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Research paper

Modelling population dynamics using age-structured system of partial differential equations

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Abstract

In this paper, an age-structured model was used to model population dynamics, and make predictions through simulation using 2019 Kenya population data. The age-structured mathematical model was developed, using partial differential equations on population densities as func-tions of age and time. The population was structured into 20 clusters each of 5 year interval, and assigned different birth and death rate pa-rameters. Crank-Nicolson numerical scheme was used to simulate the model and the 2019 initial population of 38,589,011 was found to increase by 50% to 57,956,100 by 2050. The initial economic dependency ratio was computed to be 1:2, but due to changes in technology and improvement of living standards, the new ratio is lowered to 1:1.14. The graphical presentation showed a trend of transition from ex-pansive to constrictive population pyramid.

Keywords: *Age-Structured; Constrictive; Dependency Ratio; Expansive; Population; Simulation*.

1. Introduction

Population dynamics is defined as the study of changes in the number and composition of individuals in a population, and the important factors that influence those changes. This also includes the study of the way population is affected by birth rate, death rate, immigration and emigration, and its characterization in terms of distribution, gender, aging and population structure [1]. Mathematical modeling of population dynamics is a central topic in theoretical mathematical biology [2]. Population modeling is seen as a tool used to keep track of the components of population changes and a means to extract important parameters and determine trends from complex processes, to permit analysis of the causes of processes acting on the system and to make the prediction about the future of a given population. The continued increase in population has posed a great challenge to the whole World, because its growth rate is rapid and far outpacing the capacity of our planet to support it, given current practices [3]. This has led to uncontrolled urbanization, which has produced over-crowding, destitute settlements, crime and pollution, dismal health-care, pressure on resources and facilities, exacerbating food and water shortages reducing resilience of nature, climate change and making it harder for the most vulnerable communities to rise of generational poverty [4].

Every government requires accurate information on its population in order to put realistic plans and management of its government in terms of provision of quality services and products for its citizens. This is always obtained through census, which is time consuming, expensive and done at intervals of 10 years, giving a very long span of time that much unaccounted for changes happen. In addition, the knowledge of future population is required for the purpose of planning and budgeting of citizen's provision of services. In order to bridge this gap, the use of mathematical modeling provides information on current and projected population dynamics that can reliably be used for planning [5]. Demographic studies helps in effective provision of services to the citizens, so every government always needs accurate data about the current and future size of the population, and its structural characteristics Appropriate mathematical analytic and simulation methods are then used to draw conclusion and make predictions of future human population size, distribution, and simulation of future trends [6]. Modeling of dynamics interactions in nature can provide a convenient way of understanding how numbers change over time or in relation to each other. Many patterns can be noticed by using population models as a tool [7].

Population models are also used to determine optimal harvest rate of livestock, projection in agriculture, to understand the dynamics of biological invasions, and for environmental conservation, climatic and weather forecast, epidemiology, spread of parasites, viruses, and impact of epidemics, among others.

Factors that directly affect population growth include; Immigration and emigration, birth and death rate. Others which indirectly affect population growth are social factors which include; economic, education, health, and housing, among others. As much as high population is discouraged, it has positive effects which is not limited to availability of cheap labour, increased market of products, innovation and spurred economic activities.

However, high population brings negative effects like; environmental degradation, pollution, pressure on available resources, poverty, food crisis, rise of slums, increased aging dependency, dilapidated housing and health facilities, and increased insecurity. This can be avoided by determining the amount of efforts required to support and control optimal population.

Many strategies of controlling population are available and they include family planning, use of contraceptives, public health education awareness campaign, use of incentives, use of policies just to mention but a few.

1.1. Population mathematical modelling

Mathematical modeling of population is based on compartmental model, where population is considered to be boxed in a compartment (Figure 1), with inlets and outlets representing factors which either increase or decrease population as time changes.

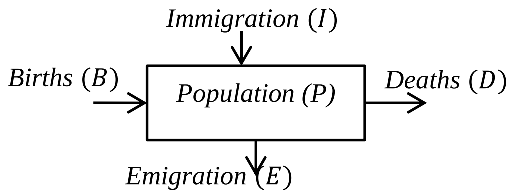

Fig. 1: Compartmental Model Showing Changes in Population Compartment.

This yields a book-keeping equation to keep track of the four components of population dynamics or change, that's a type of mathematical model that's applied to the study of population dynamics. In mathematical symbolism, population model equation is expressed as [8],

$$P_{t+1} = P_t + B_t - D_t + I_t - E_t \tag{1}$$

Where population size (P) at time $t + 1$is equal to the population at time t plus births (B) minus deaths (D) plus immigrants (I) minus emigrants(E).

1.2. Discrete mathematical population models

This is the simplest structured models where both time and the structure are discrete. The population is divided into two, the current and the future sub-populations, and the rates of growth are appended to the current population to give the next future populations. The mathematical equation is given by;

$$P_{t+1} = (B - D)P_t + M_t \tag{2}$$

Where the subscripts denote the time, M represents the net migration, B births and D deaths. Recursive substituting for $t = 0, 1, 2, \dots n$ yields the close form solution,

$$P_n = (\beta - \mu)^n P_0 + \sum_{i=1}^{n-1} (\beta - \mu)^{n-i} M_{i-1} \tag{3}$$

where P_0 is the initial population. The population grows if the growth rate $r := \beta - \mu > 0$. Here, β is the birth rate and μ is the death rate.

1.3. Instantaneous exponential model

Here, the population dynamics is modelled using ordinary differential equations, and the growth rate is assumed to be constant throughout the simulation. This is represented by the equation

$$P'(t) = rP(t) + M;\ P(0) = P_0 \tag{4}$$

Whose solution by integration yields,

$$P(t) = \left(P_0 + \frac{M}{r}\right) e^{rt} - \frac{M}{r} \tag{5}$$

Notice that if by policy or any other means migration is stopped $(M = 0)$, the population will grow exponentially $P(t) = P_0 e^{rt}$.

1.4. Age-structured model

Since population is always heterogeneous, it is ideal to divide the large population into homogeneous groups according to some significant parameters such as age, sex, size, maturity or proliferative state and study interactions within the given group's population. Models of this type are called structured and they describe the time evolution of the distribution of the population according to the fixed parameters. A continuous age-structured population at a given time reveals a set of individuals who were born over a range of past time and whose fertility and probability of survival depend on their age. For instance, in the human population dynamics, the fertility and probability of survival depend on age. This area of mathematical biology has been investigated by many authors [9]. Considering the population growth model depending on age, and assuming homogeinity of gender in terms of survival, birth and death rate, the population can be represented mathematically by $P(t, a)$, which is a function of time and age. Such a model is called age-structured as explicitly described by Von-Foester and McKendrick [10] and [11].
The dynamics of a continuous age – structured population is given by,

$$\frac{\partial P(t,x)}{\partial t} + \frac{\partial P(t,x)}{\partial a} = -\mu(a)P(t,a) \tag{6a}$$

Subject to the conditions,

$\mathrm{P}(\mathrm{t},0) = \int_0^\infty \mathrm{b}(\mathrm{a})\mathrm{P}(\mathrm{t},\mathrm{a})\mathrm{da}$ (6b) $\mathrm{P}(0,\mathrm{a}) = \mathrm{f}(\mathrm{a})$ (6c)

Where $P(t,a)$ is the population at time t and age a, $b(a)$ is the specific birth rate of the population cluster at age a, $\mu(a)$ is age–specific death rate and $P(0,a) = f(a)$ is the initial age distribution while $\int_0^\infty P(t,a)da$ is the total population at time t.

Most mathematical models of this type and those based on partial differential equations, are solved using appropriate numerical scheme, which include Finite Element Method (FEM) and Finite Difference Methods (FDM) [12].

This paper it is noted that equation (6) assumes homogeinity of people with respect to death rate, birth rate, contrary to the reality where infants and the aged are known to be vulnerable and nore susceptible to death. Similarly, equation (6) is only valid with the assumption that the entire population can give birth. This is not true from the fact that children under the age of 10 on average and adults (women) over 45 years are unproductive. Other age groups like school going ages (6 - 25) have very low birth rate, while above 35 – 45 are known to have lower fertility rate. With this in mind, this paper presents an age-structured model with partitions as described in the next section. The results of this study are significant contribution to the body of knowledge to the researchers, and the model can be used by the government or any organization who is interested in simulating population of a certain species, and analysis of societal and environmental impacts of their interaction [13].

2. Model formulation and analysis

This section looks at derivation of mathematical model of the factors that influences the human population dynamics. This is discussed under the following subsections: formulation of models.

The study of population dynamics is an area that has been of much interest to researchers, politicians, planning and provision of services. Of much important or interest is the ability to project the future population of a region, cluster or an ecosystem. Here we develop a model to describe the population dynamics of humans in region over a given period of time, some natural questions related to population problems, with the major challenges being: What will be the population of a region over a given period of time. Leslie model is a powerful tool used to determine the growth of a population as well as the age- structured distribution within a population over time [14]. This model has been used to describe the population dynamics of a wide variety of organisms and also humans [15]. Leslie model is a discrete, age-structured model of population growth that is popular in population ecology. It is one of the best-known ways to describe the growth of populations and their projected age-distribution, in which a population is closed to migration and where only one sex, usually female is considered [16]. In this paper, the same ideas are utilized using a system of partial differential equations with regard to this problem.

2.1. Model flow chart and model equations

In this section, the modeling of population dynamics is discussed. First, we commence with the general modeling of population with population parameters such as birth rate, death rate and net migration rate

Let $\rho(t,a)$ be the age-density function at time t with $a \in [0,a_+]$, where $a_+ < \infty$, is the maximum age of individuals, or with $a \in [0,\infty)$. Then, we have

$$\int_{a_1}^{a_2} \rho(t,a)da = P(t,a_2) - P(t,a_1) \tag{7}$$

Is the number of individuals having ages in the interval $[a_1,a_2]$ at time t, and the total population is given by;

$$\int_0^\infty \rho(t,a)da = P(t,a) \tag{8}$$

Let $\beta(a)$ be the age- specific birth rate. The number of births produced by individuals with ages in $[a_1,a_2]$ at any given time t is given by,

$$\int_{a_1}^{a_2} \beta(a)\rho(t,a)da \tag{9}$$

The total number of new-born of the entire productive population, at time t, is therefore given by:

$$\int_{a_k}^{a_{k+n}} \beta(a)\rho(t,a)da = B(t,a) \tag{10}$$

Where n is the length of productive period, and k is the starting age of productivity.

Let $\mu(a)$ be the age- specific mortality rate, then the total number of deaths at time t, occurring over a whole lifespan period $[0,\infty]$, is given by the equation:

$$\int_0^\infty \mu(a)\rho(t,a)da \tag{11}$$

In this paper, heterogeinity of the age characteristics is accounted by dividing the entire population into 20 clusters each covering a span of 5 years. These clusters fall into ages [0 – 4], [5 – 9], 10 – 14], …, [95 – 99]. Represented by the population densities $\rho_0(t,a), \rho_1(t,a), \rho_2(t,a), \dots, \rho_{19}(t,a)$ respectively, with $\rho_k(t,a)$ having corresponding differential $\beta_k(a), \mu_k(a)$ birth and death rates, together with its associated net migration rate $m_k(a)$. In this paper, it is assumed that age dependent fertility rate $\beta_k(a)$ is zero for immature clusters $\rho_0(a) - \rho_1(a)$ of ages [0 – 9] and zero for the aged clusters $\rho_9(a) - \rho_{19}(a)$ of ages [45 – 99]. Let $s_k(a)$ be the survival rate of individuals transiting from cluster $\rho_k(a)$ to the next cluster $\rho_{k+1}(a)$ with $s_{-1}(a) = \rho(t,0)$ being the total new borns from productive clusters.

With these assumptions and definitions, the flow chart illustrating the relationship between all the population clusters is presented in Figure 2.

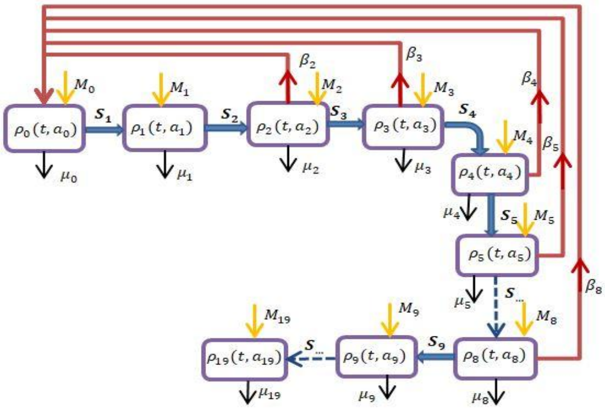

Fig. 2: Model Flow Chart Showing Interaction of Population Clusters. the Population Densities Denoted by $\rho_0(t,a_0), \rho_1(t,a_1), \ldots, \rho_{19}(t,a_{19})$, Death Rates Is Given By $\mu_0, \mu_1, \ldots, \mu_{19}$, Migration Rates Into the Cluster Is $M_0, M_1, M_2, \ldots, M_{19}$Transition Rates $S_1, S_2, \ldots, S_{19}$, Birth Rates $\beta_2, \beta_3, \ldots, \beta_8$.

From the flow chart, we formulate a one sex model using a system of partial differential equations as follows. The equation for the first cluster $k = 0$ is given by;

$$\frac{\partial \rho_0(t,a)}{\partial t} + \frac{\partial \rho_0(t,a)}{\partial a} = -\mu(a)\rho_0(t,a) + m_0(a) - \int_0^{a_0} s_0 \rho_0(t,a)da \tag{12a}$$

This equation (12a) shows the population density change with respect to time of individuals of cluster [0 – 4]. The corresponding boundary condition is

$$\rho_0(t,0) = \sum_{k=2}^{7} \int_{a_k}^{a_{k+1}} \beta_k(a)\rho_k(t,a)da \tag{12b}$$

Which accounts for the total births from six actively reproductive clusters, and the initial condition

$$\rho_0(0,a_0) = K_0(a_0), \tag{12c}$$

Describing the initial population of cluster [0 – 4] at the start of simulation time $t = 0$.
Similar equations for clusters $k = 1,2,3,\ldots,19$ are given in compact form as;

$$\frac{\partial \rho_k(t,a)}{\partial t} + \frac{\partial \rho_k(t,a)}{\partial a} = -\mu_k(a)\rho_k(t,a) + m_k(a) + S_k \tag{13a}$$

Where $S_k = \int_{a_{k-1}}^{a_k} s_{k-1}\rho_{k-1}(t,a)da - \int_{a_k}^{a_{k+1}} s_k \rho_k(t,a)da$ is the net transition. The initial condition of cluster k overlap with the boundary Condition of the next cluster $k+1$, and thus we have the following 19 conditions,

$$\rho_k(0,a_k) = K_k(a_k),\ k = 1,2,\ldots,19 \text{ and } \rho(t,\infty) = 0 \tag{13b}$$

The numerical solution of the system of equations (12) – (13) is presented in the next section.

3. Numerical solutions

Non-linear phenomenon appears in many scientific areas such as human population, physics, ecosystems, among other fields and can be modelled by systems of non-linear partial differential equations. The process of finding exact solutions of such systems is hectic, if not unfeasible. The approximate numerical solutions offers the alternative way forward, which is adopted in this paper. An unconditionally stable [17] Crank-Nicholson finite difference scheme is hereby adopted.

Finite difference scheme is a class of numerical techniques for solving differential equations by approximating derivatives using finite differences [18]. Both the spatial and temporal domains are discretized into a finite number of steps, and the value of the solution at these discrete points is approximated by solving algebraic equations containing finite differences and values from nearby points. The use of the Finite Difference numerical method, results in the generation of a set of algebraic equations that can be solved for dependent variables. The set of algebraic equations are solved at the discrete grid points in the physical domain under consideration. The difference relates the values of variables at each grid points to its neighbouring points.

Taylors Series expansion of partial derivatives in equation (12) – (13) gives rise to the following discretization from the definition.

$$\frac{\rho_{a_k}^{n+1} - \rho_{a_k}^{n}}{\Delta t} + \frac{(\rho_{a_{k+1}}^{n+1} - \rho_{a_{k-1}}^{n+1}) + (\rho_{a_{k+1}}^{n} - \rho_{a_{k-1}}^{n})}{2\Delta a} = -\mu(k)\rho_{a_k} + m_k - S_k\rho_{a_k} \tag{14}$$

Let $\gamma = \frac{\Delta t}{2\Delta a}$ and let $\eta = \Delta t$, then equation (14) reduces to;

$$\rho_{a_k}^{n+1} - \rho_{a_k}^{n} + \gamma(\rho_{a_{k+1}}^{n+1} - \rho_{a_{k-1}}^{n+1}) + \gamma(\rho_{a_{k+1}}^{n} - \rho_{a_{k-1}}^{n}) = -\eta\mu(n,k)\rho_{a_k} + \eta m_k - \eta S_k\rho_{a_k}$$

Rearranging the elements to have the future time elements on the left and current time on the right yields,

$$\rho_{a_k}^{n+1} + \gamma\left(\rho_{a_{k+1}}^{n+1} - \rho_{a_{k-1}}^{n+1}\right) = \rho_{a_k}^{n} - \gamma\left(\rho_{a_{k+1}}^{n} - \rho_{a_{k-1}}^{n}\right) - \eta\mu(n,k)\rho_{a_k} + \eta m_k(k) - \eta s(n,k)\rho_{a_k} + \eta s(n,k-1)\rho_{a_{k-1}} \tag{15}$$

Equation (15) forms a set of 22 equations with 20 unknowns representing the population dynamics for cluster $k = 0,1,2,\dots,19$. The extra two equations dexcribe the initial and boundary conditions and can be obtained from the information given in (12b), (12c) and (13b).
In matrix form, equation (15) can be expressed in compact form as;

$$\begin{pmatrix} -\gamma & 1 & \gamma & 0 & 0 & \dots & 0 \\ 0 & -\gamma & 1 & \gamma & 0 & \dots & 0 \\ 0 & 0 & -\gamma & 1 & \gamma & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \gamma \end{pmatrix} \begin{pmatrix} \rho_{a_{-1}}^{n+1} \\ \rho_{a_0}^{n+1} \\ \rho_{a_1}^{n+1} \\ \vdots \\ \rho_{20}^{n+1} \end{pmatrix} = \begin{pmatrix} \gamma & a_{11} & -\gamma & 0 & 0 & \dots & 0 \\ 0 & b_{21} & a_{22} & -\gamma & 0 & \dots & 0 \\ 0 & 0 & b_{32} & a_{33} & -\gamma & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & -\gamma \end{pmatrix} \begin{pmatrix} \rho_{a_{-1}}^{n} \\ \rho_{a_0}^{n} \\ \rho_{a_1}^{n} \\ \rho_{a_2}^{n} \\ \vdots \\ \rho_{a_{20}}^{n} \end{pmatrix} + \eta \begin{pmatrix} m_0 \\ m_1 \\ m_2 \\ \vdots \\ m_{19} \end{pmatrix}$$

Where $a_{11} = 1 - \eta(\mu_0^n + S_0^n)$, $a_{22} = 1 - \eta(\mu_1^n + S_1^n)$, $a_{33} = 1 - \eta(\mu_2^n + S_2^n)$, $\dots, a_{k,k} = 1 - \eta(\mu_{k-1}^n + S_{k-1}^n)$ and $b_{21} = \gamma + \eta S_0^n$, $b_{32} = \gamma + \eta S_1^n$, $b_{43} = \gamma + \eta S_2^n$, $\dots, b_{k,k-1} = \gamma + \eta S_{k-2}^n$ where $k = 1,2,3,\dots,20$.

The above system of matrices is in the form

$$A\rho_{a_k}^{n+1} = B\rho_{a_k}^{n} + \eta M_k \tag{16}$$

Note that matrix A and matrix B are not square matrices. The extra two unknown values $\rho_{a_{-1}}^{n}$and $\rho_{a_{20}}^{n}$ on matrix A on the left, together with $\rho_{a_{-1}}^{n+1}$ and $\rho_{a_{20}}^{n+1}$ on matrix B on the right-hand side are obtained from the boundary and initial conditions. The system (16) can be split into square matrix coefficients added to a vector of column matrix to obtain,

$$\hat{A}\rho_{a_k}^{n+1} + L_{a_k} = \hat{B}\rho_{a_k}^{n} + M_k + R_{a_k} \tag{17}$$

Which is represented in matrix form by,

$$\begin{pmatrix} 1 & \gamma & 0 & 0 & \dots & 0 \\ -\gamma & 1 & \gamma & 0 & \dots & 0 \\ 0 & -\gamma & 1 & \gamma & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix} \begin{pmatrix} \rho_{a_0}^{n+1} \\ \rho_{a_1}^{n+1} \\ \rho_{a_2}^{n+1} \\ \vdots \\ \rho_{19}^{n+1} \end{pmatrix} + \begin{pmatrix} -\gamma\rho_{a_{-1}}^{n+1} \\ 0 \\ 0 \\ \vdots \\ \gamma\rho_{20}^{n+1} \end{pmatrix} = \begin{pmatrix} a_{11} & -\gamma & 0 & 0 & \dots & 0 \\ b_{21} & a_{22} & -\gamma & 0 & \dots & 0 \\ 0 & b_{32} & a_{33} & -\gamma & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & a_{20,20} \end{pmatrix} \begin{pmatrix} \rho_{a_0}^{n} \\ \rho_{a_1}^{n} \\ \rho_{a_2}^{n} \\ \vdots \\ \rho_{a_{19}}^{n} \end{pmatrix} + \begin{pmatrix} \gamma\rho_{a_{-1}}^{n} \\ 0 \\ 0 \\ \vdots \\ -\gamma\rho_{a_{20}}^{n} \end{pmatrix} + \eta \begin{pmatrix} m_0 \\ m_1 \\ m_2 \\ \vdots \\ m_{19} \end{pmatrix}$$

In order to determine the future population, $\rho_{0_i}^{n+1}$ of cluster k and age a, equation (317) is rearranged by multiplying both sides of the system by the inverse of matrix $\hat{A}$, that is $\hat{A}^{-1}$, to obtain;

$$\rho_{a_k}^{n+1} = \hat{A}^{-1}\hat{B}\rho_{a_k}^{n} + \hat{A}^{-1}M_k + \hat{A}^{-1}R_{a_k} - \hat{A}^{-1}L_{a_k} \tag{18}$$

Note that from the boundary and initial conditions $\rho_{a_{20}}^{n} = 0 = \rho_{a_{20}}^{n+1} = \rho_{a_{-1}}^{n+1} = \rho(t,\infty) = 0$ and $\rho_{a_{-1}}^{n} = \int_{a_2}^{a_8} \beta(a,t)\rho(t,a)da$. Using these initial and boundary conditions, equation (18) reduces to,

$$\rho_{a_k}^{n+1} = F\rho_{a_k}^{n} + G_k \tag{19}$$

Where $G = \hat{A}^{-1}(M + R + L)$. The above equation (19)can be solved with appropriate data of any desired place.

4. Model simulation

In this section, the numerical model simulation is run and solutions presented by graphical means. The population data used in this model are the Kenyan population census results of 2019. The base year of 2019 is chosen as the initial time $t = 0$, and simulation is run for 31 years to predict the position in 2050. Validation of the model is done by comparing the simulation results and the available data of the year 2023.

4.1. The kenya population data of 2019

The following Table shows the data collected and used in the numerical analysis of this research thesis. In this model, population is structured into 20 clusters, of age bracket $[0-4, 5-9, 10-14, \dots, 95-99]$ each of 5 years interval. The initial population of each cluster is denoted by $K_i, i = 0,1,2,\dots,19$ and are given by; K_0 =5,939,306; K_1 =5,597,716; K_2 =5,034,855; K_3 =4,169,543; K_4 =3,775,103; K_5 =3201226; K_6 =2519506; K_7 =2008632; K_8 =1476169; K_9 =1272745; K_{10} =956206; K_{11} =711953; K_{12} =593778; K_{13} =390763; K_{14} =339301; K_{15} =218508; K_{16} =157900; K_{17} =95267; K_{18} =75834 and K_{19} =54700.

Other parameters used in the model are presented in Table 4.1.

Table 4: 1 Model Parameters for 2019 Kenyan Population Data

Item	Symbol	Description	Value
1	β	Birth rate (assumed constant for all)	0.02639
2	μ_0	Infant mortality rate	0.02786
3	m_i	Net migration rate at the i^{th} cluster	-0.00019
4	S	Transition rate across clusters	0.65
5	$\boldsymbol{\mu_i}$	General mortality rate of the $\boldsymbol{i^{th}}$ cluster	0.0313

Model equation (19) is numerically simulated as discussed below with the assumption that the ratio of female and male is 1:1, and thus the female population structure only will be subsequently discussed.

4.2. Model simulation results

The discretized model represented by equation (19) is simulated using data in Table 1. The initial population structure of Kenya, as per 2019 population census is depicted in Figure 3. Notice that the Kenyan population is pyramid like, a characteristic of high birth rate, poor transition rate due to poor health, and high death rate.

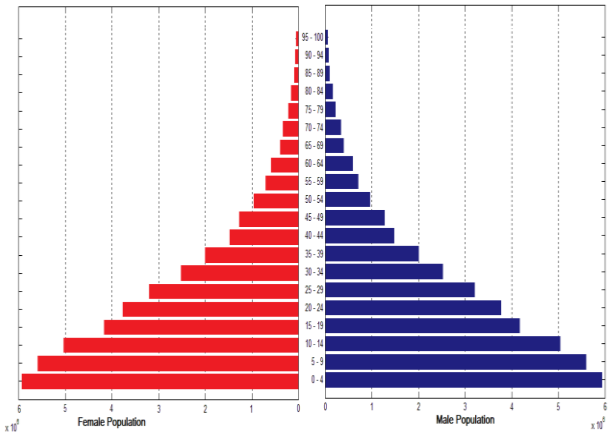

Fig. 3: The Kenya Population Pyramid for 2019 Population Census.

In Figure 3, the population of age $[0-4]$ is the largest for both male and female, and narrows down as age advances. The population of the youngest class is K_0 =5,939,306 while that of the oldest cluster $[95-99]$ is a less than 1% of the young, standing at K_{19} =54,700. Note that the dependent population of age bracket $[0-24]$ are school and college going, together with the age bracket $[65-99]$ the group that retired from service, add up to 25,848,796 while the working population in the age bracket $mid = [25-64]$ are 12,740,215. If we assume that the middle age are working and supporting all the others, then the dependency ratio of $1:2$ is obtained, that is, every working person supports two unemployed people. Simulated results after 31 years is presented in Figure 4.

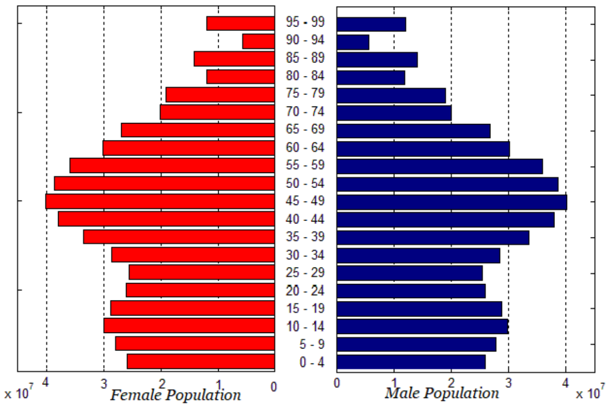

Fig. Error! No text of specified style in document. Kenyan 2050 Population Pyramid Simulated for A Period of 31years from 2019.

Simulated results of 2050 clearly indicates the change in population structure. It is noted that the population is tending from expansive type of pyramid towards constrictive type, indicating positive increase literacy levels, access to quality health, low mortaity, low fertility, and improved living standards. In this model, population has increased from 38,589,011 to 57,956,100, a 50% increase. Using the same crite-rion, the dependency ratio will have dropped to 1:1.14, an admirable ratio.

5. Conclusion

For population dynamics of humans, we have in this study made an effort to develop an age-structured model using a system of 20 partial differential equations. It is noted that the model is able to make predictions which can be used to make vital decisions. However, the model used various assumptions, in which actual data need to be incorporated for exact results. These include the use of constant fertility rate across the clusters, constant net migration, and absence of the effect of social factors and limitation of resources.

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