International Journal of Applied Mathematical Research, 11 (2) (2022) 43-47

International Journal of Applied Mathematical Research

Website: www.sciencepubco.com/index.php/IJAMR

doi:

Research paper

Analytical solutions of reaction-diffusion-convection type equations from porous media by the Laplace-Adomian method

Yanick Alain Servais WELLOT[1*] **and Gires Dimitri NKAYA**[2]

[1] *Université Marien NGOUABI Congo Brazzaville*
[2] *Université Marien NGOUABI Congo Brazzaville*
[*] *Corresponding author E-mail: wellotyanick@gmail.com*

Abstract

The aim of this paper is to solve analytically fluid flow problems in a porous medium, the Laplace-Adomian method gives algorithms that converge faster to achieve the exact solution when it exists.

Keywords: *Laplace-Adomian method, media porous, Reaction-diffusion-convection equation.*

1. Introduction

The mathematical models associated with the flow in porous medium, with possible transport of solutes are represented by the systems of partial differential equations based on reaction- diffusion-convection. The study of flows in porous media is central to the oil industry during the exploitation of an oil or gas deposit, in the management of water resources, pollution by chemical, agricultural or radioactive wastes, and also many environmental problems.
Natural porous media are heterogeneous at several scales, which complicates experimental studies, if not impossible, while predictions are vital and the need for reliable numerical simulation models remains [10, 15].
In this paper, we focus on nonlinear partial differential equations and the use of Laplace transforms and the Adomian decompositional method to analytically solve these equations[2, 4, 5].

2. About the Laplace-Adomian method

The mathematical modeling of physical systems leads to functional equations (ordinary differential equations (ODE), partial differential equations (PDEs), integro-differential equations and integral equations, ...). The search for exact or approximate solutions, when they exist, uses several methods. Among then, we find the Adomian decomposition methods of homotopy perturbation method, Laplace-Adomian method , variational iterations method, ...
Laplace transforms do not allow us to solve nonlinear equations, because there is no Laplace transform of nonlinear terms[6, 7, 8, 9]. To circumvent the difficulty or to overcome this insufficiency of the Laplace transform, a coupling is made between the Laplace transforms and the Adomian decomposition method. It is this coupling that gives the Laplace-Adomian method[11, 12, 13, 14].
The purpose of this work is to experiment this Laplace-Adomian method by avoiding the linearization and the discretization of the space and of time for better to solve the models of the partial differential equations of the porous media problems.
The presentation below is shows how the Laplace-Adomian method works. It is the algorithm of the method.[1, 4, 5]
Consider a functional equation

$$Au = h \tag{1}$$

With

$$A = L + R + N \tag{2}$$

The equation (1) becomes:

$$Lu + Ru + Nu = h \tag{3}$$

Where u is a unknown function of H into H (H is a Hilbert space), L and R are linear operators; and L invertible, with L^{-1} as inverse. N is a nonlinear operator from a Hilbert space H into H. h is a given fonction in H.
By applying the transform $\mathscr{L}$ of Laplace at the equation (3), we have:

$$\mathscr{L}(Lu) + \mathscr{L}(Ru) + \mathscr{L}(Nu) = \mathscr{L}(h) \tag{4}$$

Case of a partial differential equation

Let's set $L_{tt}(.) = \dfrac{\partial^2}{\partial t^2}(.)$ with the initial and boundary conditions, we have the following relation:

$$u(x,0) = \alpha_1(x) \quad et \quad u_t(x,0) = \alpha_2(x) \tag{5}$$

Finally, with the conditions (5), the Laplace transform, into the equation (4) we have:

$$\mathscr{L}(Lu(x,t))+\mathscr{L}(Ru(x,t))+\mathscr{L}(Nu(x,t))=\mathscr{L}h(x,t) \tag{6}$$

Let:

$$\begin{aligned} s^2\mathscr{L}(u(x,t)) = su(x,0)+u_t(x,0)+\mathscr{L}(h(x,t)) \\ -\mathscr{L}(Ru(x,t)-\mathscr{L}(Nu(x,t)) \end{aligned} \tag{7}$$

This gives the expression:

$$\begin{aligned} \mathscr{L}(u(x,t)) = \frac{1}{s}u(x,0)+\frac{1}{s^2}u_t(x,0)+\frac{1}{s^2}\mathscr{L}(h(x,t)) \\ -\tfrac{1}{s^2}\mathscr{L}(Ru(x,t))-\tfrac{1}{s^2}\mathscr{L}(Nu(x,t)) \end{aligned} \tag{8}$$

Or again

$$\begin{aligned} \mathscr{L}(u(x,t)) = \frac{1}{s}\alpha_2(x)+\frac{1}{s^2}\alpha_1(x)+\frac{1}{s^2}\mathscr{L}(h(x,t)) \\ -\tfrac{1}{s^2}\mathscr{L}(Ru(x,t))-\tfrac{1}{s^2}\mathscr{L}(Nu(x,t)) \end{aligned} \tag{9}$$

We look for the solution u when it exists, the equation (1) in the form of a series:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t) \tag{10}$$

The non-linear part is also expressed as a series of polynomials:

$$N(u(x,t))=\sum_{n=0}^{\infty}A_n(x,t) \tag{11}$$

where A_n are Adomian polynomials defined by the formula . Substituting (10) and (11) in (9) we obtain the expression:

$$\begin{aligned} \sum_{n=0}^{\infty}\mathscr{L}(u_n(x,t)) = \frac{1}{s}\alpha_1(x)+\frac{1}{s^2}\alpha_2(x)+\frac{1}{s^2}\mathscr{L}(h(x,t)) \\ -\sum_{n=0}^{\infty}\left(\frac{1}{s^2}\mathscr{L}(Ru_n(x,t))+\frac{1}{s^2}\mathscr{L}(A_n(x,t))\right) \end{aligned} \tag{12}$$

We deduce the following Laplace-Adomian algorithm:

$$\begin{cases} \mathscr{L}(u_0(x,t)) = \frac{1}{s}\alpha_1(x)+\frac{1}{s^2}\alpha_2(x)+\frac{1}{s^2}\mathscr{L}(h(x,t)) \\ \\ \mathscr{L}(u_{n+1}(x,t)) = \\ -\left(\frac{1}{s^2}\mathscr{L}(Ru_n(x,t))+\frac{1}{s^2}\mathscr{L}(A_n(x,t))\right), n\geqslant 0 \end{cases} \tag{13}$$

By applying the inverse $\mathscr{L}^{-1}$ of the Laplace transform into the expressions of $u_0(x,t)$ and $u_{n+1}(x,t)$ are established:

$$\begin{cases} u_0(x,t) = \mathscr{L}^{-1}\left(\frac{\alpha_1(x)}{s}\right) \\ \\ \quad +\mathscr{L}^{-1}\left(\frac{1}{s^2}\alpha_2(x))+\frac{1}{s^2}\mathscr{L}[h(x,t)]\right) \\ u_{n+1}(x,t) = \\ \\ \quad -\mathscr{L}^{-1}\left(\frac{1}{s^2}\mathscr{L}[Ru_n(x,t)]-\frac{1}{s^2}\mathscr{L}[A_n(x,t)]\right), n\geqslant 0 \end{cases} \tag{14}$$

3. Numerical Applications

3.1. Example 1

Let us consider the following equation [2]

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = u^2(x,t)\frac{\partial^2 u(x,t)}{\partial x^2}+u^3(x,t)+\frac{\partial u(x,t)}{\partial x}+u(x,t) \\ \\ u(x,0)=\sin(x) \end{cases} \tag{15}$$

Or:

$$\frac{\partial u(x,t)}{\partial t} = u^2(x,t)\frac{\partial^2 u(x,t)}{\partial x^2}+u^3(x,t)+\frac{\partial u(x,t)}{\partial x}+u(x,t) \tag{16}$$

By applying the Laplace transform in relation witch (16), we have:

$$\mathscr{L}\left(\frac{\partial u(x,t)}{\partial t}\right)=\mathscr{L}\left(u^2(x,t)\frac{\partial^2 u(x,t)}{\partial x^2}+u^3(x,t)+\frac{\partial u(x,t)}{\partial x}+u(x,t)\right) \tag{17}$$

We obtain:

$$\mathscr{L}(u(x,t)=\frac{u(x,0)}{s-1}+\frac{1}{s-1}\mathscr{L}(Nu(x,t))+\frac{1}{s-1}\mathscr{L}\left(\frac{\partial u(x,t)}{\partial x}\right) \tag{18}$$

With:

$$Nu(x,t)=u^2(x,t)\frac{\partial^2 u(x,t)}{\partial x^2}+u^3(x,t) \tag{19}$$

The application of the inverse Laplace transform gives the following result:

$$u(x,t)=e^t\sin(x)+\mathscr{L}^{-1}\left(\frac{1}{s-1}\mathscr{L}(Nu(x,t))+\frac{1}{s-1}\mathscr{L}\left(\frac{\partial u(x,t)}{\partial x}\right)\right) \tag{20}$$

Let us look for the solution of $u(x,t)$ in the form (21) below:

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t) \tag{21}$$

And:

$$Nu(x,t)=\sum_{n=0}^{\infty}A_n(x,t) \tag{22}$$

We obtain the following algorithm:

$$\begin{cases} u_0(x,t)=e^t\sin(x) \\ u_{n+1}(x,t)=\mathscr{L}^{-1}\left(\frac{1}{s-1}\mathscr{L}(A_n(x,t))+\frac{1}{s-1}\mathscr{L}\left(\frac{\partial u_n(x,t)}{\partial x}\right)\right) \end{cases} \tag{23}$$

with:

$$A_0(x,t)=u_0^2\frac{\partial^2 u_0}{\partial x^2}+u_0^3=0 \tag{24}$$

By substitution process, we have:

$$u_1(x,t)=\mathscr{L}^{-1}\left(\frac{1}{s-1}\mathscr{L}(A_0(x,t))+\frac{1}{s-1}\mathscr{L}\left(\frac{\partial u_0(x,t)}{\partial x}\right)\right) \tag{25}$$

Let:

$$u_1(x,t)=\mathscr{L}^{-1}\left(\frac{1}{s-1}\mathscr{L}\left(\frac{\partial u_0(x,t)}{\partial x}\right)\right) \tag{26}$$

Hence:

$$u_1(x,t)=\cos(x)\mathscr{L}^{-1}\left(\frac{1}{(s-1)^2}\right) \tag{27}$$

Thus:

$$u_1(x,t)=te^t\cos(x) \tag{28}$$

By the same process, the other terms are obtained.

$$u_2(x,t) = \mathscr{L}^{-1}\left(\frac{1}{s-1}\mathscr{L}(A_1(x,t)) + \frac{1}{s-1}\mathscr{L}(\frac{\partial u_1(x,t)}{\partial x})\right) \quad (29)$$

$$A_1(x,t) = 2u_1u_0\frac{\partial^2 u_0}{\partial x^2} + u_0^2\frac{\partial^2 u_1}{\partial x^2} + 3u_1u_0^2 = 0 \quad (30)$$

$$u_2(x,t) = -\sin(x)\mathscr{L}^{-1}\left(\frac{1}{(s-1)^3}\right) \quad (31)$$

$$u_2(x,t) = -\frac{1}{2}t^2e^t\sin(x) \quad (32)$$

and:

$$A_2(x,t) = 2u_2u_0\frac{\partial^2 u_0}{\partial x^2} + u_1^2\frac{\partial^2 u_0}{\partial x^2} + 2u_1u_0\frac{\partial^2 u_1}{\partial x^2} + u_0^2\frac{\partial^2 u_2}{\partial x^2} + 3u_1^2u_0 + 3u_0^2u_2 = 0 \quad (33)$$

$$u_3(x,t) = -\cos(x)\mathscr{L}^{-1}\left(\frac{1}{(s-1)^4}\right) \quad (34)$$

$$u_3(x,t) = -\frac{1}{6}t^3\cos(x) \quad (35)$$

$$\begin{cases} A_3(x,t) = 0 \\ u_4(x,t) = \frac{1}{4!}t^4\sin(x) \end{cases} \quad (36)$$

Recursively, we have :

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + u_4(x,t) + \dots \quad (37)$$

Therefore:

$$u(x,t) = e^t\sin(x)\left(1 - \tfrac{1}{2}t^2 + \dots + (-1)^n\frac{t^{2n}}{2n!}\right) + e^t\cos(x)\left(t - \tfrac{1}{6}t^3 + \dots + (-1)^n\frac{t^{2n+1}}{(2n+1)!}\right) \quad (38)$$

That is:

$$u(x,t) = e^t(\sin(x)\cos(t) + \cos(x)\sin(t)) = e^t\sin(x+t) \quad (39)$$

The exact solution of the problem 1 is:

$$u(x,t) = e^t\sin(x+t) \quad (40)$$

3.2. Example 2

Let us consider the following equation [8]

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = \left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right)_x + 3\left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right) + 2\left(u(x,t) - u^2(x,t)\right) \\ u(x,0) = 2\sqrt{e^x - e^{-4x}} \end{cases} \quad (41)$$

Let's solve this equation by the Laplace-Adomian method.
Let's ask:

$$\frac{\partial u(x,t)}{\partial t} = \left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right)_x + 3\left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right) + 2\left(u(x,t) - u^2(x,t)\right) \quad (42)$$

Now:

$$\frac{\partial u(x,t)}{\partial t} = Nu(x,t) + 2u(x,t) \quad (43)$$

With :

$$Nu(x,t) = \left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right)_x + 3\left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right) - 2u^2(x,t) \quad (44)$$

Applying the Laplace transform $\mathscr{L}$ to the equation (42), generates the following:

$$\mathscr{L}\left(\frac{\partial u(x,t)}{\partial t}\right) = \mathscr{L}(Nu(x,t)) + 2\mathscr{L}(u(x,t)) \quad (45)$$

Or:

$$s\mathscr{L}(u(x,t)) - 2\mathscr{L}(u(x,t)) = u(x,0) + \mathscr{L}(Nu(x,t)) \quad (46)$$

Or again:

$$\mathscr{L}(u(x,t)) = \frac{1}{s-2}u(x,0) + \frac{1}{s-2}\mathscr{L}(Nu(x,t)) \quad (47)$$

Applying the inverse Laplace transform $\mathscr{L}^{-1}$ gives:

$$u(x,t) = \mathscr{L}^{-1}(\frac{1}{s-2}u(x,0)) + \mathscr{L}^{-1}(\frac{1}{s-2}\mathscr{L}(Nu(x,t))) \quad (48)$$

Now,

$$\mathscr{L}^{-1}\left(\frac{1}{s-2}\right) = e^{2t} \quad (49)$$

therefore:

$$u(x,t) = 2e^{2t}\sqrt{e^x - e^{-4x}} + \mathscr{L}^{-1}(\frac{1}{s-2}\mathscr{L}(Nu(x,t))) \quad (50)$$

Let's find the solution of $u(x,t)$ in the form

$$u(x,t) = \sum_{n=0}^{+\infty} u_n(x,t) \quad (51)$$

Knowing that:

$$Nu(x,t) = \sum_{n=0}^{+\infty} A_n(x,t) \quad (52)$$

We obtain the following algorithm:

$$\begin{cases} u_0(x,t) = 2e^{2t}\sqrt{e^x - e^{-4x}} \\ u_{n+1}(x,t) = \mathscr{L}^{-1}\left(\frac{1}{s-2}\mathscr{L}(A_n(x,t))\right) \end{cases} \quad (53)$$

With:

$$A_0(x,t) = \left(u_0(x,t)\frac{\partial u_0(x,t)}{\partial x}\right)_x + 3\left(u_0(x,t)\frac{\partial u_0(x,t)}{\partial x}\right) - 2u_0^2(x,t) \quad (54)$$

and:

$$A_1(x,t) = \left(u_0(x,t)\frac{\partial u_1(x,t)}{\partial x} + u_1(x,t)\frac{\partial u_0(x,t)}{\partial x}\right)_x + 3\left(u_0(x,t)\frac{\partial u_1(x,t)}{\partial x} + u_1(x,t)\frac{\partial u_0(x,t)}{\partial x}\right) - 4u_0u_1(x,t) \quad (55)$$

The calculations made give:

$$\begin{cases} A_0(x,t) = 0 \\ u_1(x,t) = 0 \\ A_1(x,t) = 0 \\ u_2(x,t) = 0 \\ A_2(x,t) = 0 \\ u_3(x,t) = 0 \\ . \\ . \\ . \\ u_n(x,t) = 0 \end{cases} \tag{56}$$

Therefore, the exact solution of the problem is then :

$$u(x,t) = 2e^{2t}\sqrt{e^x - e^{-4x}} \tag{57}$$

3.3. Example 3

Let us consider the following equation [3]

$$\begin{cases} \dfrac{\partial w(x,t)}{\partial t} = 2w(x,t)\dfrac{\partial^2 w(x,t)}{\partial x^2} + 2\left(\dfrac{\partial w(x,t)}{\partial x}\right)^2 - \beta w(x,t) \\ w(x,0) = x \end{cases} \tag{58}$$

$$\begin{cases} \dfrac{\partial w(x,t)}{\partial t} = 2w(x,t)\dfrac{\partial^2 w(x,t)}{\partial x^2} + 2\left(\dfrac{\partial w(x,t)}{\partial x}\right)^2 - \beta w(x,t) \\ w(x,0) = x \end{cases} \tag{59}$$

The application of the Laplace transform $\mathscr{L}$ to the equation (59), generates the following:

$$\begin{aligned} &\mathscr{L}\left(\frac{\partial w(x,t)}{\partial t}\right) = \\ &\mathscr{L}\left(2w(x,t)\frac{\partial^2 w(x,t)}{\partial x^2} + 2\left(\frac{\partial w(x,t)}{\partial x}\right)^2 + \beta w(x,t)\right) \end{aligned} \tag{60}$$

Let's put

$$N\left(w(x,t)\right) = 2w(x,t)\frac{\partial^2 w(x,t)}{\partial x^2} + 2\left(\frac{\partial w(x,t)}{\partial x}\right)^2 \tag{61}$$

We get:

$$s\mathscr{L}\left(w(x,t)\right) - w(x,0) = \mathscr{L}\left(Nw(x,t)\right) + \beta\mathscr{L}\left(w(x,t)\right) \tag{62}$$

This gives:

$$(s-\beta)\mathscr{L}\left(w(x,t)\right) = w(x,0) + \mathscr{L}\left(Nw(x,t)\right) \tag{63}$$

Afterwards,

$$\mathscr{L}\left(w(x,t)\right) = \frac{1}{(s-\beta)}w(x,0) + \frac{1}{(s-\beta)}\mathscr{L}\left(Nw(x,t)\right) \tag{64}$$

The application of the inverse Laplace transform gives the following result:

$$w(x,t) = xe^{\beta t} + \mathscr{L}^{-1}\left(\frac{1}{s-\beta}\mathscr{L}(Nw(x,t))\right) \tag{65}$$

Let's look for the solution of $u(x,t)$ in the form (66) below :

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{66}$$

and :

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n(x,t) \tag{67}$$

We obtain the following algorithm:

$$\begin{cases} w_0(x,t) = xe^{\beta t} \\ w_{n+1}(x,t) = \mathscr{L}^{-1}(\dfrac{1}{s-2}\mathscr{L}(A_n((x,t)) \end{cases} \tag{68}$$

With :

$$A_0(x,t) = \left(w_0(x,t)\frac{\partial^2 w_0(x,t)}{\partial x^2}\right) + \left(\frac{\partial w_0(x,t)}{\partial x}\right)^2 \tag{69}$$

and :

$$A_1(x,t) = \left(w_0(x,t)\frac{\partial^2 w_1(x,t)}{\partial x^2} + w_1(x,t)\frac{\partial^2 w_0(x,t)}{\partial x^2}\right) + 2\left(\frac{\partial w_1(x,t)}{\partial x}\right)\left(\frac{\partial w_0(\cdot}{\partial x}\right. \tag{70}$$

The calculations made give:

$$\begin{cases} A_0(x,t) = e^{2\beta t} \\ w_1(x,t) = -\dfrac{1}{\beta}(e^{\beta t} - e^{2\beta t}) \\ A_1(x,t) = 0 \\ w_2(x,t) = 0 \\ A_2(x,t) = 0 \\ w_3(x,t) = 0 \\ . \\ . \\ . \\ w_n(x,t) = 0, \qquad \forall n \geqslant 2 \end{cases} \tag{71}$$

Finally, the sum of the terms of the series gives:

$$w(x,t) = w_0(x,t) + w_1(x,t) = (x - \frac{1}{\beta})e^{\beta t} + \frac{1}{\beta}e^{2\beta t} \tag{72}$$

The exact solution of the problem is then:

$$w(x,t) = \left(x - \frac{1}{\beta}\right)e^{\beta t} + \frac{1}{\beta}e^{2\beta t} \tag{73}$$

4. Conclusion

The application of the Laplace-Adomian method has allowed us to obtain the exact solutions of the problems studied in this article. The manipulation of Adomian polynomials and Laplace transforms have allowed us to obtain efficient algorithms that converge faster to the exact solution of the problem. We can conclude that this method is well suited for solving nonlinear partial differential equations not only in the case of flows in porous media, but also in many physical phenomena of high complexity.

References

[1] Arshad Ali, Humaira, Laila, Kamal Shah, "Analytical Solution of General Fisher's Equation by using Laplace Adomian decomposition method", *J. Pur. Appl. Math.*, Vol.2, No.3, (2018), pp.1-4.

[2] S. Nuseir Ameina, "Power Series Solutions for Nonlinear Systems of Partial Differential Equations", *Journal of Mathematics Sciences: Advances and Applications*, Vol.61, (2020), pp.17-35.

[3] Jonas Berx and Joseph O. Indekeu, "The BLUES function method applied to partial differential equations and analytic approximants for interface growth under shear", *Institute for Theoretical Physics, KULeuven, Belgium (Dated: August 5. 2021)*, (2021)

[4] B. Beyi, J. Bonazebi-Yindoula, L. Somé, G. Bissanga, "Application of the Laplace-Adomian Method annd the SA Method to Solving Tthe Partial Differential and Integro-differential Equations" , *Applied Mathematical Sciences*, Vol.6, No.104, (2012), pp.5147-5159.

[5] J. Bonazebi-Yindoula, A. Massamba, G. Bissanga, "Solving of Klein-Gordon Equation by Two Methods of Numerical Analysis", *Journal of Applied Mathematics and Physics*, Vol.4, (2016), pp.1916-1929.

[6] Jasem Fadaei, "Application of Laplace-Adomian Decomposition Method on Linear and Nonlinear System of PDEs", *Applied Mathematical Sciences*, Vol.5, (2011), pp.1307-1315.

[7] M. Hussain and M. MajidKhan, "Modified Laplace Decomposition Method", *Applied Mathematical Sciences*, Vol.4, No.36, (2010), pp.1769-1783.

[8] A. Khalouta and A. Kadem, "New analytical method for solving nonlinear time-fractional reaction-diffusion-convection problems", *Revista Colombiana de Matemàticas*, Vol.54, (2020), pp.1-11.

[9] Onur Kiymaz, "An Algorithm for Solving Initial Value Problems Using Laplace-Adomian Decomposition Method", *Applied Mathematical Sciences*, Vol.3, No.30, (2009), pp.1453-1459.

[10] Vincent Le Maout, "Modélisation d'écoulements multiphasiques de fluides viscoélastiques en milieux poreux", *Thèse de doctorat Université de Bordeaux*, Bordeaux le 03 Novembre, 2020.

[11] Shahid Mahmood, Rasool Shan, Hassan Khan and Muhammad Arif, "Laplace Adomian Decomposition Method for Multi Dimensional Time Fractional Model of Navier-Stokes Equation", *Symmetry MDPI*, Vol.11 No.149, (2019), pp.1-15.

[12] A. Naghipour, J. Manafian, "Application of the Laplace Adomian Decomposition and implicit Methods for solving Burger's Equation", *TWMS J. Appl. Math.*, Vol.6, No.1, (2015), pp.68-77.

[13] P. Pue-on, "Laplace Adomian Decomposition Method for Solving Newell-Whitehead-Segel Equation", *Applied Mathematical Sciences*, Vol.7, No.132, (2013), pp.6593-6600.

[14] A. M. Wazwaz and M. S. Mehanna, "The Combined Laplace-Adomian Method for handling Singular Integral Equation of Heat Transfer", *International Journal of Nonlinear Sciences*, Vol.10, No.2, (2010), pp.248-252.

[15] F. Zami-Pierre, R. de Loubens, M. Quintard and Y. Davit, "Polymer Flow Through Porous Media: Numerical Prediction of the Contribution of Slip to the Apparent Viscosity", *Transport in Porous Media*, Vol.119, No.3, (2010), pp.521-538.