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Research Paper

Some results of a class of univalent functions with negative coiffitions

Mustafa Abbas Fadhel [1]*, Rafid Habib Buti [2]

[1] *Department of General Sciences,College of Basic Education,University of Al-Muthanna, Iraq*
[2] *Department of Mathematics and Computer Applications, College of Sciences,University of Al-Muthanna, Iraq*
**Corresponding author E-mail: mustaffa8@hotmail.com*

Abstract

In this paper, we study a new subclass of univalent analytic functions with positive coefficients in the unit disk; we obtain main result, distortion theorem and some properties of this subclass.

Keywords: *Univalent Functions, Distortion Theorem, Linear Combination.*

1. Introduction

Let R denote the class of functions of the form:

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n \tag{1}$$

Which are analytic and univalent in the unit disk $U=\{z\in C:|z|<1\}$?

Let R^* be a subclass of a class H consisting of functions of the form:

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n\ , \qquad a_n\ge 0\ . \tag{2}$$

A function $f\in R^*$ is said to be starlike function of order γ if and only if

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\}>\gamma\ ,\ \ (0\le\gamma<1; z\in U)\ . \tag{3}$$

Definition 1: A function $f\in H^*$ is said to be in the class $RM(B,\tau,\theta)$ if f satisfies the condition:

$$\left|\frac{\left[\left(\frac{zf'(z)}{f(z)}-1\right)(\tau+\theta)+B\left(\frac{zf'(z)}{f(z)}-1\right)\right]}{(1-\theta)+\theta\left(\frac{zf'(z)}{f(z)}-1\right)}\right|<1\ , \tag{4}$$

Where, $0\le B\le 1,\ 0\le\theta<1,\ 0\le\tau\le 1$.

In the following theorem, we obtain a sufficient condition for the function f to be in the class $RM(B,\tau,\theta)$.

Theorem 1: *A function f defined by (2) be in the class $RM(B,\tau,\theta)$ if*

$$\sum_{n=2}^{\infty}\left[(n-2)\theta+1+(n-1)(\tau+\theta+B)\right]a_n\le 1-\varepsilon\ . \tag{5}$$

The result is sharp.

Proof: For $|z|=1$, we have $\left|\left[zf'(z)-f(z)(\tau+\theta)+B\left(zf'(z)-f(z)\right)\right]\right|$

$$-\left|f(z)(1-\theta)+\theta\left(zf'(z)-f(z)\right)\right|$$

$$=\left|\sum_{n=2}^{\infty}(n-1)(\tau+\theta+B)a_n z^n\right|$$

$$-\left|z(1-\theta)+\sum_{n=2}^{\infty}\left[(n-2)\theta+1\right]a_n z^n\right|$$

$$=\sum_{n=2}^{\infty}\left[(n-2)\theta+1+(n-1)(\tau+\theta+B\right]a_n-(1-\theta)\leq 0\,.$$

This by maximum modulus theorem $f\in RM(B,\tau,\theta)$

The result is sharp for the function f given by the form

$$f_n(z)=z+\frac{1-\theta}{\left[(n-2)\theta+1+(n-1)(\tau+\theta+B)\right]}z^n \tag{6}$$

There are many authors who have studied the various interesting properties of the classes, H. Silvrerman [4], H. J. A. Hussein and R. H. Buti[3], K. K. Dixit and Y.K. Mishra[2] , N. E. Cho , S. H. Lee and S. Owa [1].

In the next, we obtain distortion theorem for the class $RM(B,\tau,\theta)$.

Theorem 2: Let $f(z)$ defined by (2) be in the class $RM(B,\tau,\theta)$. Then

$$|f(z)|\leq|z|+\frac{1-\theta}{\left[1+\tau+\theta+B\right]}|z|^2 \tag{7}$$

and

$$|f(z)|\geq|z|-\frac{1-\theta}{\left[1+\tau+\theta+B\right]}|z|^2\,. \tag{8}$$

The inequalities in (7) and (8) are attuned for the function

$$f(z)=z+\frac{1-\theta}{\left[1+\tau+\theta+B\right]}z^2 \tag{9}$$

Proof: By using Theorem 1, we have

$$\sum_{n=2}^{\infty}a_n\leq\frac{1-\theta}{\left[1+\tau+\theta+B\right]} \tag{10}$$

So by using (2) and (10), we have

$$|f(z)|\leq|z|+|z|^2\sum_{n=2}^{\infty}a_n$$

$$\leq|z|+\frac{1-\theta}{\left[1+\tau+\theta+B\right]}|z|^2,$$

Which gives (7), we also have

$$|f(z)|\geq|z|-|z|^2\sum_{n=2}^{\infty}a_n$$

$$\geq|z|-\frac{1-\theta}{\left[1+\tau+\theta+B\right]}|z|^2$$

Which gives (8)?

Now, we shall prove that class $RM(B,\tau,\theta)$ is closed under convex linear combinations.

Let the function $f_k\ \ (k=1,2,...,m)$ be defined by

$$f_k(z)=z+\sum_{n=1}^{\infty}a_{n,k}z^n\quad,\quad(a_{n,k}\geq 0, n\geq 2)\ . \tag{11}$$

Theorem 3: *Let the function* $f_k(z)$ *defined by (11) be in the class* $RM(B,\tau,\theta)$, $(0\leq\theta<1)$*.Then the following function* g *defined by*

$$g(z)=z+\frac{1}{m}\sum_{n=2}^{\infty}\left[\sum_{k=2}^{m}a_{n,k}\right]z^n\ ,\ (k=1,2,...,m) \tag{12}$$

Is in the class $RM(B,\tau,\theta)$, where $\theta=\min_{2\leq k\leq m}\{\theta_k\}$. (13)

Proof: Since $f_k\in RM(B,\tau,\theta)$ for each $(k=1,2,...,m)$, we note that

$$\sum_{n=2}^{\infty}\left[(n-2)\theta_k+1+(n-1)(\tau+\theta_k+B)\right]a_{n,k}\leq 1-\varepsilon_k\ .$$

Therefore

$$\sum_{n=2}^{\infty}\left[(n-2)\theta_k+1+(n-1)(\tau+\theta_k+B)\right]\left[\frac{1}{m}\sum_{k=2}^{m}a_{n,k}\right]$$
$$=\frac{1}{m}\sum_{k=2}^{m}\left[\sum_{n=2}^{\infty}\left[(n-2)\theta_k+1+(n-1)(\tau+\theta_k+B)\right]a_{n,k}\right]$$
$$\le\frac{1}{m}\sum_{k=2}^{m}(1-\theta_k)\le(1-\theta)\ .$$

Thus, we get

$$\sum_{n=2}^{\infty}\left[(n-2)\theta_k+1+(n-1)(\tau+\theta_k+B)\right]\left[\frac{1}{m}\sum_{k=2}^{m}a_{n,k}\right]\le 1-\theta\ .$$

Hence, by Theorem 1, we have $g\in RM(B,\tau,\theta)$.

In the next we show that the integral operator in the class $RM(B,\tau,\theta)$.

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