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Research paper

Modelling silicon etching using inverse methods

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Abstract

This paper considers a real-world application of a recently presented alternative form of the Gelfand-Levitan equation. Here is considered the case of potential in the plasma above silicon during the etching process. It is shown that although standard methods have significant challenges, the alternative form of the Gelfand-Levitan equation gives a straightforward way to determine the reflection coefficient from an assumed potential.

Keywords: *Inverse Scattering; Gelfand-Levitan Equation; Reflection Coefficient; One-Dimensional Scattering.*

1. Introduction

The problem of modelling the potential in the plasma above silicon during the etching process is of interest for obvious reasons. This paper presents a method to determine this potential structure using recently published results. The method used here is not what would traditionally be done, so some background is important. First, this paper looks at the experimental structure of the etching process and the assumed structure of the potential used by experimentalists. This background discussion includes a survey of theoretical techniques commonly used for analysis and why their application here is problematic. Then, a different approach is presented using recently developed methods that give the expected reflection coefficient for the potential assumed by the experimentalists. This potential's form is general enough that verification through a scattering experiment is reasonable.

2. Background on plasma etching and theoretical methods

An interesting problem arises in the fabrication of semi-conductors.[1] An instantaneous potential difference is set up between two plates with a low-density plasma between the plates. The problem is to determine the potential in the region between the plates, so that the etching process can be controlled and predicted.

This could be done by trying to solve the dynamics of the plasma, but this is probably not possible with current methods. Further, the calculations which can be performed make assumptions, such as thermal equilibrium, lack of instabilities, inviscid plasmas, etc., which may not be applicable.

What is desired is a time averaged electric potential, so a model will be used which allows direct measurement of the potential using inverse scattering methods. [2], [3] This corresponds to electro-magnetic wave propagation in the plasma region.

Experimentalists believe [1] that a constant plus a sine function, probably with a phase, is a good approximation to the potential between the plates. Since matter boils off at what is defined as x = 0, something other than a sine function will be needed at x = e, where e is a small distance above the plate. A contribution of ld(x-e) is therefore added to the sine function as the simplest perturbation which leads to significant modifications of the spectral data and reflection coefficient for the model. The sine function part of the potential is:

$$V_o = -a + b\sin\left(\frac{\pi x}{L} + c\right)\ 0 \leq x \leq L \tag{1a}$$

where a, b and c are constants. V_o is zero elsewhere. The modified potential for this model is then:

$$V_1 = ld(x\text{-}e) + V_o \tag{1b}$$

Following the methods of previous authors [4-7], Maxwell's equations for a linear, isotropic, inhomogeneous, medium are now considered. For the case without sources or polarization changing processes, Maxwell's equations can be modelled with transverse solutions by the equations:

$$[\nabla^2 + k^2\epsilon_T(\vec{x})]u(\vec{x}) = 0 \tag{2a}$$

$$[\nabla^2 - \frac{1}{c^2}\partial_t^2]U(\overrightarrow{x,t}) = 0 \tag{2b}$$

where $\epsilon_T(\vec{x})$ is the inhomogeneous dielectric function and u & U are the magnitudes of the electric field in the fixed polarization direction.
When there are no magnetic fields present, and the electric field density is low enough that electron-electron collisions are negligible, the dielectric function may be approximated by:

$$\epsilon_T(\vec{x}) = k^2\left[1 - \frac{\epsilon_R(\vec{x})}{k^2}\right] = k^2 - V(\vec{x}) \tag{3}$$

where $\epsilon_R(\vec{x})$ is the reduced dielectric function and $V(\vec{x})$ is the spatial energy distribution of electrons.
Substitution of expression (3) into equation (2b) yields the Schrodinger equation:

$$[\nabla^2 + k^2 - V(\vec{x})]u(\vec{x}) = 0 \tag{4a}$$

And the plasma wave equation:

$$[\nabla^2 - \frac{1}{c^2}\partial_t^2 - V(\vec{x})]U(\overrightarrow{x,t}) = 0 \tag{4b}$$

There has been important work done with equation (4a) by Newton [8], who has done rigorous studies of the Marchenko inverse problem, Since this approach requires a five variable data set to construct a three variable potential, Cheney et al [9, 10] and Defacio et al [7] have explored the time domain, which is represented by equation (4b). They find related inversions which require only a three variable data set for a more restricted class of potentials. However, if the plasma is transversely homogeneous, (i.e., $\epsilon(\vec{x}) = \epsilon(x_1)$, where x_1 is a single cartesian coordinate) then the full structure of equations (4a) and (4b) is not required since ∇^2 reduces to d^2/dx_1^2. In this case, the plasma wave equation reduces to the case studied by Jordan and Ahn [5]:

$$[d^2/dx_1^2 + k^2 - V(x_1)]\varphi(x_1, k) = 0 \tag{5}$$

The rigorous structure if (5) has been studied extensively by Faddeev [11], Newton [12] and especially by Dieft et al [13] and Sabatier [14].
Calculational aspects of equation (5) have been carried out by Jordan and Ahn [5]. However, like Sabatier [14], all of their calculations are for potentials with rational coefficients. Thus, they are not applicable in equation (1).
With the previous assumptions, one model for an electromagnetic wave propagation through a plasma is

$$[d^2/dx_1^2 + k^2 - V_1(x_1)]\varphi(x_1, k) = 0 \tag{6}$$

Where V_1 is the potential given in equation (1).

3. Recent work and applications to plasma etching

Recent work presented in this journal [15], looked at an alternative method of analysis using assumed separable characteristics in the Gelfand-Levitan equation. In summary, this paper found that the relationship between the reflection coefficient, R(k), and the potential, V(r) can be expressed as:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} R(k)e^{-ik(r+s)}dk = \frac{d}{dr}\exp\{-\frac{1}{2}\int_0^r\int_0^s V(s')ds'\,ds\} \tag{7}$$

This can be used to find the reflection coefficient for the potential of equation (1a). To do this equation (7) is first rewritten as:

$$R(k) = \int_{-\infty}^{+\infty} dr\, e^{2ikr}\frac{d}{dr}exp\{-\frac{1}{2}\int_0^r\int_0^s V(s')ds'\,ds\}$$

Applying the form of the potential form (1a) gives:

$$R(k) = \int_{-\infty}^{+\infty} dr\, e^{2ikr}\frac{d}{dr}exp\{-\frac{1}{2}\int_0^r\int_0^L\left(-a + bsin(\frac{\pi s'}{L} + c)\right)ds'\,ds\} \tag{8}$$

The right had side can be evaluated, and yields:

$$R(k) = -z^2/(z^2 + k^2) \tag{9}$$

Where:

$$z = -aL - \frac{bL}{\pi} cos(\pi + c) - \frac{bL}{\pi} cos\ (c)$$

This r(k) is the reflection coefficient that an experimentalist should look for to determine the general parameters of V_o in equation (1a).
It is important to note that V_1 is probably the actual potential, so a delta function needs to be added to the potential of equation (8), as mentioned earlier. This reflection coefficient can still be determined analytically.

4. Conclusion

This paper presents an application to a recently published alternative form of the Gelfand-Levitan equation. Here presented was how this alternative form gives a much more direct method for finding the reflection coefficient for a real-world potential in the case of a plasma above silicon in the etching process.

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