# Extension of the TOPSIS method to group decision-making 

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#### Abstract

TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) is a very practical decision support method used in several areas of life. This method already exists in the literature in the context of a single decision maker. In order to adapt this method to group decision making, which can be easily applied in various situations, this work extended the TOPSIS method to group decision making using the quadratic mean and the geometric mean. In this work, numerical applications have been made and interesting results have been obtained.


Keywords: Group decision; extension-TOPSIS; geometric mean; quadratic mean,decision-making.

## 1. Introduction

Decision support is generally requested by industrialists and researchers when they are faced with complex decision-making problems [12] . Thus for [13] Multi-criteria support aims, as its name indicates, to provide a decision-maker with tools enabling him to progress in the resolution of the decision problem where several points of view, often contradictory, must be taken into account. Decision making, i.e. the selection of the optimal solution from a feasible set, is a very common task present in almost all human activities [9]. Thus, in all domains, from everyday life to the world of work, a large number of decisions are made, either individually or collectively. They lead to happy or unhappy outcomes, they are the object of regret or contentment, they give rise to progress or regression [2] .According to the idea of synergy, decisions made collectively tend to be more effective than decisions made individually, so according to [10] , Multiple Attribute Group Decision Making (MAGDM) plays an important role in the real world. According to [8] , a group decision is the selection by several decision makers of one or more alternatives from a large set of alternatives. A large number of methods, theories and applications have been proposed for solving MAGDM problems. For [1] formally, multi-actor decision problems are characterised by the existence of at least 2 decision makers, each with their own perceptions, attitudes, motivations and personalities towards the decision alternatives and who are motivated to reach a collective choice. For [3] it is therefore necessary to establish a consensus-building process in order to arrive at a decision that best reconciles local preferences with the choice made by the group. For this purpose, several aggregation methods related to group decision already exist in the scientific literature such as TOPSIS (Technique for Order Performance by Similarity to Ideal Solution). TOPSIS is used in various fields of life for decision making. Its great flexibility allows us to extend it further to group decision making in order to make better choices in various situations, which is the object of our work. In this paper we have provided a new method for solving group decision problems by extending the TOPSIS method (Technique for Order Performance by Similarity to Ideal Solution). The literature on group decision contains many aggregation methods but the use of some methods leads to compensation of weak criteria by stronger ones, the use of others is complex; it is in this context that we adapt here the TOPSIS method in the context of a single decision maker to group decision. After the State of the Art we will write our new method, then we will make a numerical application, comparisons and we will finish with a conclusion.

## 2. State of the art

### 2.1. Application of the arithmetic mean to group decision-making: the MACASP method

This section is inspired by [4]
Consider $n$ values $x_{i}$, then the arithmetic mean is given by :
$\bar{x}=\frac{\sum_{i=1}^{i=n} x_{i}}{n}$
a) let $N$ be the number of decision makers and $D$ their set $D=\left\{d_{1}, d_{2}, \ldots, d_{N}\right\}$
b) Let $m$ be the number of criteria whose index set is $\{1,2, \ldots, m\}$
c) Let $M$ be the number of actions and their set ; $A=\left\{a_{1}, a_{2}, \ldots . a_{M}\right\}$

Let $G_{k}$ denote the additive value aggregation function for decision maker $d_{k}$. We consider the following arithmetic mean:
$U_{j}\left(a_{i}\right)=\sum_{i=1}^{N} G_{j}\left(a_{i}\right) / N ; i=1, \ldots, N ; j=1, \ldots, m ;$
$G_{k}\left(a_{i}\right)=\sum_{j=1}^{j=m} w_{j}^{k} g_{j}^{k}\left(a_{i}\right)$.
The collective aggregation function based on the arithmetic mean is called MACASP (Collective Aggregation Model Using Weighted Sum), denoted $U$ defined by :
$U\left(a_{i}\right)=\sum_{k=1}^{k=N} G_{k}\left(a_{i}\right) / N$.
and
$U\left(a_{i}\right)=\sum_{k=1}^{k=N} \sum_{j=1}^{j=m} w_{j}^{k} g_{j}^{k}\left(a_{i}\right) / N$.
$U\left(a_{i}\right)=\sum_{j=1}^{j=m} \sum_{k=1}^{k=N} w_{j}^{k} g_{j}^{k}\left(a_{i}\right) / N$
After determining the aggregation function, we can then assign a score to each action.

### 2.2. Presentation of the harmonic mean applied to group decision: the Lon-Zo method

This part follows from [5]
In this part $w_{j}^{k}$ represents the weight assigned to criterion $j$ by decision maker $k$. Consider the harmonic mean $\bar{x}_{h}$ of the $n$ values $x_{i}$ : 99
$\bar{x}_{h}=\frac{n}{\sum_{i=1}^{n} \frac{1}{x_{i}}}$
Let us note: :
a) $N$ the number of decision makers and $D$ the set; $D=\left\{d_{1}, d_{2}, \ldots, d_{N}\right\}$;
b) $m$ the number of criteria whose index set is $\{1,2, \ldots, m\}$;
c) $M$ the number of actions and their set $A=\left\{a_{1}, a_{2}, \ldots a_{M}\right\}$.

Let $G_{k}$ denote the additive value aggregation function for decision maker $d_{k}$. The collective aggregation function based on the harmonic mean is called the Lon-Zo (Longin-Zoinabo) method and is defined as follows:
$U\left(a_{i}\right)=\frac{N}{\sum_{k=1}^{N} \frac{1}{G_{k}\left(a_{i}\right)}}$.
with
$G_{k}\left(a_{i}\right)=\sum_{j=1}^{j=m} w_{j}^{k} g_{j}^{k}\left(a_{i}\right) ; i=1, \ldots, M ; j=1, \ldots, m$.

### 2.3. TOPSIS Description

This part is from [7]
TOPSIS is based upon the concept that the chosen alternative should have the shortest distance from the ideal solution and farthest from the negative ideal solution.Assume that each alternative takes the monotonically increasing (or decreasing) utility. It is then easy to locate the ideal solution, which is a combination of all the best attribute value attainable, while the negative ideal solution is a combination of all the worst attribute values attainable. One approach is to take an alternative that has the minimum (weighted) Euclidean distance to the ideal solution of the TOPSIS method consists of the following steps:

Step 1: Calculate the normalized decision matrix. The normalized value $N_{i j}$ is calculated as

$$
N_{i j}=\frac{x_{i j}}{\sum_{i=1}^{m} x_{i j}^{2}}, i=1, \ldots, m, j=1, \ldots, n . \text { (2) }
$$

Step 2: Calculate the weighted normalized decision matrix. The weighted normalized value $V_{i j}$ is calculated as $V_{i j}=W_{j} \times N_{i j}, i=$ $1, \ldots, m, j=1, \ldots, n$, where $W_{j}$ is the weight of i th attribute and $\sum_{j=1}^{n} W_{j}=1$
Step 3: Actually $A^{+}, A^{-}$are not absolute values but represent the best or worst evaluation among the different alternatives analyzed in the matrix V .

$$
\begin{equation*}
A^{+}=\left\{V_{1}^{+} ; V_{2}^{+} ; \ldots ; V_{n}^{+}\right\} \text {where } V_{j}^{+}=\max _{i} V_{i j} \mid j \in J \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
A^{-}=\left\{V_{1}^{-} ; V_{2}^{-} ; \ldots ; V_{n}^{-}\right\} \text {where } V_{j}^{-}=\min _{i} V_{i j} \mid j \in J \tag{4}
\end{equation*}
$$

where J is associated with benefit attributes.
Step 4: Compute the distance of each alternative from the positive ideal solution by

$$
\begin{align*}
& D_{i}^{+}=\sqrt{\sum_{j=1}^{n}\left(V_{i j}-V_{j}^{+}\right)^{2}}, \text { for } i=1, \ldots, m  \tag{5}\\
& D_{i}^{-}=\sqrt{\sum_{j=1}^{n}\left(V_{i j}-V_{j}^{-}\right)^{2}}, \text { for } i=1, \ldots, m \tag{6}
\end{align*}
$$

Step 5: Compute the relative combined measure of goodness for the alternative $A_{i}$ with respect to $A^{+}$as:

$$
R_{i}=\frac{D_{i}^{-}}{D_{i}^{-}+D_{i}^{+}}, i=1, \ldots, m
$$

Since $D_{i}^{-} \geq 0$ and $D_{i}^{+} \geq 0$ then $R_{i} \in[0,1]$
Step 6: Rank the preference order. For ranking alternatives using this index, we can rank alternatives in decreasing order. The basic principle of the TOPSIS technique, chosen the alternative should have the "shortest distance" from the positive ideal solution and the "farthest distance" from the negative ideal solution.

## 3. Presentation of the new method : EMETOP

To solve a group decision problem by extending the TOPSIS: EMETOP method, the members of the group determine together the actions and criteria to be retained in a consensual manner; on the basis of a chosen scale, each decision-maker evaluates the criteria and actions considered and assigns a weight to each criterion according to the importance given.
Let us note:
a) $N$ the number of decision makers and $D$ the set ; $D=\left\{d_{1}, d_{2}, \ldots, d_{N}\right\}$;
b) $m$ the number of criteria whose set is $\left\{c_{1}, c_{2}, \ldots, c_{m}\right\}$;
c) $M$ the number of actions and their set $A=\left\{a_{1}, a_{2}, \ldots a_{M}\right\}$.
d) $w_{j}^{k}$ the weight affected to criterion $c_{j}$ by decision maker $d_{k}$.
e) $g_{j}^{k}\left(a_{i}\right)$ the partial evaluation of action $a_{i}$ with respect to criterion $c_{j}$ by decision maker $d_{k}$.
f) $W_{j}$ the global weight of the criterion $c_{j}$
g) Let us note $G_{j}\left(a_{i}\right)$ the global performance of the action $a_{i}$ with respect to the criterion $c_{j}$.

Thus the judgment matrix of the decision maker $d_{k}$ is:

|  | $c_{1}$ | $c_{2}$ | $\ldots$ | $c_{m}$ |
| :---: | :---: | :---: | :---: | :---: |
| weight | $w_{1}^{k}$ | $w_{2}^{k}$ | $\ldots$ | $w_{m}^{k}$ |
| $a_{1}$ | $g_{1}^{k}\left(a_{1}\right)$ | $g_{2}^{k}\left(a_{1}\right)$ | $\ldots$ | $g_{m}^{k}\left(a_{1}\right)$ |
| $a_{2}$ | $g_{1}^{k}\left(a_{2}\right)$ | $g_{2}^{k}\left(a_{2}\right)$ | $\ldots$ | $g_{m}^{k}\left(a_{2}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $a_{M}$ | $g_{1}^{k}\left(a_{M}\right)$ | $g_{2}^{k}\left(a_{M}\right)$ | $\ldots$ | $g_{m}^{k}\left(a_{M}\right)$ |

Table 1: Judgment matrix of the decision maker K

We therefore use the root mean square to determine the overall preferences of the decision makers for each alternative and the geometric mean for each weight. Then we apply the TOPSIS method in the framework of a single decision maker.

### 3.1. Root mean square

The root mean square $q$ of $n$ values $x_{i}$ is obtained as follows:

$$
q=\sqrt{\frac{\sum\left(x_{i}^{2}\right)}{n}}
$$

### 3.2. Principle of the new method

Step 1 : For the weights we do

$$
W_{j}=\sqrt[N]{\prod_{k=1}^{N}\left(w_{j}^{k}\right)}
$$

For example, for 3 decision makers, the global weight of the criterion $c_{1}$ is:

$$
W_{1}=\sqrt[3]{w_{1}^{1} \times w_{1}^{2} \times w_{1}^{3}}
$$

Step 2 : For the overall performance of the stock $a_{i}$ with respect to the criterion $c_{j}$ we have:

$$
G_{j}\left(a_{i}\right)=\sqrt{\frac{1}{N} \sum_{k=1}^{N}\left(g_{j}^{k}\left(a_{i}\right)\right)^{2}}
$$

For example, for three decision-makers, the overall performance of the stock $a_{1}$ with respect to the criterion $c_{2}$ is:

$$
G_{2}\left(a_{1}\right)=\sqrt{\frac{1}{3}\left[\left(g_{2}^{1}\left(a_{1}\right)\right)^{2}+\left(g_{2}^{2}\left(a_{1}\right)\right)^{2}+\left(g_{2}^{3}\left(a_{1}\right)\right)^{2}\right]}
$$

we thus obtain the global performance matrix of the $N$ decision-makers

|  | $c_{1}$ | $c_{2}$ | $\ldots$ | $c_{m}$ |
| :---: | :---: | :---: | :---: | :---: |
| weight | $W_{1}$ | $W_{2}$ | $\ldots$ | $W_{m}$ |
| $a_{1}$ | $G_{1}\left(a_{1}\right)$ | $G_{2}\left(a_{1}\right)$ | $\ldots$ | $G_{m}\left(a_{1}\right)$ |
| $a_{2}$ | $G_{1}\left(a_{2}\right)$ | $G_{2}\left(a_{2}\right)$ | $\ldots$ | $G_{m}\left(a_{2}\right)$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ |
| $a_{M}$ | $G_{1}\left(a_{M}\right)$ | $G_{2}\left(a_{M}\right)$ | $\ldots$ | $G_{m}\left(a_{M}\right)$ |

Table 2: Overall performance matrix of $N$ decision makers

Step 3 : Let's calculate the normalized decision matrix.
The normalized value $N_{i j}$ is calculated as follows:

$$
r_{i j}=\frac{G_{j}\left(a_{i}\right)}{\sqrt{\sum_{i=1}^{M}\left(G_{j}\left(a_{i}\right)\right)^{2}}}
$$

Step 4 :In this step, we simply multiply all the entries $r_{i j}$ of the normalised matrix by the weighting associated with each criterion, so we proceed by column.

$$
V_{i j}=W_{j} \times r_{i j}
$$

Step 5 : Define the positive ideal $A^{+}$and the negative ideal $A^{-}$:

$$
\begin{gathered}
A^{+}=\left(\text {best }_{i}\left(V_{i 1}\right) ; \operatorname{best}_{i}\left(V_{i 2}\right) ; \ldots ; \text { best }_{i}\left(V_{i m}\right)\right) \\
A^{+}=\left\{r_{j}^{+} ; j=1,2, . ., m\right\} \\
A^{-}=\left(\text {worst }_{i}\left(V_{i 1}\right) ; \text { worst }_{i}\left(V_{i 2}\right) ; \ldots ; \text { worst }_{i}\left(V_{i m}\right)\right) \\
A^{-}=\left\{r_{j}^{-} ; j=1,2, . ., m\right\}
\end{gathered}
$$

with

$$
\begin{gathered}
\operatorname{best}_{i}\left(V_{i j}\right)=\left\{\begin{array}{r}
\max _{i}\left(V_{i j}\right) \text { si } c_{j} \text { est beneficial } \\
\min _{i}\left(V_{i j}\right) \text { si } c_{j} \text { est non beneficial }
\end{array}\right. \\
\text { worst }_{i}\left(V_{i j}\right)=\left\{\begin{array}{r}
\min _{i}\left(V_{i j}\right) \text { si } c_{j} \text { est beneficial } \\
\max _{i}\left(V_{i j}\right) \text { si } c_{j} \text { est non beneficial }
\end{array}\right.
\end{gathered}
$$

$\star$ For each criterion (attribute) the most favourable associated value $A^{+}$is calculated according to the nature of the criterion (favourable or unfavourable). If the criterion is favourable, the largest value in each column is selected. If the criterion is unfavourable, the smallest value in each column is selected.
$\star$ For each criterion (attribute) the least favourable associated value $A^{-}$is calculated according to the nature of the criterion (favourable or unfavourable). If the criterion is favourable, the smallest value in each column is selected. If the criterion is unfavourable, the largest value in each column is selected.
$\underline{\text { Step } 6: L e t ~ u s ~ c a l c u l a t e ~ f o r ~ e a c h ~ a l t e r n a t i v e, ~ t h e ~ E u c l i d e a n ~ d i s t a n c e ~ b e t w e e n ~ t h e ~ p o s i t i v e ~ i d e a l ~ a n d ~ t h e ~ n e g a t i v e ~ i d e a l, ~ n o t e d ~} d^{+}$ and $d^{-}$respectively

$$
\begin{aligned}
& d_{i}^{+}=\sqrt{\sum_{j=1}^{m}\left(r_{j}^{+}-V_{i j}\right)^{2}} \\
& d_{i}^{-}=\sqrt{\sum_{j=1}^{m}\left(r_{j}^{-}-V_{i j}\right)^{2}}
\end{aligned}
$$

Step 7 : Let us calculate the degree of proximity to the positive ideal $D_{i}^{+}$. The larger $D_{i}^{+}$is, the closer the alternative $i$ is to the positive ideal and the further it is from the negative ideal:

$$
D_{i}^{+}=\frac{d_{i}^{-}}{d_{i}^{-}+d_{i}^{+}}
$$

Step 8 : Finally, let us sort the solutions with respect to $D_{i}^{+}$and rank the alternatives in order of preference.

## 4. Digital experiences

### 4.1. Digital experience 1

This numerical experiment is taken from the PhD thesis of the University of Law and Science of Aix-Marseille presented by Rasmi Ginting [11].

### 4.1.1. Problem statement

There are four products and five selection criteria. Three decision-makers each have an individual opinion. The task is to find a compromise product. The problem is to find the best product.
$A=\{$ Produit 1, Produit 2, Produit 3, Produit 4$\}$ all of the actions.
The set of criteria is $\mathrm{F}=\left\{C_{1}, C_{2}, C_{3}, C_{4}, C_{5}\right\}$ where :
$C_{1}$ : Production price (Francs/litre),
$C_{2}$ : Longevity (years)
$C_{3}$ : Harmfulness of paint (Very slightly harmful, Moderately harmful, Very harmful),
$C_{4}$ : Drying time
$C_{5}$ : Paint smell (not strong, medium, strong, very strong).
The data is provided by three (3) decision-makers (or assigned to the criteria) in the form of scores between 0 and 10 . The range of the scoring scales may differ from one decision-maker to another, and each criterion is subject to a weighting coefficient expressing the importance of the criterion. The result is a subset A of A containing alternatives (products) that outperform the others: The product that outperforms the others must be accepted by a majority of decision-makers, and must not be rejected too clearly, even by a single decision-maker. Each decision-maker constructs its own judgement matrix. Let us assume that the different profiles are as follows:

|  | price | Longevity | smell | Drying | Harmfulness |
| :---: | :---: | :---: | :---: | :---: | :---: |
| weight | 6 | 3 | 2 | 4 | 3 |
| produit 1 | 6 | 5 | 2 | 4 | 5 |
| produit 2 | 5 | 6 | 3 | 3 | 4 |
| produit 3 | 7 | 5 | 4 | 6 | 3 |
| produit 4 | 6 | 4 | 5 | 3 | 6 |

Table 3: Decision-maker's judgement matrix 1

|  | price | Longevity | smell | Drying | Harmfulness |
| :---: | :---: | :---: | :---: | :---: | :---: |
| weight | 7 | 5 | 3 | 3 | 4 |
| produit 1 | 7 | 6 | 2 | 3 | 3 |
| produit 2 | 6 | 5 | 2 | 5 | 3 |
| produit 3 | 5 | 7 | 3 | 6 | 4 |
| produit 4 | 5 | 4 | 4 | 4 | 3 |

Table 4: Decision-maker's judgement matrix 2

|  | price | Longevity | smell | Drying | Harmfulness |
| :---: | :---: | :---: | :---: | :---: | :---: |
| weight | 6 | 4 | 2 | 3 | 3 |
| produit 1 | 6 | 5 | 2 | 4 | 4 |
| produit 2 | 7 | 6 | 3 | 5 | 3 |
| produit 3 | 6 | 5 | 4 | 3 | 5 |
| produit 4 | 5 | 4 | 3 | 6 | 4 |

Table 5: Decision-maker's judgement matrix 3

### 4.1.2. Resolution by the method of LON-ZO

Using this example with the LON-ZO method we get the following:

|  | $\sum_{j=1}^{j=5} w_{j}^{1} g_{j}^{1}\left(a_{i}\right)$ | $\sum_{j=1}^{j=5} w_{j}^{2} g_{j}^{2}\left(a_{i}\right)$ | $\sum_{j=1}^{j=5} w_{j}^{3} g_{j}^{3}\left(a_{i}\right)$ | $U\left(a_{i}\right)=\frac{3}{\sum_{k=1}^{3} \frac{1}{G_{k}\left(a_{i}\right)}}$ | Rang |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Produit 1 | 86 | 106 | 84 | 91.00 | 2 e |
| Produit 2 | 78 | 100 | 96 | 90.26 | 3 e |
| Produit 3 | 98 | 113 | 88 | 98.63 | 1 er |
| Produit 4 | 88 | 91 | 82 | 86.83 | 4 e |

Thus, the overall scores of the actions are:

| Produit 1 | 91.00 |
| :--- | :--- |
| Produit 2 | 90.26 |
| Produit 3 | 98.63 |
| Produit 4 | 86.83 |

By this method we find that Product 3 is better.

### 4.1.3. Résolution Par la méthode MACASP

|  | $\sum_{k=1}^{3} w_{1}^{k} g_{1}^{k}\left(a_{i}\right)$ | $\sum_{k=1}^{3} w_{2}^{k} g_{2}^{k}\left(a_{i}\right)$ | $\sum_{k=1}^{3} w_{3}^{k} g_{3}^{k}\left(a_{i}\right)$ | $\sum_{k=1}^{3} w_{4}^{k} g_{4}^{k}\left(a_{i}\right)$ | $\sum_{k=1}^{3} w_{5}^{k} g_{5}^{k}\left(a_{i}\right)$ | $\frac{\sum_{j=1}^{5} \sum_{k=1}^{3} w_{j}^{k} g_{j}^{k}\left(a_{i}\right)}{N}$ | Rang |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Produit 1 | 121 | 65 | 14 | 37 | 39 | $276 / 3$ | 2 e |
| Produit 2 | 114 | 67 | 18 | 42 | 33 | $274 / 3$ | 3 e |
| Produit 3 | 113 | 70 | 25 | 51 | 40 | $299 / 3$ | 1 er |
| Produit 4 | 101 | 48 | 28 | 42 | 42 | $261 / 34 \mathrm{e}$ |  |

the table shows that product 3 is the best compared to the other products
4.1.4. Resolution by the new method: EMETOP

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ | $c_{4}$ | $c_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| weight | 6.316 | 3.915 | 2.289 | 3.302 | 3.302 |
| $a_{1}$ | 6.351 | 5.354 | 2.000 | 3.697 | 4.082 |
| $a_{2}$ | 6.055 | 5.686 | 2.708 | 4.435 | 3.367 |
| $a_{3}$ | 6.055 | 5.745 | 3.697 | 5.196 | 4.082 |
| $a_{4}$ | 5.354 | 4.000 | 4.082 | 5.509 | 4.509 |

Table 6: Overall performance matrix of the three decision makers

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ | $c_{4}$ | $c_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| weight | 0.330 | 0.205 | 0.120 | 0.173 | 0.173 |
| $a_{1}$ | 0.532 | 0.510 | 0.310 | 0.412 | 0.506 |
| $a_{2}$ | 0.508 | 0.542 | 0.420 | 0.494 | 0.418 |
| $a_{3}$ | 0.508 | 0.548 | 0.573 | 0.579 | 0.506 |
| $a_{4}$ | 0.449 | 0.381 | 0.632 | 0.502 | 0.559 |

Table 7: Standardisation of the global performance matrix

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ | $c_{4}$ | $c_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| weight | 0.330 | 0.205 | 0.120 | 0.173 | 0.173 |
| $a_{1}$ | 0.176 | 0.105 | 0.037 | 0.071 | 0.087 |
| $a_{2}$ | 0.168 | 0.111 | 0.050 | 0.085 | 0.072 |
| $a_{3}$ | 0.168 | 0112 | 0.069 | 0.100 | 0.087 |
| $a_{4}$ | 0.148 | 0.078 | 0.076 | 0.087 | 0.097 |

Table 8: Normalized and weighted matrix


Table 9: Ideal favourable solutions $A^{+}$.

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ | $c_{4}$ | $c_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A^{-}$ | 0.176 | 0.078 | 0.037 | 0.071 | 0.097 |

Table 10: Ideal unfavourable solutions $A^{-}$.

| alternative | $E^{+}$ |
| :---: | :---: |
| $a_{1}$ | 0.058 |
| $a_{2}$ | 0.035 |
| $a_{3}$ | 0.026 |
| $a_{4}$ | 0.044 |

Table 11: Calculation of the deviation of the solution $A^{+}$from each row of the matrix

| alternative | $E^{-}$ |
| :---: | :---: |
| $a_{1}$ | 0.028 |
| $a_{2}$ | 0.046 |
| $a_{3}$ | 0.056 |
| $a_{4}$ | 0.050 |

Table 12: Calculation of the deviation of the solution $A^{-}$from each row of the matrix

| alternative | $S^{*}$ | Odre de choix |
| :---: | :---: | :---: |
| $a_{1}$ | 0.325 | 4 e |
| $a_{2}$ | 0.567 | 2 e |
| $a_{3}$ | 0.685 | 1 er |
| $a_{4}$ | 0.532 | 3 e |

Table 13: Calculation of the proximity coefficient of the ideal solution and arrangement in order of choice

CONCLUSION : It can be seen that with EMETOP the best choice is produit 3.

### 4.2. Digital experience 2

This part is from [6]

### 4.2.1. Problem statement

Three low-income brothers want to team up to buy a second-hand android mobile phone for their uncle. Among the mobile phones found at the retailers, three caught the attention of these three people. The criteria that were selected by the latter are: The price of the phone, the duration of use of the phone(very short duration of use, short duration of use, medium, long duration of use, very long duration of use) and the condition of the phone (very bad condition, bad condition, medium, good condition and very good condition). The information on the length of use of each device is given honestly as well as its condition. All three decision-makers want to have a complete stock list before making a choice. The performance matrices of the three decision makers are as follows: We have three actions;
$A=\{$ samsung galaxy S5, Tecno Spark 3, Tecno sapark 3 pro $\}$
Let's call it:

- $a_{1}$, the phone samsung galaxy S5;
- $a_{2}$, the phone Tecno Spark 3;
- $a_{3}$, the phone Tecno Spark 3 pro.

A coherent family of criteria was chosen: $\mathrm{F}=\left\{c_{1}, c_{2}, c_{3}\right\}$ where: $g_{1}$ is the price of the phone, $g_{2}$ is the duration of use of the phone and $g_{3}$ is the condition of the phone. Each of these three decision-makers ( $D_{1}, D_{2}$ and $D_{3}$ ) constructs its judgment matrix.

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| Scale min | 0 | 0 | 0 |
| Scale max | 10 | 10 | 10 |
| weight | 2 | 1 | 3 |
| $a_{1}$ | 4 | 5 | 3 |
| $a_{2}$ | 4 | 5 | 3 |
| $a_{3}$ | 2 | 7 | 8 |

Table 14: Decision maker's judgment matrix $1\left(D_{1}\right)$ ( example 2)

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| Scale min | 0 | 0 | 0 |
| Scale max | 10 | 10 | 10 |
| weight | 2 | 4 | 2 |
| $a_{1}$ | 5 | 2 | 1 |
| $a_{2}$ | 3 | 6 | 4 |
| $a_{3}$ | 6 | 1 | 5 |

Table 15: Decision Maker Judgement Matrix $2\left(D_{2}\right)$ (example 2)

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| scale min | 0 | 0 | 0 |
| scale max | 10 | 10 | 10 |
| weight | 2 | 3 | 5 |
| $a_{1}$ | 4 | 5 | 5 |
| $a_{2}$ | 1 | 4 | 2 |
| $a_{3}$ | 3 | 6 | 1 |

Table 16: Decision maker's judgement matrix $3\left(D_{3}\right)$ (example 2)

### 4.2.2. Résolution par la méthode Lon-Zo

By successively applying the formulas $g_{, l}\left(a_{i}\right)=\sum_{j=1}^{j=n} w_{j, l} g_{j, l}\left(a_{i}\right)$ et

$$
g\left(a_{i}\right)=\frac{k}{\sum_{l=1}^{k} \frac{1}{g_{l l}\left(a_{i}\right)}}
$$

. The results are shown in the following table:
By the Lon-Zo method the action $a_{3}$ : the Tecno Spark 3 pro phone is the best.

### 4.2.3. Resolution by MACASP

$\operatorname{De} g\left(a_{i}\right)=\frac{1}{k} \sum_{j=1}^{n} g_{j}\left(a_{i}\right)=\frac{1}{k} \sum_{j=1}^{n}\left(\sum_{l=1}^{k} w_{j, l} g_{j, l}\left(a_{i}\right)\right)$ the results are shown in the following table:
We can see that with MACASP we cannot make the best choice.

| Alternatives $\left(a_{i}\right)$ | Overall performance $\left(g\left(a_{i}\right)\right)$ | Rank |
| :---: | :---: | :---: |
| $a_{1}$ | 25.798 | 3rd |
| $a_{2}$ | 28.931 | 2 nd |
| $a_{3}$ | 29.552 | 1 st |

Table 17: Overall Performance and rank (exemple 2)

| Alternatives $\left(a_{i}\right)$ | Overall performance $\left(g\left(a_{i}\right)\right)$ | Rank |
| :---: | :---: | :---: |
| $a_{1}$ | 30 | 1 st |
| $a_{2}$ | 28 | 2 nd |
| $a_{3}$ | 30 | 1 st |

Table 18: Overall performance and rank ( exemple 2)
4.2.4. Resolution with the new method: EMETOP

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| Weight | 2.000 | 2.289 | 3.107 |
| $a_{1}$ | 4.359 | 4.243 | 3.416 |
| $a_{2}$ | 2.944 | 5.066 | 3.109 |
| $a_{3}$ | 4.041 | 5.354 | 5.477 |

Table 19: Overall performance matrix of the three decision makers $\left(D_{2}\right)$ (example 2)

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| Weight | 0.270 | 0.310 | 0.420 |
| $a_{1}$ | 0.657 | 0.499 | 0.477 |
| $a_{2}$ | 0.444 | 0.596 | 0.434 |
| $a_{3}$ | 0.609 | 0.630 | 0.764 |

Table 20: Standardisation of the global performance matrix

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| Weight | 0.270 | 0.310 | 0.420 |
| $a_{1}$ | 0.178 | 0.154 | 0.200 |
| $a_{2}$ | 0.120 | 0.184 | 0.182 |
| $a_{3}$ | 0.165 | 0.195 | 0.321 |

Table 21: Normalized and weighted matrix

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| $A^{+}$ | 0.120 | 0.154 | 0.321 |

Table 22: Ideal favourable solutions $A^{+}$

|  | $c_{1}$ | $c_{2}$ | $c_{3}$ |
| :---: | :---: | :---: | :---: |
| $A^{-}$ | 0.178 | 0.195 | 0.182 |

Table 23: Ideal unfavourable solutions $A^{-}$.

| alternative | $E^{+}$ |
| :---: | :---: |
| $a_{1}$ | 0.134 |
| $a_{2}$ | 0.142 |
| $a_{3}$ | 0.060 |

Table 24: Calculation of the deviation of the solution $A^{+}$from each row of the matrix

| alternative | $E^{-}$ |
| :---: | :---: |
| $a_{1}$ | 0.044 |
| $a_{2}$ | 0.059 |
| $a_{3}$ | 0.139 |

Table 25: Calculation of the deviation of the solution $A^{-}$from each row of the matrix

| alternative | $S^{*}$ | Odre de choix |
| :---: | :---: | :---: |
| $a_{1}$ | 0.248 | 3 e |
| $a_{2}$ | 0.292 | 2 e |
| $a_{3}$ | 0.698 | 1 er |

Table 26: Calculation of the proximity coefficient of the ideal solution and arrangement in order of choice

From the previous results we obtain: $a_{3}>a_{2}>a_{1}$ from which $a_{1}$ : the spark 3 pro mobile phone is better. We can see that by applying our method, we obtain exactly the same ranking as the Lon-Zo method: $a_{3}>a_{2}>a_{1}$.

## 5. comparison

In the first example, the Lon-Zo method and our method gave the same choice: produit 3 is the best. In the second example, the problems dealt with are those of ranking and choice. The arithmetic mean alone gave ties but the Lon-Zo method and our method gave exactly the same results: same ranking and same choice.

## 6. Conclusion

The new method, known as the Extension of the TOPSIS Method using the quadratic and geometric mean (EMETOP), made it possible to separate all the actions, but the arithmetic mean produced ties in certain cases. It reduces the shortcomings observed when using the arithmetic mean. In addition, it requires fewer calculations and deals with almost all the problems involved in decision support, since the ranking obtained by the aforementioned method can be used to make a choice, sort and so on. It has given results as good as the Lon-Zo method. From all the above we believe that EMETOP seems to be suitable for solving group decision problems.

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