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Research Paper

Cobweb heuristic for multi-objective vehicle routing problem

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Abstract

Solving a classical vehicle routing problem (VRP) by exact methods presents many difficulties for large dimension problem. Consequently, in multi-objective framework, heuristic or metaheuristic methods are required. Due to particular VRP structure, it seems that a dedicated heuristicis more suitable than a metaheuristic. The aim of this article is to collapse different heuristics solving classical VRP and adapt them for to solve the multi-objective vehicle routing problem (MOVRP). The so-called Cobweb Algorithm simulates spider's behavior when weaving cobweb. This paper presents the algorithm, a didactic example, concluding remarks and way for further researches.

Keywords: *Savings; Heuristics; Multiobjective; Vehicle Routing Problem.*

1. Introduction

The vehicle routing problem (VRP) [1] is one of the most attractive topics in operation research, which is useful for logistics, and supply-chain management (see [2], [3]). Indeed one of the real-life multi-objective optimization problem applications [4]. VRP deals with minimizing the total cost of logistics systems [5]. VRPs are well-known combinatorial optimization problems arising in transportation logistic that usually involve scheduling in constrained environments (see [6], [8]). In transportation management [9], there is a requirement to provide services from a supply point (depot) (see [10], [11] to various geographically dispersed points (customers) with significant economic implications; many researchers have developed solution approaches for those problems (see [12], [15]).

We devote this paper to the hybridization of some heuristics dedicated to classical VRP problems for solving the multi-objective Vehicle Routing Problem (MOVRP) (see [16], [19]). There are:

i) The economics heuristic of Clarke & Wright [20];
ii) Insertion heuristic (see [21], [22]);
iii) The two phases Heuristic [23];
iv) The heuristics of local research [24].

A much more complete description of this classical VRP heuristics, with a comparative analysis of their performance, can be found in chapter 5 of Toth et al. [22] and in Brasseur et al. [25]. All these heuristics are hybridized with the preferential reference mark of predominance method (see [26], [27], [28]).For this purpose, our paper is organized as follows: section 2 presents the mathematical formulation, section 3 presents some incidental definitions, next in section 4 we describe the so-called preferential reference mark of dominance method (PRMD), section 5 outlines Cobweb Algorithm. A didactic example is provided to validate our step.

2. Mathematical formulation of multi-objective vehicle routing problem

Let be considered m objectives functions and v the number of delivery vehicles with a maximal capacity Q intended to serve all customers indicated by the set V from the central deposit during a maximal duration time T. The mathematical formulation of this multi-objective problem of vehicles routing is the following:

$$
(P) \equiv \begin{cases}
\text{"min"} \sum_{k=1}^{v} \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}{}^{k} x_{ijk} \quad k = 1, 2, \dots, m & \\
\sum_{k=1}^{v} \sum_{j=0}^{n} x_{ijt} = 1 \ \forall\, i \in V \backslash \{0\} & (1) \\
\sum_{i=0}^{n} x_{ijt} = \sum_{j=0}^{n} x_{jlt} \ \forall\, j \in V \backslash \{0\}, \qquad t = 1, 2, \dots, v & (2) \\
\sum_{j=1}^{n} x_{ojt} = 1, \qquad t = 1, 2, \dots v & (3) \\
\sum_{i=0}^{n} x_{iot} = 1 \quad t = 1, 2, \dots, v & (4) \\
\sum_{i=1}^{n} \sum_{j=0}^{n} q_i x_{ijt} \le Q, \qquad t = 1, 2, \dots, v & (5) \\
\sum_{i=1}^{n} \sum_{j=0}^{n} s_i x_{ijt} + \sum_{i=0}^{n} \sum_{j=0}^{n} t_{ij} x_{ijt} \le T, \quad t = 1, 2, \dots, v & (6) \\
\sum_{i \in U} \sum_{j \in U} x_{ijt} \ge \sum_{j=1}^{n} x_{ijt}, \forall U \subset V \backslash \{0\}, l \in U; \ t = 1, 2, \dots, v & (7) \\
x_{ijt} \in \{0, 1\} \ \forall\, i, j \in V, i \neq j, \qquad t = 1, 2, \dots, v & (8)
\end{cases}
$$

Interpretation of the different constraints of the vehicle routing problem is:
(1) Each customer $i \in V\text{-}\backslash \{0\backslash\}$ is visited one and only one time,
(2) Each vehicle l arriving at the customer j leaves from there.
(3) and (4): each vehicle l leaving the depot comes back to it,
(5) Respect of the maximal capacity Q of vehicles,
(6) Respect of the maximal duration time T of routing,
(7) Elimination of the under-tours to guarantee the connection of the different vehicle routing,
(8) Precise that it is a combinatorial optimization.

3. Definitions

1) Reference mark of dominance of arailable solution a is often referred to an orthonormal reference mark of origin a, dividing the space in four areas of preference in accordance to the diagram of figure 1 below.

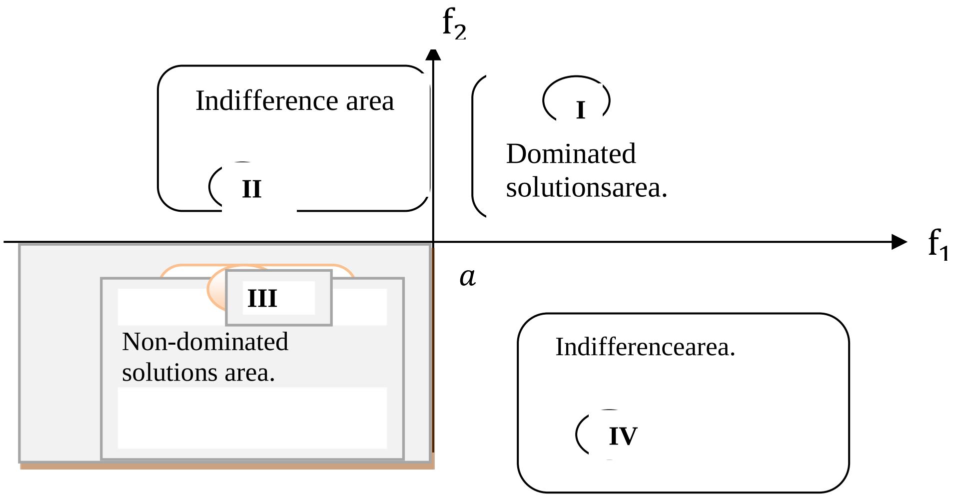

Fig. 1: Preferences Zones in the Dominance Relation

2) Let us now consider the objectives' space O of a multi-objective combinatorial optimization problem, $z_1, z_2 \in O$ and $V(z_1)$ a neighborhood of z_1. It is said that the solution $z_2 \in V(z_1)$ certainly improves z_1 if z_2 is situated in the non-dominated solutions area of the preferential reference mark of z_1. In this case, the acceptance probability of z_2 equals 1. It improves z_1 with an acceptance probability ρ, $0 < \rho < 1$ when it is situated in an indifference area of

the $z_1$preferential reference mark, and with a nil acceptance probability in the dominated solutions area. In other words, if $\rho \equiv \mathbb{P}(\text{acceptance of neighborhoods } z_1 \text{ of } z_2)$ then :

$$\begin{cases} \rho = 1 \text{ if } z_2 \in \text{III} \\ 0 < \rho < 1 \text{ if} z_2 \in \text{II} \cup \text{IV} \\ \rho = 0 \text{ if } z_2 \in \text{I} \end{cases}$$

4. Description stages of preferential reference mark of dominance method

```
Input:
D: Set of admissible solutions
O = F (D) = (f_i(a))_{i=1,...,m}, a ∈ D.
Output:E(P) : Set of efficient solutions.
Start
E (P)← ∅
RepresentgraphicallyO
Do while O≠ ∅do
    Choose z in O
    Draw the preferential reference mark of dominance of z
    Forz′ in O\ {z} do
        If z′ is situated in thenon-dominated solutions area then
            E(P)  ←E (P)∪ {z′}
            O ←O \ {z′}
        End if
        If z′is situated in the dominated solutions areathen
            O ←O \ {z′}
        End if
        If the non-dominated solutions areais emptythen
            E(P)  ←E (P)∪ {z′}
            O ←O \ {z′}
        End if
    Next
    If z′ is situated inindifference areathen
        z ← z′
        E(P)  ←E (P)∪ {z′}
        O ←O \ {z′}
    End if
Loop
Choose z in E (P)
Draw the preferential reference mark of dominance of z
    For z′ in E (P) \ {z} do
        If  z′ is situated in thenon-dominated solutions area then
            E (P)  ←E (P\ {z}
        End if
        If z′ is situated in the dominated solutions areathen
            E(P)  ←E (P\ {z′}
        End if
    Next
Display E(P)
End
```

5. Description stages of cobweb algorithm

Following functions are used in the algorithm:

- RePref(A,B)return the efficient solutions set of the saving distance matrix Aand the saving priority matrix B obtain by preferential reference mark of dominance method
- Card(A) return the numberof element of A
- Insert (x,y)return road z in which y is inserted in road x based on the insertion heuristic
- Capacity(r) return the sum of customers’ request
- RechLoc(P) return a fleet P ameliorated by local research heuristic
- Len(r) return road length t

- Priority(r) return the sum of road t visited-customers' priorities

```
Input:
        A the set of n customers
        (d_ij) the matrix of distances between customers (i=0, …, n; j=o, …, n)
        (p_0j) the matrix of customers' priorities (i=1, …, n)
        C the vehicles' capacity
        (d_i) the requests' vector of customers, (i=1, …, n)
Output:
        P : Set of compromises' road
        Triplets (length, prior, size) corresponding to fleets' length, the sum of customers' priorities and its' size
Start
For i =1 to n do
    For j=1 to n do
        δ_ij ← d_i0+d_0j − d_ij
        p_ij ← p_0I+p_0j
    Next j
Next i
E ← RePref ((δ_ij), (p_ij))
B ← {x/x is a customer visited by a road t ∈ E}
Part ← {P/P is a partiion of B}
A ← A \B
Choose P ϵ Part
taille← Card (P)
Do while A≠ ∅ do
    Choose a in A
    testinsert ← false
    Foreach r in P do
        If Capacity (Insert (r,a))<= C
            r ← Insert (r, a)
            testinsert ← true
        End if
    Next
    If testinsert = false then
        r′ ← Insert (depart, a)
        P ← P ∪ {r′}
        Size ← Size + 1
    End if
    A ← A \ {a}
Loop
P ← RechLoc (P)
Length← 0
Prior ← 0
Foreach r in P do
    Length← Length + Len(r)
    Prior← Prior + Priority(r)
Next r
Display (Length, Prior, Size)
End
```

6. Didactic example

6.1. Facts of the case

A pharmaceutical industry having a warehouse (0) launches a new product on the market; it lays out of an offer of delivery vehicles of eight tons maximum capacity. The requests d_i (i =1,2…,15) of the customers arise in the following table, the distances being symmetrical and checking the triangular inequality. The customers' priorities are quantified from 1 to 15 and are allotted according to descending order of the requestsarrivals.

Table 1: Stamp Distances (Km) And Demands (Ton) : C^1_{ij}

N°	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	-	15	28	30	22	27	21	22	20	36	63	120	22	63	12	27
1		-	21	32	32	41	35	32	22	52	25	37	18	25	22	22
2			-	18	30	46	47	50	42	45	54	40	54	54	18	20
3				-	18	36	43	52	50	24	42	56	49	42	54	36
4					-	18	27	40	43	21	12	43	38	12	49	63
5						-	16	33	42	15	51	72	55	51	38	37
6							-	18	30	32	23	65	52	23	58	40
7								-	15	35	53	37	39	53	55	56
8									-	28	52	38	64	52	52	43
9										-	43	25	42	43	39	72
10											-	40	53	35	64	65
11												-	62	26	42	37
12													-	33	53	38
13														-	62	25
14															-	40
d_i	-	3	3	4	2	4	2	3	4	5	3	4	2	5	4	3

Table 2: Customer priorities: C^2_{ij}

Customer	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Priority	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

6.2. Concerns of the decision maker

Organize roads of distribution which

- Minimize the distances covered;
- Minimize height of the fleet;
- Maximize the customers priorities.

So this is a multiple objective vehicle routing problem with three criteria. Solving this problem consist to find all non-dominated solutions.

6.3. Solving problem

To find the set of efficient solutions we proceed sequentially. We consider initially the distance and priority to illuminate some solutions with superfluous components to remain only solutions with components significant. To solve this problem we use Cobweb algorithm (see §4). Following table 3 summarize values $\delta^k_{ij} = C^k_{io} + C^k_{oj} - C^k_{ij}$ in distance and priority.

Table 3: Saving in Distance and Priorities : $(\delta^1_{ij}, \delta^2_{ij})$

	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	(22,29)	(13,28)	(5,27)	(1,26)	(1,25)	(5,24)	(13,23)	(-1,22)	(53,21)	(98,20)	(19,19)	(53,18)	(5,17)	(20,16)
2	-	(40,27)	(20,26)	(9,25)	(2,24)	(0,23)	(6,22)	(14,21)	(47,20)	(108,19)	(-4,18)	(47,17)	(22,16)	(35,15)
3	-	-	(34,25)	(21,24)	(8,23)	(0,22)	(0,21)	(42,20)	(51,19)	(94,18)	(3 ,17)	(51,16)	(-12,15)	(21,14)
4	-	-	-	(31,23)	(16,22)	(4,21)	(-1,20)	(37,19)	(73,18)	(99,17)	(6,16)	(73,15)	(-15,14)	(-14,13)
5	-	-	-	-	(32,21)	(16,20)	(5,19)	(48,18)	(39,17)	(75,16)	(-6,15)	(39,14)	(1,13)	(17,12)
6	-	-	-	-	-	(25,19)	(11,18)	(25,17)	(61,16)	(76,15)	(-9,14)	(61,13)	(25,12)	(8,11)
7	-	-	-	-	-	-	(27,17)	(23,16)	(32,15)	(105,14)	(5,13)	(32,12)	(-21,11)	(-7,10)
8	-	-	-	-	-	-	-	(28,15)	(31,14)	(102,13)	(-22,12)	(31,11)	(-20,10)	(4,9)
9	-	-	-	-	-	-	-	-	(56,13)	(131,12)	(16,11)	(56,10)	(-1,9)	(-9,8)
10	-	-	-	-	-	-	-	-	-	(143,11)	(32,10)	(91,9)	(11,8)	(25,7)
11	-	-	-	-	-	-	-	-	-	-	(80,9)	(157,8)	(90,7)	(110,6)
12	-	-	-	-	-	-	-	-	-	-	-	(52,7)	(-19,6)	(11,5)
13	-	-	-	-	-	-	-	-	-	-	-	-	(13,5)	(65,4)
14	-	-	-	-	-	-	-	-	-	-	-	-	-	(-1,3)

For example couple (22, 29) intersection of line 1 and column 2 is obtained by:

$$\delta^1_{12} = C^1_{10} + C^1_{02} - C^1_{12} = 15 + 28 - 21 = 22$$

$$\delta^2_{12} = C^2_{10} + C^2_{02} - C^2_{12} = 15 + 14 - 0 = 29$$

6.3.1. Sequentially efficient solutions

The set of sequentially efficient solutions is in conformity with the table 3: $E(P) = \{(22,29),(40,27),(53,21),(98,20),(108,19),(143,11)\}$.Corresponding roads are respectively (0-1-2-0), (0-2-3-0), (0-1-10-0), (0-1-11-0), (0-2-11-0) and (0-10-11-0) of respective capacities $6,7,6,7,7,7$, these roads are incompatible because only customer 3 is visited once.

6.3.2. Roads building under capacity constraint

The set of customers corresponding to $E_1(P)$ is B= $\{1,2,3,10,11\}$. A partition of these customers is formed of the sets $\{1,2\},\{3\},\{10,11\}$ corresponding to roads (0-1-2-0), (0-3-0), (0-10-11-0). Taking a non-affected customer randomly, 5 for example, single possible insertion is (0-3-5-0) of a total request for 8 tons. Taking another customer randomly, for example 12, a good possible insertion is (0-12-1-2-0) of a 8-tons capacity. Following the same step, one finally obtains (0-6-4-14-0), (0-7-8-0), (0-13-15-0) and (0-9-0) corresponding respectively to the requests of 8, 7, 8 and 5 tons each one. An improvement of this solution is the permutation and reintegration of the customers 3 and 14. The final result is (0-12-1-2-0), (0-5-14), (0-10-11-0), (0-3-4-6-0), (0-7-8-0), (0-13-15-0) and (0-9-0).

Table 4: Recapitulation of Rounds with Capacity, Length, Priority and Size of the Fleet

N°	Roads	Length in Km	Capacity in Ton	Priority	fleet height
1	(0-12-1-2-0)	89	8	41	1
2	(0-5-14-0)	77	8	24	1
3	(0-10-11-0)	223	7	11	1
4	(0-3-4-6-0)	96	8	16	1
5	(0-7-8-0)	57	7	17	1
6	(0-13-15-0)	115	8	4	1
7	(0-9-0)	72	5	7	1
8	TOTAL	729	51	120	7

6.3.3. Obtaining efficient solutions set

With a similar reasoning applied on the partition $\{1,2\},\{3,10\},\{11\}$, we have two solutions which are: (862, 120,7) and (887,120,7). With the partition $\{1\},\{2,3\},\{10,11\}$ we have the solution: (782,120,7); with $\{1,3\},\{2,10\},\{11\}$ we obtain two additional solutions: (800,120,7) and (792,120,7); with $\{1,3\},\{2,11\},\{10\}$, the solution found is (759,120,7); for $\{1,10\},\{2,11\},\{3\}$, one has (723,120,7) and finally, for $\{2,10\},\{1,11\},\{3\}$ we have (750,120,7).
The decision maker must choose between the following nine solutions:
(729,120,7), (862,120,7), (887,120,7), (782,120,7), (800,120,7), (792,120,7), (759,120,7), (723,120,7) and (750,120,7).

7. Conclusion

The originality of this method lies in the fact that it always keeps on the multi-objective aspect of the studied problem and that it has never dealt with any classical optimization problem. Yet, the majority of results found in the literature incorporate various objectives in a single objective thanks to an aggregation function and to the weights provided by the decision maker; nevertheless the subjectivity of situations weight interpretation still problematic in any. A trail of research opened here is the implementation of this algorithm in a suitable computer programming language.

References

[1] Wei Zhou, Tingxin Song, Fei He, and Xi Liu. Multiobjective Vehicle Routing Problem with Route Balance Based on Genetic Algorithm.*Discrete Dynamics in Nature and Society*, volume 2013, 2013. http://dx.doi.org/10.1155/2013/325686.

[2] M. M. Solomon, "Algorithms for the vehicle routing and scheduling problems with time window constraints," Operations Research, vol. 35, no. 2, pp. 254–265, 1987. View at Publisher • View at Google Scholar • View at Zentralblatt MATH • View at MathSciNet http://dx.doi.org/10.1287/opre.35.2.254.

[3] R. Thangiah, K. E. Nygard, and P. L. Juell, "GIDEON: a genetic algorithm system for vehicle routing with time windows," in Proceedings of the 7th IEEE Conference on Artificial Intelligence Applications, pp. 322–328, Miami Beach, Fla, USA, February 1991. View at Scopus http://dx.doi.org/10.1109/caia.1991.120888.

[4] B. Ombuki, B. J. Ross, and F. Hanshar, "Multi-objective genetic algorithm for vehicle routing problem with time windows," Applied Intelligence, vol. 24, pp. 17–33, 2006. http://dx.doi.org/10.1007/s10489-006-6926-z.

[5] M. A. Figliozzi, "An iterative route construction and improvement algorithm for the vehicle routing problem with soft time windows," Transportation Research C, vol. 18, no. 5, pp. 668–679, 2010. View at Publisher • View at Google Scholar • View at Scopus http://dx.doi.org/10.1016/j.trc.2009.08.005.

[6] K. Pang, "An adaptive parallel route construction heuristic for the vehicle routing problem with time windows constraints," Expert Systems with Applications, vol. 38, no. 9, pp. 11939–11946, 2011. View at Publisher • View at Google Scholar • View at Scopus http://dx.doi.org/10.1016/j.eswa.2011.03.088.

[7] K. Ghoseiri and S. F. Ghannadpour, "Multi-objective vehicle routing problem with time windows using goal programming and genetic algorithm," Applied Soft Computing Journal, vol. 10, no. 4, pp. 1096–1107, 2010. View at Publisher • View at Google Scholar • View at Scopus http://dx.doi.org/10.1016/j.asoc.2010.04.001.

[8] K. C. Tan, L. H. Lee, Q. L. Zhu, and K. Ou, "Heuristic methods for vehicle routing problem with time windows," Artificial Intelligence in Engineering, vol. 15, no. 3, pp. 281–295, 2001. View at Publisher • View at Google Scholar • View at Scopus http://dx.doi.org/10.1016/S0954-1810(01)00005-X.

[9] W. Chiang and R. A. Russell, "Simulated annealing metaheuristics for the vehicle routing problem with time windows," Annals of Operations Research, vol. 63, pp. 3–27, 1996. View at Scopus. http://dx.doi.org/10.1007/BF02601637.

[10] E. D. Taillard, P. Badeau, M. Gendreau, F. Guertin, and J. Potvin, "A tabu search heuristic for the vehicle routing problem with soft time windows," Transportation Science, vol. 31, no. 2, pp. 170–186, 1997. View at Scopus. http://dx.doi.org/10.1287/trsc.31.2.170.

[11] S. Thangiah, "Vehicle routing with time windows using genetic algorithms," in Applications Handbook of Genetic Algorithms: New Frontiers, Volume II, pp. 253–277, CRC Press, Boca Raton, Fla, USA, 1995.

[12] N. Jozefowiez, F. Semet, and E. Talbi, "Target aiming Pareto search and its application to the vehicle routing problem with route balancing," Journal of Heuristics, vol. 13, no. 5, pp. 455–469, 2007. View at Publisher • View at Google Scholar • View at Scopus. http://dx.doi.org/10.1007/s10732-007-9022-6.

[13] G. B. Alvarenga, G. R. Mateus, and G. de Tomi, "A genetic and set partitioning two-phase approach for the vehicle routing problem with time windows," Computers and Operations Research, vol. 34, no. 6, pp. 1561–1584, 2007. View at Publisher • View at Google Scholar • View at Scopus. http://dx.doi.org/10.1016/j.cor.2005.07.025.

[14] N. Jozefowiez, F. Semet, and E. Talbi, "Multi-objective vehicle routing problems," European Journal of Operational Research, vol. 189, no. 2, pp. 293–309, 2008. View at Publisher • View at Google Scholar • View at Zentralblatt MATH • View at MathSciNet. http://dx.doi.org/10.1016/j.ejor.2007.05.055.

[15] D. E. Goldberg, Genetic Algorithms Insearch, Optimization and Machine Learning, New York, NY, USA, Addison-Wesley edition, 1989.

[16] K. C. Tan, Y. H. Chew, and L. H. Lee, "A hybrid multiobjective evolutionary algorithm for solving vehicle routing problem with time windows," Computational Optimization and Applications, vol. 34, no. 1, pp. 115–151, 2006. View at Publisher • View at Google Scholar • View at Zentralblatt MATH • View at MathSciNet. http://dx.doi.org/10.1007/s10589-005-3070-3.

[17] S. R. Thangiah, "A hybrid genetic algorithms, simulated annealing and tabu search heuristic for vehicle routing problems with time windows," in Practical Handbook of Genetic Algorithms Complex Structures, Volume 3. L. Chambers, pp. 374–381, CRC Press, 1999.

[18] K. C. Tan, L. H. Lee, and K. Ou, "Hybrid genetic algorithms in solving vehicle routing problems with time window constraints," Asia-Pacific Journal of Operational Research, vol. 18, no. 1, pp. 121–130, 2001. View at Zentralblatt MATH • View at MathSciNet • View at Scopus.

[19] R. Ba-os, J. Ortega, C. Gil, A. L. Marquez, and F. de Toro, "A hybrid meta-heuristic for multi-objective vehicle routing problems with time windows," Computers and Industrial Engineering, vol. 65, no. 2, pp. 286–296, 2013. http://dx.doi.org/10.1016/j.cie.2013.01.007.

[20] O.Clarke ET J. Wright, Scheduling of Vehicles from a Central Depot to a Number of Delivery Points, *Operations Research*, 12(4): 568-581, 1964. http://dx.doi.org/10.1287/opre.12.4.568.

[21] R.H.Mole and S.R. Jameson,A sequential route-building algorithm employing a generalized saving criterion. *Operational Research Quaterly*, 27: 503-511, 1976. http://dx.doi.org/10.2307/3008819.

[22] Toth, Paolo ET D. Vigo: *The Vehicle Routing Problem*. Philadelphia: Society for Industrial and Applied Mathematics, 2002. http://dx.doi.org/10.1137/1.9780898718515.

[23] B.E. Gillett and L.R. Miller, *A Heuristic Algorithm for the Vehicle-Dispatch Problem, Operations Research,* vol. 21, pp. 340-349, 1974. http://dx.doi.org/10.1287/opre.22.2.340.

[24] J. Teghem, *Recherche opérationnelle Tome1: Méthodes d'optimisation*, Ellipses 2012.

[25] Basseur M., F. Seynhaeve, and E-G Talbi. *Design of multi-objective evolutionary algorithms: application to the flow shop*. In congress off evolutionary capitation, Honolulu Hawaii, IEEE service center USA, 2002.

[26] Okitonyumbe, Y.F. *Optimisation combinatoire multi-objectif: Méthodes exactes et métaheuristiques*, Master'sThesis, Université Pédagogique Nationale, Kinshasa/RD CONGO, Septembre 2012.

[27] Okitonyumbe, Y.F. and Ulungu, E.L. Nouvelle caractérisation des solutions efficaces des problèmes d'optimisation combinatoire multiobjectif. *Revue Congolaise des Sciences Nucléaires*, 27, Décembre 2013.

[28] B. Ulungu andJ. Teghem, Multi-objective Combinatorial Optimization Problems: A Survey, *Journal of Multi-criteria Decision Analysis*, 3: 83-104, 1994. http://dx.doi.org/10.1002/mcda.4020030204.