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New soliton solutions for Kaup-Boussinesq system

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Abstract

In this Letter, we study Kaup-Boussinesq system by using the well-known He's variational approach. In fact, the He's variational method is a promising method to various systems of linear and nonlinear equations.

Keywords: *Variational method, Solitary solution, Kaup-Boussinesq system*

1 Introduction

Most scientific problems and physical phenomena occur nonlinearly. In fact we can present many important phenomena and dynamic processes in physics, mechanics, chemistry, biology, nonlinear optics, the theory of shallow water waves, plasma physics and others by nonlinear partial differential equations. The study of exact solutions of nonlinear evolution equations plays an important role in soliton theory and explicit formulas of nonlinear partial differential equations play an essential role in the nonlinear science. Also, the explicit formulas may provide physical information and help us to understand the mechanism of related physical models.
Recently, there have been a multitude of methods presented for solving Nonlinear partial differential equations (NPDEs), for instance, the Adomian decomposition method [1],the homotopy perturbation method [2],the variational iteration method [3, 4] , the He's variational approach [5], the $F-$expansion method [6], three-wave method [7], extended homoclinic test approach [8, 9], the $(\frac{G'}{G})-$expansion method [6] and the exp-function method [10].

In this paper, by means of the He's variational approach, we will obtain some Solitary solutions of the following Kaup-Boussinesq system given in [11]

$$\begin{aligned} &u_t - v_{xxx} - 2\,v\,u_x - 2\,u\,v_x = 0, \\ &v_t - u_x - 2\,v\,v_x = 0. \end{aligned} \tag{1}$$

2 He's variational method

In order to seek its travelling wave solution, we introduce a transformation

$$u(x,t) = u(\xi) \quad , \quad v(x,t) = v(\xi) \quad , \quad \xi = x - c\,t \tag{2}$$

by substituting Eqs. (2) into Eq. (1), we have

$$-c\,u^{'} - v^{'''} - 2\,v\,u^{'} - 2\,u\,v^{'} = 0, \tag{3}$$

$$-c\,v^{'} - u^{'} - 2\,v\,v^{'} = 0. \tag{4}$$

We can rewrite Eq. (4) in the form

$$u^{'} + c\,v^{'} + 2\,v\,v^{'} = 0, \tag{5}$$

where prime denotes the differential with respect to ξ. Integrating Eq. (5) with respect to ξ and taking the integration constant as zero yields

$$u = -c\,v - v^2. \tag{6}$$

Now, inserting Eq. (6) into Eq. (3), yields

$$c^2\,v^{'} + 6c\,v\,v^{'} + 6v^2\,v^{'} - v^{'''} = 0. \tag{7}$$

Now, Integrating Eq. (7) with respect to ξ and taking the integration constant as zero yields

$$v^{''} - c^2\,v - 3c\,v^2 - 2v^3 = 0. \tag{8}$$

According to Ref. [12], By He's semi-inverse method [13], we can arrive at the following variational formulation:

$$J(\phi) = \int_0^{\infty} \left[-\frac{1}{2}\,(v^{'})^2 - \frac{(c^2)}{2}\,v^2 - c\,v^3 - \frac{1}{3}\,v^4 \right] d\xi. \tag{9}$$

We assume the soliton solution in the following form

$$v(\xi) = A\,sech(\xi), \tag{10}$$

where A is an unknown constant to be further determined.
By Substituting Eq. (10) into Eq. (9) we obtain

$$J=-\frac{1}{4}\,A^3c\pi-\frac{1}{3}\,A^4-\frac{1}{2}\,c^2A^2-\frac{1}{6}\,A^2, \tag{11}$$

For making J stationary with respect to A

$$\frac{\partial J}{\partial A}=-\frac{3}{4}\,A^2c\pi-\frac{4}{3}\,A^3-c^2A-\frac{1}{3}\,A, \tag{12}$$

From Eq. (12), we have

$$A=-\frac{9}{32}\,c\,\pi+\frac{1}{32}\,\sqrt{81\,c^2\pi^2-256-768\,c^2}. \tag{13}$$

The solitary solutions are, therefore, obtained as follows:

$$v(\xi)=A\,sech(\xi), \tag{14}$$

and

$$u(\xi)=-c\,A\,sech(\xi)-A^2\,sech^2(\xi), \tag{15}$$

where $A=-\frac{9}{32}\,c\pi+\frac{1}{32}\,\sqrt{81\,c^2\pi^2-256-768\,c^2}$.
In this solutions c is an arbitrary real parameter. Fig. 1 and Fig. 2 show the graph of $u(x,t)$ and $v(x,t)$ for $c=\pi$, $-30\le x\le 30$ and $0\le t\le 5$.

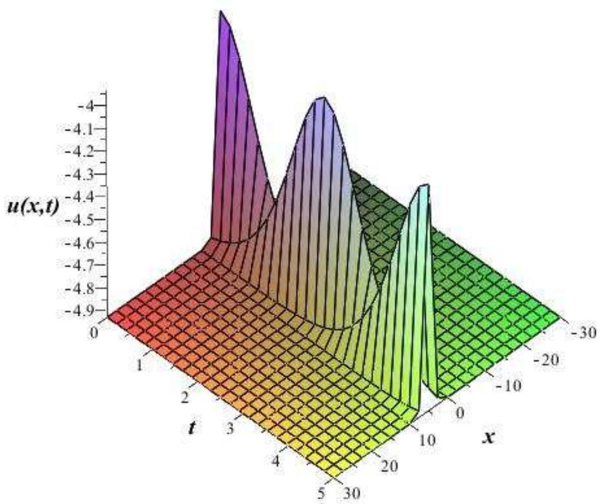

Figure 1: The Soliton solution of Eq. (14).

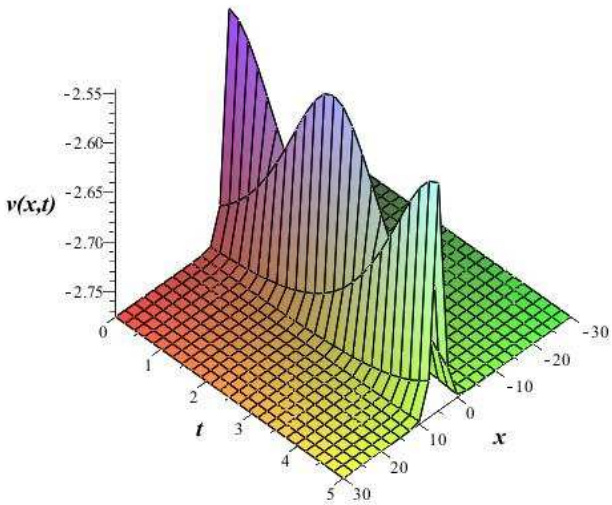

Figure 2: The Soliton solution of Eq. (15).

We search another soliton solution in the form

$$u(\xi) = D\, sech^2(\xi), \tag{16}$$

where D is an unknown constant to be further determined.
By Substituting Eq. (16) into Eq. (9) we obtain

$$J = -\frac{8}{35} D^4 - \frac{8}{15} c\, D^3 - \frac{1}{3} c^2 D^2 - \frac{4}{15} D^2, \tag{17}$$

For making J stationary with respect to D

$$\frac{\partial J}{\partial D} = -\frac{32}{35} D^3 - \frac{8}{5} c\, D^2 - \frac{2}{3} c^2 D - \frac{8}{15} D, \tag{18}$$

From Eq. (18), we have

$$D = -\frac{7}{8} c + \frac{1}{24} \sqrt{21\, c^2 - 336}, \tag{19}$$

The solitary solutions are, therefore, obtained as follows:

$$v(\xi) = D\, sech^2(\xi), \tag{20}$$

and

$$u(\xi) = -c\, D\, sech^2(\xi) - D^2\, sech^4(\xi), \tag{21}$$

where $D = -\frac{7}{8} c + \frac{1}{24} \sqrt{21\, c^2 - 336}$.
In this solutions c is an arbitrary real parameter. Fig. 3, Fig. 4 show the graph of $u(x, t)$ and $v(x, t)$ for $c = 5\,\pi$, $-30 \le x \le 30$ and $0 \le t \le 5$.

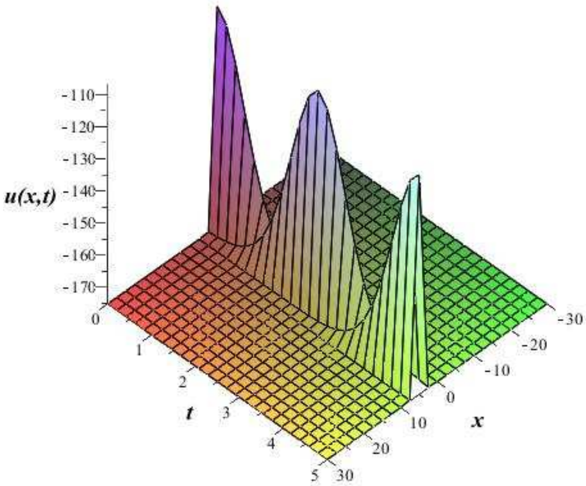

Figure 3: The Soliton solution of Eq. (20).

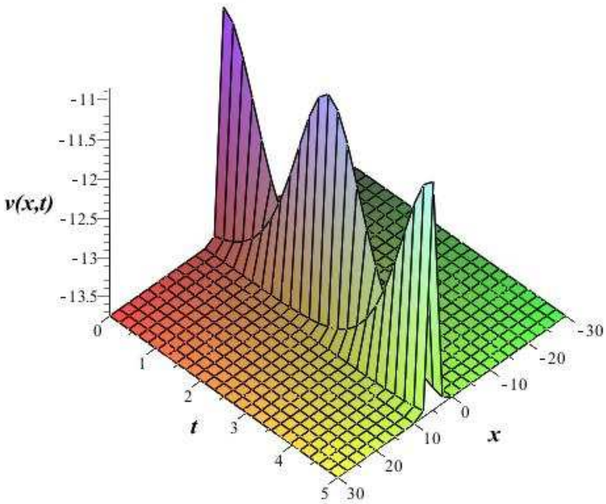

Figure 4: The Soliton solution of Eq. (21).

3 Conclusion

In this paper, we used He's variational method to search for solitary solutions of Kaup-Boussinesq system. He's variational principle is a very dominant instrument to find the solitary solutions for various nonlinear equations.

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