International Journal of Applied Mathematical Research, 4 (2) (2015) 325-331
www.sciencepubco.com/index.php/IJAMR
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doi: 10.14419/ijamr.v4i2.4351
Research Paper

On the solution of fuzzy dual linear systems of equation

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Abstract

In this paper the exact, multiple and approximation solutions of Dual fuzzy linear systems of equations(DFLSE) with triangular variable are investigated based on a 1-level expansion. To this end, 1-level of DFLSE are solved for calculating the cores of fuzzy solution and then its spreads are obtained by solving an optimization problem with a special objective function. Finally, the existence of solution of DFLSE is proved in details and some numerical examples are solved to illustrate the accuracy and capability of the method.

Keywords: Fuzzy triangular number; Linear programming; Fuzzy convex combination; Fuzzy dual linear systems.

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1. Introduction

Systems of simulations linear equations play major role in various areas such as mathematics, physics, statistics, engineering and social sciences. Since in many applications at least some of the systems parameters and measurements are represented by fuzzy rather than crisp numbers, it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy dual linear systems and solve them. The $n \times n$ fuzzy dual linear systems of equations has been studied by many authors. In [1] Friedman, Ming and Kandel proposed a general model for solving such fuzzy dual linear systems of equations by using the embedding approach and then gave the conditions for the existence of an unique fuzzy solution to $n \times n$ linear system, though there is a flaw in their main result. Following [1], some other numerical methods for calculating the solution are designed by [2]. You know finding core of this system is easy but for widths are difficult. Furthermore in this paper we consider to approximate of unknown variable widths by defined optimization problem with new target function. The paper is organized as follows. Section 2 presents the primary definitions of fuzzy numbers. In section 3 the main idea which is solving the fuzzy dual linear systems of equations by optimization problem. Section 4 introduces three examples for illustration of method.

2. Preliminary notes

Though there are a number of ways of defining fuzzy numbers, for the purposes of this paper we adopt the following definition[8], we will identify the name of the number with that of its membership function for simplicity. Throughout this paper, R stands for the set of all real numbers, E stands the set of fuzzy numbers.

Definition 2.1 *A fuzzy number is a fuzzy set like $\tilde{u}(x) : [a, d] \to [0, 1]$ which satisfies*

1. *$\tilde{u}$ is upper semi-continuous,*
2. *$\tilde{u}(x) = 0$ outside some interval $[a, d]$,*
3. *There are real numbers b, c such that $a \le b \le c \le d$ and,*
   - *$\tilde{u}(x)$ is monotonic increasing on $[a, b]$,*
   - *$\tilde{u}(x)$ is monotonic decreasing on $[c, d]$,*
   - *$\tilde{u}(x) = 1, \quad b \le x \le c$*

The membership function $\tilde{u}(x)$ can be expressed as,

$$\tilde{u}(x) = \begin{cases} f^L_{\tilde{u}}(x) & a \le x \le b \\ 1 & b \le x \le c \\ f^R_{\tilde{u}}(x) & c \le x \le d \\ 0 & otherwise \end{cases} \tag{1}$$

where $f^L_{\tilde{u}}(x) : [a, b] \to [0, 1]$ and $f^R_{\tilde{u}}(x) : [c, d] \to [0, 1]$ are left and right membership functions of fuzzy number $\tilde{u}$ respectively.

Definition 2.2 *A fuzzy number $\tilde{u}$ is said to be a LR type fuzzy number iff:*

$$\mu_{\tilde{u}}(x) = \begin{cases} L(\frac{a-x}{\alpha}), & if \quad x \in [a-\alpha, a], \\ 1, & if \quad x \in [a, b], \\ R(\frac{x-b}{\beta}), & if \quad x \in [b, b+\beta], \\ 0, & \quad otherwise, \end{cases} \tag{2}$$

L is for left and R for right reference. a, b are the means value of $\tilde{u}$. α and β are called left and right widths, respectively. Symbolically, we write:

$$\tilde{u} = (a, b, \alpha, \beta)_{LR}$$

If $L(x)$ and $R(x)$ be linear functions then the corresponding LR number is said to be a trapezoidal fuzzy number. The trapezoidal fuzzy number $\tilde{u} = (a, b, \alpha, \beta)_{LR}$, with two defuzzifier a, b and left fuzziness α and right fuzziness β is a fuzzy set where the membership function is as

$$\tilde{u}(x) = \begin{cases} 1-(a-x)/\alpha & if \quad b-\alpha \le x \le b, \\ 1 & if \quad b \le x \le c, \\ 1-(x-b)/\beta & if \quad c \le x \le c+\beta, \\ 0 & \quad otherwise, \end{cases} \tag{3}$$

and its parametric form is $\tilde{u} = (a-(1-r)\alpha, b+(1-r)\beta)$. Provided that, $a = b$ then $\tilde{u}$ is a triangular fuzzy number, and we write $\tilde{u} = (a, \alpha, \beta)_{LR}$.

Theorem 2.3 *let $\tilde{a} = (a, \alpha, \beta)_{LR}$, $\tilde{b} = (b, \lambda, \delta)_{LR}$ then*

1. $(a, \alpha, \beta)_{LR} \oplus (b, \lambda, \delta)_{LR} = (a+b, \alpha+\lambda, \beta+\delta)_{LR}$,
2. $\ominus (b, \lambda, \delta)_{LR} = (-b, \delta, \lambda)_{RL}$,
3. $(a, \alpha, \beta)_{LR} \ominus (b, \lambda, \delta)_{RL} = (a-b, \alpha+\delta, \beta+\lambda)_{LR}$.

Proof:[8].

Definition 2.4 *Scalar multiplication of fuzzy number*

$$\lambda \times (a, \alpha, \beta)_{LR} = \begin{cases} (\lambda a, \lambda\alpha\lambda\beta)_{LR} & \lambda > 0, \\ (\lambda a, -\lambda\beta, -\lambda\alpha)_{RL} & \lambda < 0. \end{cases}$$

3. Best approximation

In this section we are going to introduce the main idea. First of all the 1-level system is solved then a minimization problem is proposed to find the width of fuzzy solutions. It is clear that solving these problems often is difficult specially in nonlinear case. So we have to use some norms to linearity and simplification. Consider $A\tilde{x} \oplus \tilde{b} = C\tilde{x} \oplus \tilde{d}$ in LR form,

$$A(x, \alpha_x, \beta_x)_{LR} \oplus (b, \alpha_b, \beta_b)_{LR} = C(x, \alpha_x, \beta_x)_{LR} \oplus (d, \alpha_d, \beta_d)_{LR}. \tag{4}$$

There are some fuzzy dual linear systems of equations that have solutions but, the method in paper [1] doesn't be able to solve them. With definition (2.4) and theorem (2.3) for solving mean value of fuzzy dual linear systems of equations (4)

$$\sum_{j=1}^{n} a_{ij}x_j + b_i = \sum_{j=1}^{n} c_{ij}x_j + d_i \Rightarrow \sum_{j=1}^{n}(a_{ij} - c_{ij})x_j = d_i - b_i,\ i = 1, 2, ..., n. \tag{5}$$

This system be able to has a solution or more. If crisp linear system of equations (5) has unique solution then we will go to next system, as follow,

$$\begin{aligned} Min \quad & \{\|\textstyle\sum_{a_j>0,c_j>0}(a_j - c_j)\alpha_j - \sum_{a_j<0,c_j<0}(a_j - c_j)\beta_j - (\alpha_d - \alpha_b)\|_p \\ & +\|\textstyle\sum_{a_j>0,c_j>0}(a_j - c_j)\beta_j - \sum_{a_j<0,c_j<0}(a_j - c_j)\alpha_j - (\beta_d - \beta_b)\|_p\} \\ s.t \quad & \alpha_j, \beta_j \geq 0 \end{aligned} \tag{6}$$

where $p \in \{1, 2, ...\} \cup \{+\infty\}$

$$\begin{aligned} Min \quad & \{(\textstyle\sum_{j=1}^{n} |\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\alpha_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\beta_j - (\alpha_{di} - \alpha_{bi})|^p)^{\frac{1}{p}} \\ & +(\textstyle\sum_{j=1}^{n} |\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\beta_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\alpha_j - (\beta_{di} - \beta_{bi})|^p)^{\frac{1}{p}}\} \\ s.t \quad & \alpha_j, \beta_j \geq 0. \end{aligned} \tag{7}$$

If $p = \infty$ then (7) transform to

$$\begin{aligned} Min \quad & \{(Max|\textstyle\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\alpha_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\beta_j \\ & \qquad -(\alpha_{di} - \alpha_{bi})|) \\ & +(Max|\textstyle\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\beta_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\alpha_j \\ & \qquad -(\beta_{di} - \beta_{bi})|), i = 1,\ 2, ..., n\} \\ s.t \quad & \alpha_j, \beta_j \geq 0, \end{aligned} \tag{8}$$

by using

$$\begin{aligned} & Max|\textstyle\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\alpha_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\beta_j - (\alpha_{di} - \alpha_{bi})|\} = z_1, \\ & Max|\textstyle\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\beta_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\alpha_j - (\beta_{di} - \beta_{bi})|\} = z_2, \\ & i = 1, 2, ..., n \end{aligned} \tag{9}$$

then we will have

$$\begin{aligned} Min \quad & z_1 + z_2 \\ s.t \quad & \\ & -z_1 \leq \textstyle\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\alpha_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\beta_j - (\alpha_{di} - \alpha_{bi}) \leq z_1 \\ & -z_2 \leq \textstyle\sum_{a_{ij}>0,c_{ij}>0}(a_{ij} - c_{ij})\beta_j - \sum_{a_{ij}<0,c_{ij}<0}(a_{ij} - c_{ij})\alpha_j - (\beta_{di} - \beta_{bi}) \leq z_2, \\ & \alpha_j, \beta_j, z_1, z_2, \geq 0, \qquad i = 1, 2, ..., n. \end{aligned} \tag{10}$$

Therefor this is a LP, where it can approximate widths for $A\tilde{x} \oplus \tilde{b} = C\tilde{x} \oplus \tilde{d}$ and by solving $A[\tilde{x}]_1 = [\tilde{b}]_1$ and they compilation, we can write an approximation fuzzy number solution for fuzzy dual linear systems of equations.

Lemma 3.1 *If vectors $(\alpha_1, \beta_1, z_{11}, z_{21})$, $(\alpha_2, \beta_2, z_{12}, z_{22})$ where optimum solutions to model (10) then, all convex combination of these solutions, it is an optimal solutions.*

Proof: Assume

$$(\alpha_y, \beta_y, z_{1y}, z_{2y}) = \lambda(\alpha_1, \beta_1, z_{11}, z_{21}) + (1-\lambda)(\alpha_2, \beta_2, z_{12}, z_{22}),$$

or

$$\begin{cases} \alpha_y = \lambda\alpha_1 + (1-\lambda)\alpha_2, \\ \beta_y = \lambda\beta_1 + (1-\lambda)\beta_2, \\ z_{1y} = \lambda z_{11} + (1-\lambda)z_{12}, \\ z_{2y} = \lambda z_{21} + (1-\lambda)z_{22}, \end{cases}$$

now we have

$$-\lambda z_{11} \leq \sum_{a_{ij}>0, c_{ij}>0}(a_{ij} - c_{ij})\lambda\alpha_{1j} - \sum_{a_{ij}<0, c_{ij}<0}(a_{ij} - c_{ij})\lambda\beta_{1j} - \lambda(\alpha_{di} - \alpha_{bi}) \leq \lambda z_{11},$$

$$-(1-\lambda)z_{12} \leq \sum_{a_{ij}>0, c_{ij}>0}(a_{ij} - c_{ij})(1-\lambda)\alpha_{2j} - \sum_{a_{ij}<0, c_{ij}<0}(a_{ij} - c_{ij})(1-\lambda)\beta_{2j}$$

$$-(1-\lambda)(\alpha_{di} - \alpha_{bi}) \leq (1-\lambda)z_{12},$$

$$i = 1, 2, ..., n,$$

with addition these two equations,

$$-z_{1y} \leq \sum_{a_{ij}>0, c_{ij}>0}(a_{ij} - c_{ij})\alpha_{yj} - \sum_{a_{ij}<0, c_{ij}<0}(a_{ij} - c_{ij})\beta_{yj} - (\alpha_{di} - \alpha_{bi}) \leq z_{1y},$$
$$i = 1, 2, ..., n.$$

For z_{2y} as form up we can show too. Now to demonstrate optimality we know $z_{11} + z_{21} = z^*$, $z_{12} + z_{22} = z^*$ hence,

$$\begin{aligned} z_{1y} + z_{2y} &= [\lambda z_{11} + (1-\lambda)z_{12}] + [\lambda z_{21} + (1-\lambda)z_{22}] \\ &= \lambda(z_{11} + z_{21}) + (1-\lambda)(z_{12} + z_{22}) \\ &= \lambda z^* + (1-\lambda)z^* = z^*. \end{aligned}$$

Therefor convex combination is an optimal solution. With this model we can approximate a solution where, it is fuzzy number and it is maybe where this approximation satisfy in first fuzzy linear system.

Definition 3.2 *All convex combination of fuzzy number is shown with $C(E)$ and it will define as follow,*

$$C(E) = \{\tilde{y} | \tilde{y} = \lambda\tilde{x} \oplus (1-\lambda)\tilde{x}', 0 \leq \lambda \leq 1, \tilde{x}, \tilde{x}' \in E\}$$

Theorem 3.3 *If model(10)has multiple solutions where satisfy in $A\tilde{x} \oplus \tilde{b} = C\tilde{x} \oplus \tilde{d}$ then we can demonstrate this system has infinite number solution as form $C(E)$.*

Proof: If $A\tilde{x}_1 \oplus \tilde{b} = C\tilde{x}_1 \oplus \tilde{d}$, $A\tilde{x}_2 \oplus \tilde{b} = C\tilde{x}_2 \oplus \tilde{d}$ and $\tilde{y} = \lambda\tilde{x}_1 \oplus (1-\lambda)\tilde{x}_2$ then,

$$\begin{cases} A\lambda\tilde{x}_1 \oplus \lambda\tilde{b} = C\lambda\tilde{x}_1 \oplus \lambda\tilde{d} \\ A(1-\lambda)\tilde{x}_2 \oplus (1-\lambda)\tilde{b} = C(1-\lambda)\tilde{x}_2 \oplus (1-\lambda)\tilde{d} \end{cases}$$

$$\Rightarrow A[\lambda\tilde{x}_1 \oplus (1-\lambda)\tilde{x}_2] \oplus \tilde{b} = C[\lambda\tilde{x}_1 \oplus (1-\lambda)\tilde{x}_2] \oplus \tilde{d} \Rightarrow A\tilde{y} \oplus \tilde{b} = C\tilde{y} \oplus \tilde{d}$$

therefor it is maybe a fuzzy linear system has infinite number solutions.

Theorem 3.4 *If $\tilde{x}$ was a fuzzy number that satisfy $A\tilde{x} \oplus \tilde{b} = C\tilde{x} \oplus \tilde{d}$ then it is an optimal solution for model(10)and optimal value equal to zero.*

Proof: Assume $\tilde{x} = (x, \alpha, \beta)$ then because $\tilde{x}$ satisfy in $A\tilde{x} \oplus \tilde{b} = C\tilde{x} \oplus \tilde{d}$ hence we have;

$$\sum_{a_{ij}>0, c_{ij}>0}(a_{ij} - c_{ij})\alpha_j - \sum_{a_{ij}<0, c_{ij}<0}(a_{ij} - c_{ij})\beta_j = (\alpha_{di} - \alpha_{bi}), \quad i = 1, 2, ..., n$$

$$\sum_{a_{ij}>0, c_{ij}>0}(a_{ij} - c_{ij})\beta_j - \sum_{a_{ij}<0, c_{ij}<0}(a_{ij} - c_{ij})\alpha_j = (\beta_{di} - \beta_{bi}), \quad i = 1, 2, ..., n$$

with model(10) $z_1 = z_2 = 0$ and $z^* = 0$ therefor $\tilde{x}$ is an optimal solution.

Lemma 3.5 *If in model (10) optimal's value was no zero then $A\tilde{x} \oplus \tilde{b} = C\tilde{x} \oplus \tilde{d}$ hasn't fuzzy number solution and (10) model's solution is an approximation only.*

4. Example

4.1. Example

Consider the following linear system

$$\begin{cases} \tilde{x}_1 \ominus \tilde{x}_2 \oplus (1,1,1) = \tilde{x}_1 \oplus \tilde{x}_2 \oplus (-1,1,1) \\ 2\tilde{x}_1 \oplus \tilde{x}_2 \oplus (1,2,1) = \tilde{x}_1 \oplus 3\tilde{x}_2 \oplus (1,1,0) \end{cases}$$

by using model (5) we have

$$\begin{pmatrix} 0 & -2 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

by using model (10) we have,

$$\begin{array}{ll} Min & z_1 + z_2 \\ s.t & \\ & -z_1 \le \beta_2 - \alpha_2 \le z_1 \\ & -z_1 \le \alpha_1 - 2\alpha_2 + 1 \le z_1 \\ & -z_2 \le \alpha_2 - \beta_2 \le z_2 \\ & -z_2 \le \beta_1 - 2\beta_2 + 1 \le z_2 \\ & \alpha_1, \beta_1, \alpha_2, \beta_2, z_1, z_2 \ge 0 \end{array}$$

this LP solutions equal to

$$\begin{cases} (\alpha_1, \beta_1, \alpha_2, \beta_2, z_{11}, z_{12}) = (1,1,1,1,0,0) \\ \\ (\alpha_2, \beta_2, \alpha_2, \beta_2, z_{21}, z_{22}) = (0,0,0.5,0.5,0,0) \end{cases}$$

hence fuzzy linear system has infinite number solutions as follow,

$$\begin{cases} \tilde{x}_1 = \lambda(2,1,1) \oplus (1-\lambda)(2,0,0) \\ \\ \tilde{x}_2 = \lambda(1,1,1) \oplus (1-\lambda)(1,0.5,0.5) \end{cases}$$

4.2. Example

Consider the following fuzzy linear system,

$$\begin{cases} 3\tilde{x}_1 \oplus 2\tilde{x}_2 \oplus (2,3,1) = \tilde{x}_1 \ominus \tilde{x}_2 \oplus (10,3,9) \\ \tilde{x}_1 \ominus 3\tilde{x}_2 \qquad\qquad\quad = \tilde{x}_1 \oplus \tilde{x}_2 \oplus (-8,8,0) \end{cases}$$

by using model (5) we have

$$\begin{pmatrix} 2 & 3 \\ 0 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

by using model (10) we have,

$$\begin{array}{ll} Min & z_1 + z_2 \\ s.t & \\ & -z_1 \le 2\alpha_1 + 2\alpha_2 - \beta_2 \le z_1 \\ & -z_1 \le 3\beta_2 - \alpha_2 - 8 \le z_1 \\ & -z_2 \le 2\beta_1 + 2\beta_2 - \alpha_2 - 8 \le z_2 \\ & -z_2 \le 3\alpha_2 - \beta_2 \le z_2 \\ & \alpha_1, \beta_1, \alpha_2, \beta_2, z_1, z_2 \ge 0 \end{array}$$

this problem has unique solution equal to,

$$(\alpha_1, \beta_1, \alpha_2, \beta_2, z_1, z_2) = (0.5, 1.5, 1, 3, 0, 0)$$

hence fuzzy linear system has single solution as follow,

$$\tilde{x}_1 = (1, 0.5, 1.5)$$

$$\tilde{x}_2 = (2, 1, 3)$$

4.3. Example

Consider the following fuzzy linear system,

$$\begin{cases} \tilde{x}_1 \oplus \tilde{x}_2 \oplus (1,2,3) = (4,3,5) \\ \tilde{x}_1 \ominus 2\tilde{x}_2 \oplus \tilde{x}_3 \oplus (0,1,1) = (-2,5,3) \\ 2\tilde{x}_1 \oplus \tilde{x}_2 \ominus \tilde{x}_3 \oplus (-3,1,2) = (0,4,8) \end{cases}$$

by using model (5) we have

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & -2 & 1 \\ 2 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

by using model (10) we have,

$$\begin{array}{ll} Min & z_1 + z_2 \\ s.t & \\ & -z_1 \leq \alpha_1 + \alpha_2 - 1 \leq z_1 \\ & -z_1 \leq \alpha_1 + 2\beta_2 + \alpha_3 - 4 \leq z_1 \\ & -z_1 \leq 2\alpha_1 + \alpha_2 + \beta_3 - 3 \leq z_1 \\ & -z_2 \leq \beta_1 + \beta_2 - 2 \leq z_2 \\ & -z_2 \leq \beta_1 + 2\alpha_2 + \beta_3 - 2 \leq z_2 \\ & -z_2 \leq 2\beta_1 + \beta_2 + \alpha_3 - 6 \leq z_2 \\ & \alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3, z_1, z_2 \geq 0 \end{array}$$

this problem has unique solution equal to,

$$(\alpha_1, \beta_1, \alpha_2, \beta_2, \alpha_3, \beta_3, z_1, z_2) = (1.223, 1.667, 0, 0.334, 2.334, 0.334, 0.222, 0)$$

hence fuzzy linear system has approximation solution as follow,

$$\tilde{x}_1 = (1, 1.223, 1.667)$$

$$\tilde{x}_2 = (2, 0, 0.334)$$

$$\tilde{x}_2 = (1, 2.334, 0.334)$$

meanwhile this example show that our solution in core and right width is accurate and left width hasn't accurate solution.

5. Conclusion

However you did see this method can a strong method for proof this our fuzzy dual linear systems and equations has been solution or not and if our solution is no then what is best approximation.

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