International Journal of Applied Mathematical Research, 3 (2) (2014) 122-132
©Science Publishing Corporation
www.sciencepubco.com/index.php/IJAMR
doi: 10.14419/ijamr.v3i2.2116
Research Paper

Exact solutions of the ZK-MEW equation and the Davey-Stewartson equation

E. M. E. Zayed [1*], Ali Filiz [2], Mehmet Ekici [3], Abdullah Sonmezoglu [3], Durgun Duran [4]

[1] *Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt*
[2] *Department of Mathematics, Faculty of Science and Arts, Adnan Menderes University, 09010 Aydin, Turkey*
[3] *Department of Mathematics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey*
[4] *Department of Physics, Faculty of Science and Arts, Bozok University, 66100 Yozgat, Turkey*
**Corresponding author E-mail: e.m.e.zayed@hotmail.com*

Abstract

In this paper we introduce a new version of the trial equation method for solving non-integrable partial differential equations in mathematical physics. Some exact solutions including soliton solutions, rational and elliptic function solutions to the generalized (2+1)-dimensional ZK-MEW equation and the generalized Davey-Stewartson equation with the complex coefficients are obtained by this method.

Keywords: *Extended trial equation method, generalized (2+1)-dimensional ZK-MEW equation, Davey-Stewartson equation, soliton solution, elliptic solutions.*

1. Introduction

The investigation of exact solutions of nonlinear evolution equations (NLEEs) plays a crucial role in the analysis of some physical phenomena. It is difficult to obtain the exact solution for these problems. In recent decades, there has been great development in exact solution for nonlinear partial differential equations (PDEs). Many powerful methods, such as the Backlund transformation, the inverse scattering method [1], bilinear transformation, the tanh-sech method [2], the extended tanh method, the pseudo-spectral method [3], the trial function and the sine-cosine method [4], the Hirota method [5], the tanh-coth method [6-7], the exponential function method [8], the (G'/G)-expansion method [9-13], the homogeneous balance method [14], the F-expansion method [15-20], the trial equation method [21-31] have been used to investigate nonlinear partial differential equations problems. The types of solutions of NLEEs, that are integrated using various mathematical techniques, are very important and appear in various areas of physics, applied mathematics and engineering.
The spatially one-dimensional KdV equation

$$u_t + \alpha u u_x + u_{xxx} = 0,$$

governs the one-dimensional propagation of small-amplitude weakly dispersive waves, and plays a major role in the soliton concept. The term soliton was coined by Zabusky and Kruskal [32] who found particle-like waves which retained their shapes and velocities after collisions. The balance between the non-linear convection term uu_x and the dispersion effect term u_{xxx} in the KdV equation gives rise to solitons. Solitons are defined as non-linear waves characterized as follows [33]:

- localized waves that propagate without change of its shape, velocity, etc.;
- localized waves that are stable against mutual collisions and retain their identities to indicate that soliton has the property of a particle.

In this paper, extended trial equation methods is used to obtain a generalized soliton solution with some free parameters of the generalized (2+1)-dimensional Zakharov-Kuznetsov-Modified Equal-Width (ZK-MEW) equation [34,35]

$$u_t + \alpha(u^n)_x + (\beta u_{xt} + \gamma u_{yy})_x = 0. \tag{1}$$

and generalized Davey-Stewartson equation (DSE) that arises in the study of fluid dynamics [36,37]

$$iq_t + a(q_{xx} + q_{yy}) + b|q|^{2n} q = \alpha qr \tag{2}$$

$$r_{xx} + r_{yy} + \beta(|q|^{2n})_{xx} = 0. \tag{3}$$

Exact solutions of the ZK-MEW equation were obtained both by using the tanh and sine-cosine methods by Wazwaz [34] and the modified simple equation method by Zayed and Arnous [35]. The Cauchy problem of the generalized Davey-Stewartson systems and the global solvability and existence of self-similar solutions to a generalized Davey-Stewartson system were studied in some sense by Zhao [38]. Ebadi and Biswas studied by applying the (G'/G)-method carry out the integration of Davey-Stewartson equation [36] while Bekir and Cevikel have solved them using the sine-cosine and the exp-function methods [37]. Subsequently, using the ansatz method this equation is integrated in $(1 + 2)$-dimensions with power law nonlinearity. Here, we use the extended trial equation method to solve the soliton solutions of generalized $(1 + 2)$-dimensional ZK-MEW equation and generalized Davey-Stewartson equation with the complex coefficients. The extended trial equation method will be employed to back up our analysis in obtaining exact solutions with distinct physical structures.

2. The extended trial equation method

Step 1. For a given nonlinear partial differential equation with rank inhomogeneous

$$P(u, u_t, u_x, u_{xx}, \dots) = 0, \tag{4}$$

take the wave transformation

$$u(x_1, \dots, x_N, t) = u(\eta), \quad \eta = \lambda\left(\sum_{j=1}^{N} x_j - ct\right), \tag{5}$$

where $\lambda \neq 0$ and $c \neq 0$. Substituting Eq. (5) into Eq. (4) yields a nonlinear ordinary differential equation,

$$N(u, u', u'', ...) = 0. \tag{6}$$

Step 2. Take transformation and trial equation as follows:

$$u = \sum_{i=0}^{\delta} \tau_i \Gamma^i, \tag{7}$$

in which

$$(\Gamma')^2 = \Lambda(\Gamma) = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} = \frac{\xi_\theta \Gamma^\theta + ... + \xi_1 \Gamma + \xi_0}{\zeta_\epsilon \Gamma^\epsilon + ... + \zeta_1 \Gamma + \zeta_0}, \tag{8}$$

where $\tau_i \,(i = 0, ..., \delta)$, $\xi_i \,(i = 0, ..., \theta)$ and $\zeta_i \,(i = 0, ..., \epsilon)$ are constants. Using the relations (7) and (8), we can find

$$(u')^2 = \frac{\Phi(\Gamma)}{\Psi(\Gamma)}\left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1}\right)^2, \tag{9}$$

$$u'' = \frac{\Phi'(\Gamma)\Psi(\Gamma) - \Phi(\Gamma)\Psi'(\Gamma)}{2\Psi^2(\Gamma)} \left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1} \right) + \frac{\Phi(\Gamma)}{\Psi(\Gamma)} \left(\sum_{i=0}^{\delta} i(i-1)\tau_i \Gamma^{i-2} \right), \tag{10}$$

where $\Phi(\Gamma)$ and $\Psi(\Gamma)$ are polynomials. Substituting these terms into Eq. (6) yields an equation of polynomial $\Omega(\Gamma)$ of Γ :

$$\Omega(\Gamma) = \varrho_s \Gamma^s + ... + \varrho_1 \Gamma + \varrho_0 = 0. \tag{11}$$

According to the balance principle we can determine a relation of θ, ϵ, and δ. We can take some values of θ, ϵ, and δ.

Step 3. Let the coefficients of $\Omega(\Gamma)$ all be zero will yield an algebraic equations system:

$$\varrho_i = 0, \quad i = 0, ..., s. \tag{12}$$

Solving this equations system (12), we will determine the values of $\xi_0, ..., \xi_\theta$; $\zeta_0, ..., \zeta_\epsilon$ and $\tau_0, ..., \tau_\delta$.

Step 4. Reduce Eq. (8) to the elementary integral form,

$$\pm(\eta - \eta_0) = \int \frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}} = \int \sqrt{\frac{\Psi(\Gamma)}{\Phi(\Gamma)}} d\Gamma. \tag{13}$$

Using a complete discrimination system for polynomial to classify the roots of $\Phi(\Gamma)$, we solve the infinite integral (13) and obtain the exact solutions to Eq. (6). Furthermore, we can write the exact traveling wave solutions to Eq. (4) respectively.

3. Applications

To illustrate the necessity of our new view concerning the trial equation method, we introduce two case studies.

Example 3.1 *Application to the generalized (2+1)-dimensional ZK-MEW equation.*

The generalized (2+1)-dimensional ZK-MEW equation [34,35] is in the form of

$$u_t + \alpha(u^n)_x + (\beta u_{xt} + \gamma u_{yy})_x = 0, \quad (n > 1)$$

where α, β and γ are arbitrary constants.

In order to look for travelling wave solutions of Eq. (1), we make the transformation

$$u(x, y, t) = u(\eta), \quad \eta = \kappa_1 x + \kappa_2 y - ct,$$

where κ_1, κ_2 and c are real constants. Then, integrating the resulting equation with respect to η and setting the integration constant to zero yield the ordinary differential equation

$$-cu + \alpha\kappa_1 u^n + (\gamma\kappa_1\kappa_2^2 - c\beta\kappa_1^2)u'' = 0, \tag{14}$$

Eq. (14), with the transformation

$$u = \omega^{\frac{1}{n-1}}, \tag{15}$$

reduces to

$$(\gamma\kappa_2^2 - c\beta\kappa_1)Q\omega\omega'' + (\gamma\kappa_2^2 - c\beta\kappa_1)W(\omega')^2 - c\omega^2 + \alpha\kappa_1\omega^3 = 0, \tag{16}$$

where

$$Q = \kappa_1/(n-1), \quad W = \kappa_1(2-n)/(n-1)^2.$$

Substituting Eqs. (9) and (10) into Eq. (16) and using balance principle yields $\theta = \epsilon + \delta + 2$. If we take $\theta = 3$, $\epsilon = 0$ and $\delta = 1$, then

$$(\omega')^2 = \frac{\tau_1^2(\xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0)}{\zeta_0},$$

where $\xi_3 \neq 0$, $\zeta_0 \neq 0$. Respectively, solving the algebraic equation system (12) yields

$$\xi_0 = \frac{\tau_0(2\alpha\kappa_1\tau_0(\beta\kappa_1\xi_1\tau_1(Q+W)-\zeta_0\tau_0)-\gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{2\tau_1^2(3\alpha\beta\kappa_1^2\tau_0(Q+W)-\gamma\kappa_2^2(3Q+2W))}, \quad \xi_3 = \frac{\alpha\kappa_1\tau_1(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))}{\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W)-\gamma\kappa_2^2(3Q+2W))},$$
$$\xi_2 = \frac{6\alpha\kappa_1\tau_0(\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))-\gamma\kappa_2^2\xi_1\tau_1(3Q+2W)}{6\alpha\beta\kappa_1^2\tau_0^2(Q+W)-2\gamma\kappa_2^2\tau_0(3Q+2W)}, \quad c = \frac{(Q+W)(6\alpha\kappa_1\zeta_0\tau_0^2+\gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{(3Q+2W)(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))},$$
$$\xi_1 = \xi_1, \quad \zeta_0 = \zeta_0, \quad \tau_0 = \tau_0, \quad \tau_1 = \tau_1.$$

Substituting these results into Eq. (8) and Eq. (13), we can write

$$\pm(\eta-\eta_0) = \sqrt{\frac{\zeta_0\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W)-\gamma\kappa_2^2(3Q+2W))}{\alpha\kappa_1\tau_1(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))}} \times \int \frac{d\Gamma}{\sqrt{\Gamma^3+\ell_2\Gamma^2+\ell_1\Gamma+\ell_0}}, \tag{17}$$

where

$$\ell_2 = \frac{6\alpha\kappa_1\tau_0(\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))-\gamma\kappa_2^2\xi_1\tau_1(3Q+2W)}{2\alpha\kappa_1\tau_1(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))},$$

and

$$\ell_1 = \frac{\xi_1\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W)-\gamma\kappa_2^2(3Q+2W))}{\alpha\kappa_1\tau_1(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))}, \quad \ell_0 = \frac{\tau_0^2(2\alpha\kappa_1\tau_0(\beta\kappa_1\xi_1\tau_1(Q+W)-\zeta_0\tau_0)-\gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{2\alpha\kappa_1\tau_1^3(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))}.$$

Integrating Eq. (17), we obtain the solutions to the Eq. (1) as follows:

$$\pm(\eta-\eta_0) = -2\sqrt{A}\frac{1}{\sqrt{\Gamma-\alpha_1}}, \tag{18}$$

$$\pm(\eta-\eta_0) = 2\sqrt{\frac{A}{\alpha_2-\alpha_1}}\arctan\sqrt{\frac{\Gamma-\alpha_2}{\alpha_2-\alpha_1}}, \quad \alpha_2>\alpha_1, \tag{19}$$

$$\pm(\eta-\eta_0) = \sqrt{\frac{A}{\alpha_1-\alpha_2}}\ln\left|\frac{\sqrt{\Gamma-\alpha_2}-\sqrt{\alpha_1-\alpha_2}}{\sqrt{\Gamma-\alpha_2}+\sqrt{\alpha_1-\alpha_2}}\right|, \quad \alpha_1>\alpha_2, \tag{20}$$

$$\pm(\eta-\eta_0) = 2\sqrt{\frac{A}{\alpha_1-\alpha_3}}F(\varphi,l), \quad \alpha_1>\alpha_2>\alpha_3,$$

where

$$A = \frac{\zeta_0\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W)-\gamma\kappa_2^2(3Q+2W))}{\alpha\kappa_1\tau_1(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))}, \quad F(\varphi,l) = \int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},$$

and

$$\varphi = \arcsin\sqrt{\frac{\Gamma-\alpha_3}{\alpha_2-\alpha_3}}, \quad l^2 = \frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}.$$

Also α_1, α_2 and α_3 are the roots of the polynomial equation

$$\Gamma^3+\frac{\xi_2}{\xi_3}\Gamma^2+\frac{\xi_1}{\xi_3}\Gamma+\frac{\xi_0}{\xi_3}=0.$$

Substituting the solutions (18)-(20) into (7) and (15), denoting $\bar{\tau}=\tau_0+\tau_1\alpha_1$, and setting

$$v = \frac{(Q+W)(6\alpha\kappa_1\zeta_0\tau_0^2+\gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{(3Q+2W)(2\zeta_0\tau_0+\beta\kappa_1\xi_1\tau_1(Q+W))},$$

we get, respectively,

$$u(x,y,t) = \left[\bar{\tau}+\frac{4\tau_1 A}{(\kappa_1 x+\kappa_2 y-vt-\eta_0)^2}\right]^{\frac{1}{n-1}}, \tag{21}$$

$$u(x,y,t)=\left\{\bar{\tau}+\tau_1(\alpha_2-\alpha_1)\left[1-\tanh^2\left(\mp\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}(\kappa_1x+\kappa_2y-vt-\eta_0)\right)\right]\right\}^{\frac{1}{n-1}}, \tag{22}$$

$$u(x,y,t)=\left\{\bar{\tau}+\tau_1(\alpha_1-\alpha_2)\mathrm{cosech}^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}(\kappa_1x+\kappa_2y-vt)\right)\right\}^{\frac{1}{n-1}}. \tag{23}$$

If we take $\tau_0=-\tau_1\alpha_1$, that is $\bar{\tau}=0$, and $\eta_0=0$, then the solutions (21)-(23) can reduce to rational function solution

$$u(x,y,t)=\left[\frac{2\sqrt{\tau_1A}}{\kappa_1x+\kappa_2y-vt}\right]^{\frac{2}{n-1}}, \tag{24}$$

1-soliton solution

$$u(x,y,t)=\frac{A_1}{\cosh^{\frac{2}{n-1}}\left[\mp B(\kappa_1x+\kappa_2y-vt)\right]}, \tag{25}$$

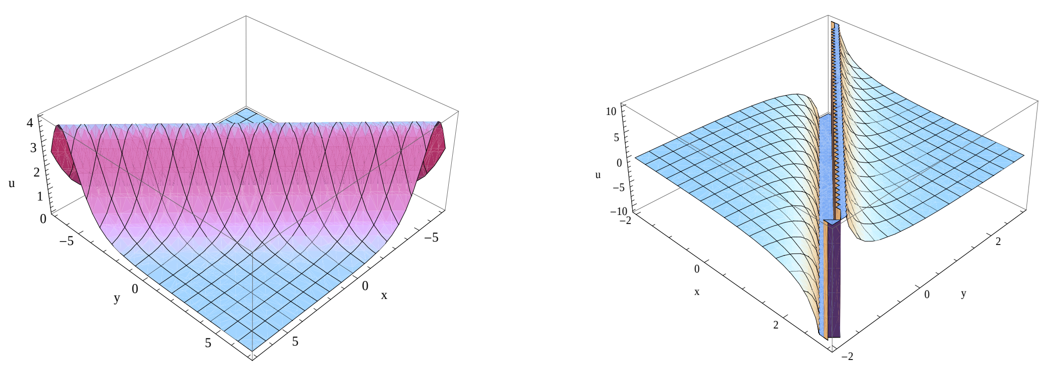
and singular soliton solution

$$u(x,y,t)=\frac{A_2}{\sinh^{\frac{2}{n-1}}\left[B(\kappa_1x+\kappa_2y-vt)\right]}, \tag{26}$$

where

$$A_1=[\tau_1(\alpha_2-\alpha_1)]^{\frac{1}{n-1}},\quad A_2=[\tau_1(\alpha_1-\alpha_2)]^{\frac{1}{n-1}},\quad B=\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A}}.$$

Here, A_1 and A_2 are the amplitudes of the solitons, κ_1 is the inverse width of solitons in the $x-$direction and κ_2 is the inverse width of solitons in the $y-$direction and v is the velocity of the solitons. Thus, we can say that the solitons exist for $\tau_1>0$.



(a) Profile of 1-soliton solution

(b) Profile of singular soliton solution

Figure 1: Figure 1 respectively is shown numerical solutions of 1-soliton solution and singular soliton solution at $n=3,\ \kappa_1=\kappa_2=1,\ A_1=A_2=4,\ B=1$ while $vt=1$.

Example 3.2 *Application to the DSE in (1+2) dimensions.*

In (2) and (3), q and r are the dependent variables while x, y and t are the independent variables. The first two of the independent variables are the spatial variables while t represents time. The exponent n is the power law parameter. It is necessary to have $n>0$. In (2) and (3), q is a complex valued function while r is a real valued function. Also, a, b, α and β are all constant coefficients. For solving the Eqs. (2) and (3) with the trial equation method, using the wave variables

$$q(x,y,t)=u(\eta)e^{i\phi},\quad r(x,y,t)=v(\eta) \tag{27}$$

$$\phi=\phi_1x+\phi_2y+\phi_3t,\quad \eta=\eta_1x+\eta_2y+\eta_3t \tag{28}$$

where $\phi_1, \phi_2, \phi_3, \eta_1, \eta_2$ and η_3 are real constants, converts (2) and (3) to the system of ODEs

$$(\eta_3 + 2a\phi_1\eta_1 + 2a\phi_2\eta_2)u(\eta) = 0, \tag{29}$$

$$-(\phi_3 + a{\phi_1}^2 + a{\phi_2}^2)u(\eta) + a({\eta_1}^2 + {\eta_2}^2)u''(\eta) + bu^{2n+1}(\eta) - \alpha u(\eta)v(\eta) = 0, \tag{30}$$

$$({\eta_1}^2 + {\eta_2}^2)v''(\eta) + \beta{\eta_1}^2(u^{2n})''(\eta) = 0 \tag{31}$$

where primes denote the derivatives with respect to η . Eq. (31) is then integrated term by term two times where integration constants are considered zero. This converts it into

$$v(\eta) = \frac{-\beta{\eta_1}^2}{{\eta_1}^2 + {\eta_2}^2}u^{2n}(\eta). \tag{32}$$

Substituting (32) into (30) gives

$$-(\phi_3 + a{\phi_1}^2 + a{\phi_2}^2)u(\eta) + a({\eta_1}^2 + {\eta_2}^2)u''(\eta) + \left(b + \alpha\beta\frac{{\eta_1}^2}{{\eta_1}^2 + {\eta_2}^2}\right)u^{2n+1}(\eta) = 0. \tag{33}$$

Eq. (33), with the transformation

$$u(\eta) = V^{\frac{1}{n}}(\eta) \tag{34}$$

reduces to

$$QVV'' + P(V')^2 - \left[\phi_3 + a{\phi_1}^2 + a{\phi_2}^2\right]RV^2 + WV^4 = 0, \tag{35}$$

where

$$Q = an\left({\eta_1}^2 + {\eta_2}^2\right)^2, \quad P = a(1-n)\left({\eta_1}^2 + {\eta_2}^2\right)^2, \quad R = n^2\left({\eta_1}^2 + {\eta_2}^2\right), \quad W = n^2\left[b\left({\eta_1}^2 + {\eta_2}^2\right) + \alpha\beta{\eta_1}^2\right].$$

Substituting Eqs. (9) and (10) into Eq. (35) and using balance principle yields $\theta = \epsilon + 2\delta + 2$. If we take $\theta = 4$, $\epsilon = 0$ and $\delta = 1$, then

$$(V')^2 = \frac{\tau_1^2(\xi_4\Gamma^4 + \xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0)}{\zeta_0},$$

where $\xi_4 \neq 0$, $\zeta_0 \neq 0$. Respectively, solving the algebraic equation system (12) yields

$$\xi_0 = \left(\frac{\tau_0}{\tau_1}\right)^2\left(\xi_2 + \frac{5\zeta_0\tau_0^2W}{P+2Q}\right), \quad \xi_1 = \frac{2\tau_0}{\tau_1}\left(\xi_2 + \frac{4\zeta_0\tau_0^2W}{P+2Q}\right), \quad \xi_2 = \xi_2, \quad \xi_3 = -\frac{4\zeta_0\tau_0\tau_1W}{P+2Q}, \quad \xi_4 = -\frac{\zeta_0\tau_1^2W}{P+2Q},$$

$$\phi_1 = \phi_1, \quad \phi_2 = \phi_2, \quad \phi_3 = \frac{\xi_2(P+Q)(P+2Q) + \zeta_0(6\tau_0^2W(P+Q) - aR(P+2Q)(\phi_1^2 + \phi_2^2))}{\zeta_0R(P+2Q)}$$

$$\zeta_0 = \zeta_0, \quad \tau_0 = \tau_0, \quad \tau_1 = \tau_1.$$

Also from Eq. (29), it can be seen that $\eta_3 = -2a(\phi_1\eta_1 + \phi_2\eta_2)$. Substituting these results into Eq. (8) and Eq. (13), we can write

$$\pm(\eta - \eta_0) = \sqrt{-\frac{P+2Q}{\tau_1^2W}} \times \int \frac{d\Gamma}{\sqrt{\Gamma^4 + \ell_3\Gamma^3 + \ell_2\Gamma^2 + \ell_1\Gamma + \ell_0}}, \tag{36}$$

where

$$\ell_3 = \frac{4\tau_0}{\tau_1}, \quad \ell_2 = -\frac{\xi_2(P+2Q)}{\zeta_0\tau_1^2W}, \quad \ell_1 = -\frac{2\tau_0(\xi_2(P+2Q) + 4\zeta_0\tau_0^2W)}{\zeta_0\tau_1^3W}, \quad \ell_0 = -\frac{\tau_0^2(\xi_2(P+2Q) + 5\zeta_0\tau_0^2W)}{\zeta_0\tau_1^4W}.$$

Integrating Eq. (36), we obtain the solutions to the Eqs. (2) and (3) as follows:

$$\pm(\eta - \eta_0) = -\frac{B}{\Gamma - \alpha_1}, \tag{37}$$

$$\pm(\eta-\eta_0)=\frac{2B}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\quad \alpha_2>\alpha_1, \tag{38}$$

$$\pm(\eta-\eta_0)=\frac{B}{\alpha_1-\alpha_2}\ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|, \tag{39}$$

$$\pm(\eta-\eta_0)=\frac{B}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}\ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|,\quad \alpha_1>\alpha_2>\alpha_3, \tag{40}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{B}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi,l),\quad \alpha_1>\alpha_2>\alpha_3>\alpha_4,$$

where

$$B=\sqrt{-\frac{P+2Q}{\tau_1^2W}},\quad F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{\sqrt{1-l^2\sin^2\psi}},$$

and

$$\varphi=\arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},\quad l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}.$$

Also α_1, α_2, α_3 and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0.$$

Substituting the solutions (37)-(40) into (7) and (34), we obtain

$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_1\pm\frac{\tau_1B}{\eta_1x+\eta_2y+\eta_3t-\eta_0}\right\}^{\frac{1}{n}}e^{i\phi}, \tag{41}$$

$$r(x,y,t)=-\frac{\beta{\eta_1}^2}{{\eta_1}^2+{\eta_2}^2}\left\{\tau_0+\tau_1\alpha_1\pm\frac{\tau_1B}{\eta_1x+\eta_2y+\eta_3t-\eta_0}\right\}^2, \tag{42}$$

$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_1+\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[(\alpha_1-\alpha_2)\left(\eta_1x+\eta_2y+\eta_3t-\eta_0\right)\right]^2}\right\}^{\frac{1}{n}}e^{i\phi}, \tag{43}$$

$$r(x,y,t)=-\frac{\beta{\eta_1}^2}{{\eta_1}^2+{\eta_2}^2}\left\{\tau_0+\tau_1\alpha_1+\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[(\alpha_1-\alpha_2)\left(\eta_1x+\eta_2y+\eta_3t-\eta_0\right)\right]^2}\right\}^2, \tag{44}$$

$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_2+\frac{(\alpha_2-\alpha_1)\tau_1}{\exp\left(\dfrac{\alpha_1-\alpha_2}{B}\left(\eta_1x+\eta_2y+\eta_3t-\eta_0\right)\right)-1}\right\}^{\frac{1}{n}}e^{i\phi}, \tag{45}$$

$$r(x,y,t)=-\frac{\beta{\eta_1}^2}{{\eta_1}^2+{\eta_2}^2}\left\{\tau_0+\tau_1\alpha_2+\frac{(\alpha_2-\alpha_1)\tau_1}{\exp\left(\dfrac{\alpha_1-\alpha_2}{B}\left(\eta_1x+\eta_2y+\eta_3t-\eta_0\right)\right)-1}\right\}^2, \tag{46}$$

$$q(x,y,t)=\left\{\tau_0+\tau_1\alpha_1+\frac{(\alpha_1-\alpha_2)\tau_1}{\exp\left(\dfrac{\alpha_1-\alpha_2}{B}\left(\eta_1x+\eta_2y+\eta_3t-\eta_0\right)\right)-1}\right\}^{\frac{1}{n}}e^{i\phi}, \tag{47}$$

$$r(x,y,t) = -\frac{\beta {\eta_1}^2}{{\eta_1}^2+{\eta_2}^2}\left\{\tau_0+\tau_1\alpha_1+\frac{(\alpha_1-\alpha_2)\tau_1}{\exp\left(\frac{\alpha_1-\alpha_2}{B}\left(\eta_1 x+\eta_2 y+\eta_3 t-\eta_0\right)\right)-1}\right\}^2, \tag{48}$$

$$q(x,y,t) = \left\{\tau_0+\tau_1\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh\left(\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{B}\left(\eta_1 x+\eta_2 y+\eta_3 t\right)\right)}\right\}^{\frac{1}{n}} e^{i\phi}, \tag{49}$$

$$r(x,y,t) = -\frac{\beta {\eta_1}^2}{{\eta_1}^2+{\eta_2}^2}\left\{\tau_0+\tau_1\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)\cosh\left(\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{B}\left(\eta_1 x+\eta_2 y+\eta_3 t\right)\right)}\right\}^2. \tag{50}$$

If we take $\tau_0 = -\tau_1\alpha_1$ and $\eta_0 = 0$, then the solutions (41)-(50) can reduce to rational function solutions

$$q(x,y,t) = \left(\pm\frac{\tau_1 B}{\eta_1 x+\eta_2 y+\eta_3 t}\right)^{\frac{1}{n}} e^{i(\phi_1 x+\phi_2 y+\phi_3 t)}, \tag{51}$$

$$r(x,y,t) = \Upsilon\left(\frac{\tau_1 B}{\eta_1 x+\eta_2 y+\eta_3 t}\right)^2, \tag{52}$$

$$q(x,y,t) = \left\{\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[(\alpha_1-\alpha_2)\left(\eta_1 x+\eta_2 y+\eta_3 t\right)\right]^2}\right\}^{\frac{1}{n}} e^{i(\phi_1 x+\phi_2 y+\phi_3 t)}, \tag{53}$$

$$r(x,y,t) = \Upsilon\left\{\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2-\left[(\alpha_1-\alpha_2)\left(\eta_1 x+\eta_2 y+\eta_3 t\right)\right]^2}\right\}^2, \tag{54}$$

traveling wave solutions

$$q(x,y,t) = \left\{\frac{(\alpha_2-\alpha_1)\tau_1}{2}\left\{1\mp\coth\left[\frac{\alpha_1-\alpha_2}{2B}\left(\eta_1 x+\eta_2 y+\eta_3 t\right)\right]\right\}\right\}^{\frac{1}{n}} e^{i(\phi_1 x+\phi_2 y+\phi_3 t)}, \tag{55}$$

$$r(x,y,t) = \Upsilon\left\{\frac{(\alpha_2-\alpha_1)\tau_1}{2}\left\{1\mp\coth\left[\frac{\alpha_1-\alpha_2}{2B}\left(\eta_1 x+\eta_2 y+\eta_3 t\right)\right]\right\}\right\}^2, \tag{56}$$

and soliton solutions

$$q(x,y,t) = \frac{A_3}{\left(D+\cosh\left[B_1(\eta_1 x+\eta_2 y+\eta_3 t)\right]\right)^{\frac{1}{n}}} e^{i(\phi_1 x+\phi_2 y+\phi_3 t)}, \tag{57}$$

$$r(x,y,t) = \Upsilon\frac{A_4}{\left(D+\cosh\left[B_1(\eta_1 x+\eta_2 y+\eta_3 t)\right]\right)^2}, \tag{58}$$

where

$$\eta_3 = -2a(\phi_1\eta_1+\phi_2\eta_2), \quad \phi_3 = \frac{\xi_2(P+Q)(P+2Q)+\zeta_0(6\tau_0^2 W(P+Q)-aR(P+2Q)(\phi_1^2+\phi_2^2))}{\zeta_0 R(P+2Q)},$$

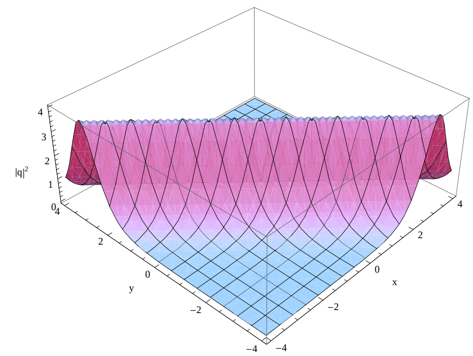
and

$$\Upsilon = -\frac{\beta {\eta_1}^2}{{\eta_1}^2+{\eta_2}^2}, \quad A_3 = \left(\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{\alpha_3-\alpha_2}\right)^{\frac{1}{n}}, \quad A_4 = A_3^{2n},$$

and

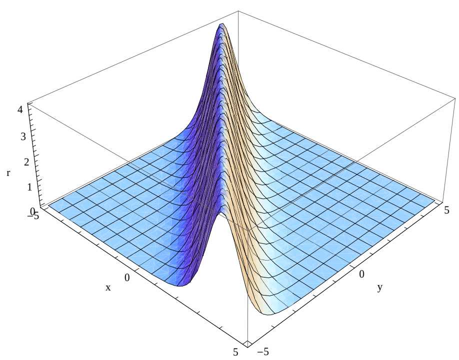
$$B_1 = \frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{B}, \quad D = \frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2}.$$

From (28), η_1 and η_2 are the widths of the solitons in the $x-$ and $y-$directions respectively while η_3 is the velocity of the solitons. From the phase component given by ϕ, ϕ_1 and ϕ_2 are the phase frequencies in the x- and y-directions respectively while ϕ_3 is the wave numbers of the solitons. Also, A_3 and A_4 are the amplitudes of the solitons. Thus, we can say that the solitons exist for $\tau_1 < 0$.

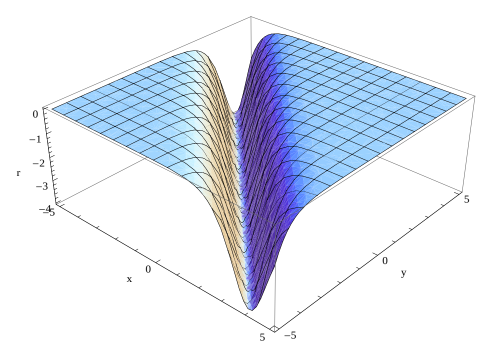


(a) Profile of (57) solution

Figure 2: Numerical solution of (57) at $n = 1,\ \eta_1 = \eta_2 = 1,\ \eta_3 t = 1,\ A_3 = 2,\ B_1 = 2$ and $D < 0$.



(a) Profile of (58) solution for $\Upsilon > 0$



(b) Profile of (58) solution for $\Upsilon < 0$

Figure 3: Numerical solutions of (58) at $\eta_1 = \eta_2 = 1,\ \eta_3 t = 1,\ A_4 = 4,\ B_1 = 1$ and $D < 0$.

4. Conclusion

In this paper we have used the extended trial equation method to derive exact solutions with distinct physical structures. This method with symbolic computation on the computer is used for constructing broad classes of periodic and soliton solutions of two nonlinear equations arising in nonlinear physics. Basic features of the 1-soliton solution and singular soliton solution were analytically and numerically discussed. We proposed more general trial equation method as an alternative approach to obtain the analytic solutions of nonlinear partial differential equations with generalized evolution in mathematical physics. We use the extended trial equation method aided with symbolic computation to construct the soliton solutions, the elliptic function and rational function solutions for generalized $(2+1)$-dimensional ZK-MEW equation and generalized Davey-Stewartson system.

References

[1] M. J. Ablowitz, P.A. Clarkson Solitons, Nonlinear Evolution Equations and Inverse Scattering Transform, Cambridge: Cambridge University press, 1991.

[2] A. M. Wazwaz, The tanh method for travelling wave solutions of nonlinear equations, Applied Mathematics and Computation 154 (2004) 713-723.

[3] P. Rosenau, J. M. Hyman, Compactons: solitons with finite wavelengths, Physical Review Letters 70 (1993) 564-567.

[4] A. M. Wazwaz, An analytic study of compactons structures in a class of nonlinear dispersive equations, Mathematics and Computers in Simulation 63 (2003) 35-44.

[5] R. Hirota, Exact solutions of the Korteweg-de-Vries equation for multiple collisions of solitons, Phys. Lett. A 27 (1971) 1192-1194.

[6] W. Malfliet, W. Hereman, The tanh method: exact solutions of nonlinear evolution and wave equations, Phys. Scr. 54 (1996) 563-568.

[7] M. A. Abdou, The extended tanh method and its applications for solving nonlinear physical models, Appl. Math. Comput. 190 (2007) 988-996.

[8] H.X. Wu, J. H. He, Exp-function method and its application to nonlinear equations, Chaos Solitons Fractals 30 (2006) 700-708.

[9] M. Wang, X. Li, J. Zhang, The ($\frac{G'}{G}$)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A 372 (2008) 417-423.

[10] G. Ebadi, A. Biswas, The ($\frac{G'}{G}$) method and topological soliton solution of the K(m,n) equation, Commun. Nonlinear Sci. Numer. Simulat. 16 (2011) 2377-2382.

[11] E. M. E. Zayed, K. A. Gepreel, The (G'/G)-expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics, Journal of Mathematical Physics 50 (2009) 013502-12.

[12] E. M. E. Zayed, M. A. S. EL-Malky, The Extended (G'/G)-expansion method and its applications for solving the (3+1)-dimensional nonlinear evolution equations in mathematical physcis, Global Journal of Science Frontier Research 11 (2011) 13 pages.

[13] M. Ekici, D. Duran, A. Sonmezoglu, Constructing of exact solutions to the (2+1)-dimensional breaking soliton equations by the multiple ($\frac{G'}{G}$)-expansion method, J. Adv. Math. Stud. 7 (2014) 27-44.

[14] M. Wang, Solitary wave solutions for variant Boussinesq equations, Phys. Lett. A 199 (1995) 169-172.

[15] H. T. Chen, H.Q. Zhang, New double periodic and multiple soliton solutions of the generalized (2 + 1)-dimensional Boussinesq equation, Chaos Soliton. Fract. 20 (2004) 765-769.

[16] D. Zhang, Doubly periodic solutions of the modified Kawahara equation, Chaos Soliton. Fract. 25 (2005) 1155-1160.

[17] Q. Liu, J. M. Zhu, Exact Jacobian elliptic function solutions and hyperbolic function solutions for Sawada-Kotere equation with variable coefficient, Phys. Lett. A 352 (2006) 233-238.

[18] X. Zhao, H. Zhi, H. Zhang, Improved Jacobi-function method with symbolic computation to construct new double-periodic solutions for the generalized Ito system, Chaos Soliton. Fract. 28 (2006) 112-126.

[19] A. Ebaid, E. H. Aly, Exact solutions for the transformed reduced Ostrovsky equation via the F-expansion method in terms of Weierstrass-elliptic and Jacobian-elliptic functions, Wave Motion 49 (2012) 296-308.

[20] A. Filiz, M. Ekici, A. Sonmezoglu, F-expansion method and new exact solutions of the Schrödinger-KdV equation, The Scientific World Journal, 2014 (2014). Article ID 534063, 14 pages.

[21] C. S. Liu, Trial equation method and its applications to nonlinear evolution equations, Acta. Phys. Sin. 54 (2005) 2505-2509.

[22] C. S. Liu, A new trial equation method and its applications, Commun. Theor. Phys. 45 (2006) 395-397.

[23] C. S. Liu, Trial equation method for nonlinear evolution equations with rank inhomogeneous: mathematical discussions and applications, Commun. Theor. Phys. 45 (2006) 219-223.

[24] C. S. Liu, Using trial equation method to solve the exact solutions for two kinds of KdV equations with variable coefficients, Acta. Phys. Sin. 54 (2005) 4506-4510.

[25] C. S. Liu, Applications of complete discrimination system for polynomial for classifications of traveling wave solutions to nonlinear differential equations, Comput. Phys. Commun. 181 (2010) 317-324.

[26] Y. Gurefe, A. Sonmezoglu, E Misirli, Application of the trial equation method for solving some nonlinear evolution equations arising in mathematical physics, Pramana-J. Phys. 77 (2011) 1023-1029.

[27] Y. Pandir, Y. Gurefe, U. Kadak, E. Misirli, Classifications of exact solutions for some nonlinear partial differential equations with generalized evolution, Abstr. Appl. Anal. 2012 (2012). Art. ID 478531, 16 pages.

[28] Y. Gurefe, E. Misirli, A. Sonmezoglu, M. Ekici, Extended trial equation method to generalized nonlinear partial differential equations, Appl. Math. Comput. 219 (2013) 5253-5260.

[29] Y. Gurefe, E. Misirli, Y. Pandir, A. Sonmezoglu, M. Ekici, New Exact Solutions of the Davey-Stewartson Equation with Power-Law Nonlinearity, Bull. Malays. Math. Sci. Soc. (2013) in press.

[30] M. Ekici, D. Duran, A. Sonmezoglu, Soliton Solutions of the Klein-Gordon-Zakharov Equation with Power Law Nonlinearity, ISRN Computational Mathematics 2013 (2013). Article ID 716279, 7 pages.

[31] A. Filiz, A. Sonmezoglu, M. Ekici, D. Duran, A New Approach for Soliton Solutions of RLW Equation and (1+2)-Dimensional Nonlinear Schrödinger's Equation, Mathematical Reports (2014) in press.

[32] N. J. Zabusky, M.D. Kruskal, Interaction of solitons in a collisionless plasma and the recurrence of initial states, Physical Review Letters 15 (1965) 240-243.

[33] M. Wadati, The modified Kortweg-de Vries equation, Journal of Physical Society of Japan 34 (1973) 1289-1296.

[34] A. M. Wazwaz, Exact solutions for the ZK-MEW equation by using the tanh and sine-cosine methods, Int. J. Comput. Math. 82 (2005) 699-708.

[35] E. M. E. Zayed, A. H. Arnous, Exact solutions of the nonlinear ZK-MEW and the Potential YTSF equations using the modified simple equation method, AIP Conference Proceedings of ICNAAM 2012 1479 (2012) 2044-2048. American Institute of Physics, USA.

[36] G. Ebadi, A. Biswas, The $(\frac{G'}{G})$ method and 1-soliton solution of the Davey-Stewartson equation, Math. Comput. Model. 53 (2011) 694-698.

[37] A. Bekir, A. C. Cevikel, New solitons and periodic solutions for nonlinear physical models in mathematical physics, Nonlinear Anal. Real World Appl. 11 (2010) 3275-3285.

[38] X. Zhao, Self-similar solutions to a generalized Davey-Stewartson system, Math. Comput. Model. 50 (2009) 1394-1399.