K)
∇	Gradient operator $\left(= i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)$
∇^2	Laplacian operator $\left(=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)$
φ	Viscous dissipation function (s2)
α	Electrical conductivity

β	Coefficient of thermal expansion, K1 $\left[\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p\right]$

DIMENSIONLESS QUANTITIES

θ	Dimensionless fluid temperature
U, V, W	Dimensionless fluid velocity
x, y, z	Dimensionless Cartesian coordinates
t	Dimensionless time
E_c	Eckert number $\left\{=\frac{U^2}{C_p\left(T-T_\infty\right)}\right\}$
Pr	Prandtl number $\left(=\frac{\mu Cp}{k}\right)$
Rm	Magnetic Reynolds number $\left(=\sigma\mu_c Lu\right)$
Nu	Nusselt number $\left(=\frac{hL}{\kappa}\right)$
S	Magnetic force number $\left(=\frac{Ho}{L}\frac{\sqrt{\mu c}}{\rho}\right)$
M	Magnetic Parameter $\left(=\sqrt{\frac{\sigma Ho^2\nu}{\mu U_m^2/\nu}}\right)$
Gr	Grashof number $\left(=\frac{g\beta\Delta T l^3}{\nu^2}\right)$

ABBREVIATIONS

MHD	Magnetohydrodynamics
FDM	Finite Difference Method