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Research Paper

Some statements for bi-pseudo-integrals and the role on reconstruction of the pseudo-additive measures

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Abstract

With the support of some very important and special generators, are given some details about the properties of bi-pseudo-integrals and above all, for the first bi-pseudo-integral the relations with integral Lebesgue are listed. Further, will be shown pseudo-linearity of bi-pseudo-integrals and some investigations in reconstructions of pseudo-additive measures by bi-pseudo-integrals synthesized the reciprocal relationship between pseudo-additive measure and bi-pseudo-integral.

Keywords: *Bi-Pseudo-Integral; Pseudo-Additive Measure; Generator, Pseudo-Operations; Reconstruction.*

1. Introduction

In section 2 are presented some of the most important generators $\bar{g}$ that play an important role in definitions of bi-pseudo-integrals and their properties. There are shown the connections between the $(\oplus_{g_{a,r}}, \odot_{g_{a,r}})$ − integral and the Lebesgue integral of non-negative function so, in the case of $(\oplus_{g_{1,r}}, \odot_{g_{1,r}}) = (\oplus_{g_{1,r}}\ ,\ \cdot\ )$, the Marinová's integral (as the first bi-pseudo-integral) is the Lebesgue integral [3], [7], [11]. In the case of ($\oplus = \vee = \max$), the Marinová's integral $\left(\mathrm{M} - \int_{\mathrm{X}}^{(\vee,\odot)}\right)$ leads to the Shilkret's integral [12]. For $(\oplus \neq \vee = \max)$ is discovered the connection between the $\left(\mathrm{M} - \int_{\mathrm{X}}^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)$ and the Lebesgue integral, but Kolesárová in [7] has explained the reasons for another definition of the $(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})$ − integral of real functions which is more appropriate than the definition that was given in [11]. The generalization of Bi-Pseudo-Integral and relations with Lebesgue integral are treated in section 3. Different properties of integrals with respect to $\overline{\oplus}_{\bar{g}}$- measure and $\overline{\vee}$-measure for f - RMF are caused by the essential difference between $\overline{\oplus}_{g_{a,r}}$ and $\overline{\vee}$ [3], [7], [11], [12]. The notion of modified pseudo-additive measure $(\overline{\oplus} - m_{\bar{g}})$ by $\bar{g}$ − transform is followed and completed with the meaning of bi-pseudo-integral of modified function $(f_{\bar{g}})$ with respect to a $m_{\bar{g}}$ in [10]. More links between different types of the bi-pseudo-integrals are noted in [10]. For the reconstruction of pseudo-additive measures some statements are treated in section 4. Further, the results are summarized in section 5.

2. Preliminary notes

Let a generator $\bar{g}: [-\infty, +\infty] \longrightarrow [-\infty, +\infty]$ be a continuous, monotone strictly increaing unbouded function of the pseudo-addition $\overline{\oplus}$ on the interval $[-\infty, +\infty]$, such that $\bar{g}(0) = 0_{\oplus}, \bar{g}(1) = 1_{\odot}$, $\bar{g}(+\infty) = +\infty$, with the convention $0 \cdot (+\infty) = 0$ and some valued undefined (or an odd extension of a given generator g from $[0, +\infty]$ to $[-\infty, +\infty]$, briefly $\bar{g}(\mathrm{x}) = \operatorname{sgn} \mathrm{x} \cdot g(|\mathrm{x}|)$, $\mathrm{x} \in [-\infty, \infty]$). The concept of pseudo-arithmetical operations $\{\oplus, \odot, \ominus, \oslash\}$ first was introduced on $[0,+\infty]$ interval and then to the whole extended real line $[-\infty, +\infty]$ [1], [3], [4], [5], [6], [7], [14], [16].

Following Mesiar and Rybárik [4], the binary operations $(\overline{\oplus}, \overline{\odot})$ (pseudo-addition, pseudo-multiplication) are respectively the binary function that [3], [7], [11] fulfill the system of axioms ($\overline{\oplus}$-A.1÷A.7) and ($\overline{\odot}$-A.1÷ A.7). The system of the axioms ($\oplus$,$\odot$-A.1÷ A.5) was formulated by the system of the Axioms of Sugeno and Murofushi [8], [15], [16]. Then, the sistem of pseudo-arithmetical operations $\{\overline{\oplus},\overline{\odot},\overline{\ominus},\overline{\oslash}\} = \{\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}},\overline{\ominus}_{\bar{g}},\overline{\oslash}_{\bar{g}}\}$ generated by this function $\bar{g}$, is said to be a consistent sistem [3], [7], [11].
So, for $x, y \in [-\infty, +\infty]$, let $\bar{g}$ be a generator on $[-\infty, +\infty]$, perhaps with some valued undefined [4], [8], [11], [13]. There are selected some most important generators $\bar{g}$ and $\bar{g} -$ calculus derived from them [2], [4], [5], [11], [13] are listed in the Table 1.

Table 1: Some of the Most Important Generators $\bar{G}$

$\bar{g}(x) = \bar{g}_{a,r}(x)$	$\{\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}},\overline{\ominus}_{\bar{g}},\overline{\oslash}_{\bar{g}}\}$	*Conditions* (a, r, x, y)
$\bar{g}_{1,1}(x) = x$	$x \overline{\oplus}_{\bar{g}_{1,1}} y = x + y$	$r = 1, a = 1$
	$x \overline{\odot}_{\bar{g}_{1,1}} y = x \cdot y$	
	$x \overline{\ominus}_{\bar{g}_{1,1}} y = x - y$	
	$x \overline{\oslash}_{\bar{g}_{1,1}} y = x / y$	$y \neq 0$
$\bar{g}_{a,1}(x) = a \cdot x$	$x \overline{\oplus}_{\bar{g}_{a,1}} y = x + y$	$r = 1, a > 0$
	$x \overline{\odot}_{\bar{g}_{a,1}} y = a \cdot (x \cdot y)$	
	$x \overline{\ominus}_{\bar{g}_{a,1}} y = x - y$	
	$x \overline{\oslash}_{\bar{g}_{a,1}} y = a^{-1} \cdot (x / y)$	$y \neq 0$
$\bar{g}_{1,r}(x) = x^r$	$x \overline{\oplus}_{\bar{g}_{1,r}} y = (x^r + y^r)^{1/r}$	$r \geq 1, a = 1$
	$x \overline{\odot}_{\bar{g}_{1,r}} y = x \cdot y$	
	$x \overline{\ominus}_{\bar{g}_{1,r}} y = (x^r - y^r)^{1/r}$	
	$x \overline{\oslash}_{\bar{g}_{1,r}} y = x / y$	$y \neq 0$
$\bar{g}_{a,r}(x) = a \cdot x^r$	$x \overline{\oplus}_{\bar{g}_{a,r}} y = (x^r + y^r)^{1/r}$	$r \geq 1, a > 0$
	$x \overline{\odot}_{\bar{g}_{a,r}} y = a^{1/r} \cdot (x \cdot y)$	
	$x \overline{\ominus}_{\bar{g}_{a,r}} y = (x^r - y^r)^{1/r}$	
	$x \overline{\oslash}_{\bar{g}_{a,r}} y = a^{-1/r} \cdot (x / y)$	$y \neq 0$

If $a = 1$, we have the follow form of the generator i.e., the normed generator $\bar{g}(x) = \bar{g}_{1,r}(x) = x^r$ and $\bar{g}(1) = \bar{g}_{1,r}(1) = 1$. Also, easily can control that $\bar{g}_{a,r}{}^{-1}(a) = 1$, $\bar{g}_{a,1}{}^{-1}(a) = 1$ and $\bar{g}_{a,r}{}^{-1}(1) = 1$, $\bar{g}_{a,1}{}^{-1}(1) = 1$ [3], [4], [7], [11], [13].

Definition 2.1.1: For a simple non-negative measurable function (in short form *s - SNNMF*) defined on X,
$s(x) = \sum_{i=1}^{n} \propto_i \cdot \mathbb{I}_{A_i}$ where sets $A_i \in \mathcal{A}, A_i \neq A_j,\ A_i \cap A_j = \emptyset$, for $i \neq j$, $i, j = \overline{1, n}$, $0 < \propto_i < +\infty$, then

$$\left(M - \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})}\right)(m, s - SNNMF) = \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})} s \odot_{g_{a,r}} dm = \oplus_{i=1}^{\infty} \propto_i \odot_{g_{a,r}} m(A_i)$$ [3], [7], [11].

Definition 2.1.2: For a non-negative measurable function (*f-NNMF*), $f: X \to [0, +\infty]$, then

$$\left(M - \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})}\right)(m,\ f - NNMF) = \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})} f \odot_{g_{a,r}} dm =$$

$$= sup\left\{\int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})} s \odot_{g_{a,r}} dm: s \leq f,\ s - \ is\ a\ SNNMF\right\},$$

And say that the function f [3], [7], [11] is integrable if

$$\left(M - \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})}\right)(m,\ f - NNMF) = \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})} f \odot_{g_{a,r}} dm < +\infty.$$

Generalized definition by Kolesárová in [7] for $(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})$ −integral, in case of *f – RMF* with respect to a $\overline{\oplus}_{\bar{g}}$- measure m where $\bar{g}_{a,r}|_{[0,+\infty]} = g_{a,r}$, $\overline{\oplus}_{\bar{g}_{a,r}}|_{[0,\infty]} = \oplus_{g_{a,r}}$, is given as bellow.

Definition 2.1.3: [7], [10] For a real measurable function (in short form f - RMF), $(\overline{\oplus} = \overline{\oplus}_{\bar{g}} \neq \overline{\vee})$,

$f: X \to (-\infty, +\infty)$, if at least one of the functions f^+, f^- is integrable, then

$$\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m, f - RMF) = \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f^+ \overline{\odot}_{\bar{g}} \; dm \, \overline{\ominus}_{\bar{g}} \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f^- \overline{\odot}_{\bar{g}} \; dm =$$

$$= \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m, f^+) \overline{\ominus}_{\bar{g}} \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m, f^-).$$

A function f is called integrable iff

$$-\infty < \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m, f - RMF) = \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f \, \overline{\odot}_{\bar{g}} \, dm < +\infty.$$

The bi-$(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}}) - integral$ in case of the $f_{\bar{g}}$ – $RMF_{\bar{g}}$ with respect to a $\overline{\oplus}_{\bar{g}} - measure \; m_{\bar{g}}$ [2], [3], [7], [10], [9], [11], [13], [14] will be given below.

Definition 2.1.4: [10] For a real measurable function $f_{\bar{g}}$ (short form $f_{\bar{g}} - RMF_{\bar{g}}$) $f_{\bar{g}}: \bar{g}^{-1}(X) \to (-\infty, +\infty)$, $(in\ case\ of\ the\ pseudo - operation\ \ \overline{\oplus} \neq \overline{\vee})$, if at least one of the functions $f_{\bar{g}}{}^+, f_{\bar{g}}{}^-$ is integrable, then

$$\left(BP_{(\bar{g}-TR)} - \int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m_{\bar{g}}, f_{\bar{g}} - RMF_{\bar{g}}) = \left(\int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f_{\bar{g}}{}^+ \, \overline{\odot}_{\bar{g}} \, dm_{\bar{g}}\right) \overline{\ominus}_{\bar{g}} \left(\int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f_{\bar{g}}{}^- \, \overline{\odot}_{\bar{g}} \, dm_{\bar{g}}\right) =$$

$$= \left(BP_{(\bar{g}-TR)} - \int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m_{\bar{g}}, f_{\bar{g}}{}^+) \, \overline{\ominus}_{\bar{g}} \left(BP_{(\bar{g}-TR)} - \int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m_{\bar{g}}, f_{\bar{g}}{}^-).$$

A function $f_{\bar{g}}$ is called integrable iff

$$-\infty < \left(BP_{(\bar{g}-TR)} - \int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m_{\bar{g}}, f_{\bar{g}} - RMF_{\bar{g}}) = \int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f_{\bar{g}} \, \overline{\odot}_{\bar{g}} \, dm_{\bar{g}} < +\infty.$$

3. The generalization of Bi-Pseudo-Integral and relations with Lebesgue integral

Proposition 3.1: Let s be a simple non-negative measurable function (short form s - SNNMF) defined on X , let m be a $\oplus - measure$ on $(X, \mathcal{A})$, $(\oplus \neq \vee)$ and let $g = g_{1,r}$ be a normed generator of the operation $\oplus_{g_{a,r}}$. Then the bi-pseudo-integral is in the form

$$\left(M - \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})}\right)(m, s - SNNMF) = g_{a,r}{}^{-1}\left(\frac{1}{a}\right) \odot_{g_{a,r}} g_{a,r}{}^{-1}\left(\int_X^{(+,\cdot)} (g_{a,r} \circ s) \cdot d(g_{a,r} \circ m)\right),$$

where the right-hand side is the Lebesgue integral $\left(L - \int_X^{(+,\cdot)}\right)(g_{a,r} \circ m, g_{a,r} \circ s - SNNMF)$ [3], [7], [10], [11].

Proof. Let $s = \sum_{i=1}^{n} \propto_i \cdot \mathbb{I}_{A_i}$, $\propto_i > 0$, $A_i \in \mathcal{A}$. Then, by definiton 3.1.1 can get:

$$\left(M - \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})}\right)(m, s - SNNMF) = \oplus_{i=1}^{\infty} \propto_i \cdot \; m(A_i).$$

If we use the equation $x \oplus y = x \oplus_{g_{a,r}} y = g_{a,r}{}^{-1}\left(g_{a,r}(x) + g_{a,r}(y)\right)$ we get:

$$\left(M - \int_X^{(\oplus_{g_{a,r}}, \odot_{g_{a,r}})}\right)(m, s - SNNMF) = g_{a,r}{}^{-1}\left(\sum_{i=1}^{n} g_{a,r}\left(\propto_i \cdot \; m(A_i)\right)\right) =$$

$$= g_{a,r}{}^{-1}\left(\sum_{i=1}^{n} a \cdot \propto_i{}^r \cdot \left(m(A_i)\right)^r\right) = g_{a,r}{}^{-1}\left(a \cdot \sum_{i=1}^{n} \propto_i{}^r \cdot \left(m(A_i)\right)^r\right) =$$

$$= g_{a,r}{}^{-1}\left(a \cdot \sum_{i=1}^{n} \frac{1}{a} \cdot g_{a,r}(\propto_i) \cdot \frac{1}{a} \cdot g_{a,r}\left(m(A_i)\right)\right) =$$

$$= g_{a,r}{}^{-1}\left(\frac{1}{a} \cdot \sum_{i=1}^{n} g_{a,r}(\propto_i) \cdot g_{a,r}\left(m(A_i)\right)\right) = g_{a,r}{}^{-1}\left(\frac{1}{a} \cdot \sum_{i=1}^{n} g_{a,r}(\propto_i) \cdot g_{a,r}\left(m(A_i)\right)\right) =$$

$$= g_{a,r}{}^{-1}\left(g_{a,r}\left(g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\right)\cdot g_{a,r}\left(g_{a,r}{}^{-1}\left(\sum_{i=1}^{n} g_{a,r}(\propto_i)\cdot g_{a,r}\left(m(A_i)\right)\right)\right)\right) =$$

$$= g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\sum_{i=1}^{n} g_{a,r}(\propto_i)\cdot g_{a,r}\left(m(A_i)\right)\right)\right) =$$

$$= g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{a,r}\circ m, g_{a,r}\circ s-SNNMF\right)\right)\right).$$

- $\left(M-\int_X^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m,s-SNNMF) = \left(M-\int_X^{(\oplus_{g_{a,r}},\cdot)}\right)(m,s-SNNMF) =$

$$= g_{a,r}{}^{-1}\left(a\cdot\sum_{i=1}^{n}\propto_i{}^{r}\cdot\left(m(A_i)\right)^{r}\right) = g_{a,r}{}^{-1}(a)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right)\right)\right).$$

- $\left(\mathrm{M}-\int_{\mathrm{X}}^{(\oplus_{g_{1,r}},\odot_{g_{1,r}})}\right)(m,s-SNNMF) = \left(\mathrm{M}-\int_{\mathrm{X}}^{(\oplus_{g_{1,r}},\cdot)}\right)(m,s-SNNMF) =$

$$= g_{1,r}{}^{-1}\left(a\oplus_{g_{1,r}},\odot_{g_{1,r}}\sum_{i=1}^{n}\propto_i{}^{r}\cdot\left(m(A_i)\right)^{r}\right) =$$

$$= g_{1,r}{}^{-1}(a)\odot_{g_{1,r}}\left(g_{1,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right)\right)\right) =$$

$$= 1\odot_{g_{1,r}}\left(g_{1,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right)\right)\right) =$$

$$= g_{1,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right)\right).$$

- $\left(\mathrm{M}-\int_{\mathrm{X}}^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m,s-SNNMF) =$

$$= \begin{cases} g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{a,r}\circ m, g_{a,r}\circ s-SNNMF\right)\right)\right) for\ r\geq 1, a>0 \\ g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right) for\ r\geq 1, a=1\right)\right) \\ g_{1,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right)\right)\ for\ r\geq 1, a=1 \end{cases}$$

- $\left(\mathrm{M}-\int_{\mathrm{X}}^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m,s-SNNMF) =$

$$= \begin{cases} g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{a,r}\circ m, g_{a,r}\circ s-SNNMF\right)\right)\right)\ for\ r\geq 1, a>0 \\ g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right) for\ r\geq 1, a=1\right)\right) \\ g_{1,r}{}^{-1}\left(\left(L-\int_X^{(+,\cdot)}\right)\left(g_{1,r}\circ m, g_{1,r}\circ s-SNNMF\right)\right)\ for\ r\geq 1, a=1 \\ g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(M-\int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m,s-SNNMF)\right)\ for\ r\geq 1, a=1\right) \\ \left(M-\int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m,s-SNNMF)\ for\ r\geq 1, a=1 \end{cases}$$

- $\left(\mathrm{M}-\int_{\mathrm{X}}^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m,s-SNNMF) =$

$$= \begin{cases} a^{-2/r} \odot_{g_{a,r}} \left({g_{a,r}}^{-1} \left(\left(L - \int_X^{(+,\cdot)} \right) (g_{a,r} \circ m, g_{a,r} \circ s - SNNMF) \right) \right) \; for\, r \geq 1, a > 0 \\ a^{-2/r} \odot_{g_{a,r}} \left({g_{a,r}}^{-1} \left(\left(L - \int_X^{(+,\cdot)} \right) (g_{1,r} \circ m, g_{1,r} \circ s - SNNMF) \; for\, r \geq 1, a = 1 \right) \right) \\ {g_{1,r}}^{-1} \left(\left(L - \int_X^{(+,\cdot)} \right) (g_{1,r} \circ m, g_{1,r} \circ s - SNNMF) \right) \; for\, r \geq 1, a = 1 \\ a^{-2/r} \odot_{g_{a,r}} \left({g_{a,r}}^{-1} \left(\left(M - \int_X^{(\oplus_{g_{1,r}},\cdot)} \right) (m, s \; - SNNMF) \right) \; for\, r \geq 1, a = 1 \right) \\ \left(M - \int_X^{(\oplus_{g_{1,r}},\cdot)} \right) (m, s \; - SNNMF) \; for\, r \geq 1, a = 1 \end{cases}$$

Proposition 3.1.2: [7], [10] Let $(X, \mathcal{A}, m)$ be a $\oplus$- measure space. The integral of a real measurable *f- RMF* with respect to a $\oplus - measure\ m$, in case $\oplus \neq \vee$ (when $\int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}$ is defined) is given by:

$$\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} \right) (m,\ f - RMF) = \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} \right) (m,\ f - RMF) =$$

$$= {\bar{g}_{a,r}}^{-1} \left({\bar{g}_{a,r}}^{-1} \left(\frac{1}{a} \right) \odot_{g_{a,r}} \left(L - \int_X^{(+,\cdot)} \right) (\bar{g}_{a,r} \circ m, \bar{g}_{a,r} \circ f - RMF) \right)$$

and the integral in the right-hand side $\left(L - \int_X^{(+,\cdot)} \right) (\bar{g}_{a,r} \circ m, \bar{g}_{a,r} \circ f - RMF)$ is Lebesgue integral, also $\bar{g}_{a,r} \circ m$ is the Lebesgue measure.
If $\bar{g}$ is the normed generator ($\bar{g} = \bar{g}_{1,r} - normed\ generator$) hold [3], [7], [10], [11]:

$$\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})} \right) (m,\ f - RMF) = \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)} \right) (m,\ f - RMF) =$$

$$= \int_X^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)} f \cdot dm = {\bar{g}_{1,r}}^{-1} \left(\left(L - \int_X^{(+,\cdot)} \right) (\bar{g}_{1,r} \circ m, \bar{g}_{1,r} \circ f - RMF) \right).$$

Proof. By using the definition 2.1.4, proposition 3.1 and the additivity of the Lebesgue integral can taken:

$$\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} \right) (m,\ f^+) \overline{\ominus}_{\bar{g}} \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} \right) (m,\ f^-) =$$

$$= \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f^+ \overline{\odot}_{\bar{g}} \ dm \ \overline{\ominus}_{\bar{g}_{a,r}} \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f^- \overline{\odot}_{\bar{g}} \ dm =$$

$$= {\bar{g}_{a,r}}^{-1} \left[\bar{g}_{a,r} \left(\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} \right) (m,\ f^+) \right) - \bar{g}_{a,r} \left(\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} \right) (m,\ f^-) \right) \right] =$$

$$= {\bar{g}_{a,r}}^{-1} \left(\bar{g}_{a,r} \left(\int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} f^+ \overline{\odot}_{\bar{g}_{a,r}} \ dm \right) - \bar{g}_{a,r} \left(\int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} f^- \overline{\odot}_{\bar{g}_{a,r}} \ dm \right) \right) =$$

$$= \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} f^+ \overline{\odot}_{\bar{g}_{a,r}} \ dm \ \overline{\ominus}_{\bar{g}_{a,r}} \ \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} f^- \overline{\odot}_{\bar{g}_{a,r}} \ dm =$$

$$= {\bar{g}_{a,r}}^{-1} \left\{ \bar{g}_{a,r} \left(\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} \right) (m,\ f^+) \right) - \bar{g}_{a,r} \left(\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}}, \overline{\odot}_{\bar{g}_{a,r}})} \right) (m,\ f^-) \right) \right\} =$$

$$= {\bar{g}_{a,r}}^{-1} \left\{ \bar{g}_{a,r} \left[{\bar{g}_{a,r}}^{-1} \left({\bar{g}_{a,r}}^{-1} \left(\frac{1}{a} \right) \overline{\odot}_{\bar{g}_{a,r}} {\bar{g}_{a,r}}^{-1} \left(\int_X^{(+,\cdot)} (\bar{g}_{a,r} \circ f^+) \cdot d(\bar{g}_{a,r} \circ m) \right) \right) \right] - \right.$$

$$\left. - \bar{g}_{a,r} \left[{\bar{g}_{a,r}}^{-1} \left(\frac{1}{a} \right) \overline{\odot}_{\bar{g}_{a,r}} {\bar{g}_{a,r}}^{-1} \left({\bar{g}_{a,r}}^{-1} \left(\int_X^{(+,\cdot)} (\bar{g}_{a,r} \circ f^-) \cdot d(\bar{g}_{a,r} \circ m) \right) \right) \right] \right\} =$$

$$= \bar{g}_{a,r}{}^{-1}\left\{\left(\bar{g}_{a,r}{}^{-1}\left(\frac{1}{a}\right)\overline{\odot}_{\bar{g}_{a,r}}\,\bar{g}_{a,r}{}^{-1}\left(\int_X^{(+,\cdot)}(\bar{g}_{a,r}\circ f^{+})\cdot d(\bar{g}_{a,r}\circ m)\right)\right)-\right.$$

$$\left.-\bar{g}_{a,r}\left(\bar{g}_{a,r}{}^{-1}\left(\frac{1}{a}\right)\overline{\odot}_{\bar{g}_{a,r}}\,\bar{g}_{a,r}{}^{-1}\left(\bar{g}_{a,r}{}^{-1}\left(\int_X^{(+,\cdot)}(\bar{g}_{a,r}\circ f^{-})\cdot d(\bar{g}_{a,r}\circ m)\right)\right)\right)\right\} =$$

$$= \bar{g}_{a,r}{}^{-1}\left\{\left[\bar{g}_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{\bar{g}_{a,r}}\left(\left(\int_X^{(+,\cdot)}(\bar{g}_{a,r}\circ f^{+})\cdot d(\bar{g}_{a,r}\circ m)\right)-\left(\int_X^{(+,\cdot)}(\bar{g}_{a,r}\circ f^{-})\cdot d(\bar{g}_{a,r}\circ m)\right)\right)\right]\right\} =$$

$$= \bar{g}_{a,r}{}^{-1}\left[\bar{g}_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{\bar{g}_{a,r}}\left(\int_X^{(+,\cdot)}(\bar{g}_{a,r}\circ f)\cdot d(\bar{g}_{a,r}\circ m)\right)\right] =$$

$$= \bar{g}_{a,r}{}^{-1}\left(\bar{g}_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(L-\int_X^{(+,\cdot)}\right)(\bar{g}_{a,r}\circ m,\bar{g}_{a,r}\circ f-RMF)\right) =$$

$$= \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}},\overline{\odot}_{\bar{g}_{a,r}})} f\,\overline{\odot}_{\bar{g}_{a,r}}\,dm = \left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}_{a,r}},\overline{\odot}_{\bar{g}_{a,r}})}\right)(m,\ f-RMF) =$$

$$= \left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}})}\right)(m,\ f-RMF).$$

If $\bar{g}$ is the normed generator of $\overline{\oplus}$, $\bar{g}|_{[0,+\infty]}=g$, $\overline{\oplus}\,|_{[0,\infty]}=\oplus$, $g=g_{1,r}-normed\ generator$, hold:

$$\left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}})}\right)(m,\ f-RMF) = \left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}_{1,r}},\overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f-RMF) =$$

$$= \left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m,\ f-RMF) = \int_X^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)} f\ \cdot dm =$$

$$= \bar{g}_{1,r}{}^{-1}\left(\bar{g}_{1,r}{}^{-1}(1)\odot_{\bar{g}_{1,r}}\left(L-\int_X^{(+,\cdot)}\right)(\bar{g}_{1,r}\circ m,\bar{g}_{1,r}\circ f-RMF)\right) =$$

$$= \bar{g}_{1,r}{}^{-1}\left(1\odot_{\bar{g}_{1,r}}\left(L-\int_X^{(+,\cdot)}\right)(\bar{g}_{1,r}\circ m,\bar{g}_{1,r}\circ f-RMF)\right) =$$

$$= \left(\left(L-\int_X^{(+,\cdot)}\right)(\bar{g}_{1,r}\circ m,\bar{g}_{1,r}\circ f-RMF)\right).$$

So, all the relations are listed below:

$$\left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}})}\right) = \begin{cases} \begin{cases} \left(M-\int_X^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m,s-SNNMF)\ for\ r\geq 1, a>0 \\ g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(M-\int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m,s-SNNMF)\right)for\ r\geq 1, a=1\right) \\ \left(M-\int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m,s-SNNMF)\ for\ r\geq 1, a=1 \end{cases} \\ \begin{cases} \left(M-\int_X^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m,\ f-NNMF)\ for\ r\geq 1, a>0 \\ g_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{g_{a,r}}\left(g_{a,r}{}^{-1}\left(\left(M-\int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m,f-NNMF)\right)for\ r\geq 1, a=1\right) \\ \left(M-\int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m,f-NNMF)\ for\ r\geq 1, a=1 \end{cases} \\ \begin{cases} \left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}_{a,r}},\overline{\odot}_{\bar{g}_{a,r}})}\right)(m,\ f-RMF)\ for\ r\geq 1, a>0 \\ \bar{g}_{a,r}{}^{-1}\left(\frac{1}{a}\right)\odot_{\bar{g}_{a,r}}\left(\bar{g}_{a,r}{}^{-1}\left(\left(M-\int_X^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m,f-RMF)\right)\ for\ r\geq 1, a=1\right) \\ \left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m,\ f-RMF)\ for\ r\geq 1, a=1 \end{cases} \end{cases}$$

4. The reconstruction of pseudo-additive measure

Lemma 4.1: *Let* $(X, \mathcal{A}, m)$ *be a* $\overline{\oplus}$*- measurable space,* $(\overline{\oplus}=\overline{\oplus}_{\bar{g}} \neq \overline{\vee})$*. Let* $f: X \to (-\infty, \infty)$ *be an integrable function (f-IF). Then the set function* $\nu_f(A) = \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})} f \, \overline{\odot}_{\bar{g}} \, dm = \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}}, \overline{\odot}_{\bar{g}})}\right)(m, f)$*, for each* $A \in \mathcal{A}$*, where the bi-pseudo-integral is given by [3], [7], [11] is :*

$$\nu_f(A) = \begin{cases} a\ finite\ \oplus_g -\ measure\ on\ A\ if\ f - SNNMF \\ a\ finite\ \oplus_g -\ measure\ on\ A\ if\ f - NNMF \\ a\ finite\ \overline{\oplus}_{\bar{g}} -\ measure\ on\ A\ if\ f - RMF \\ a\ \sigma - \overline{\oplus}_{\bar{g}} -\ measure\ on\ A\ if\ f - RMF \\ \overline{\oplus}_{\bar{g}} = (\cdot) - homogeneous\ set\ function\ on\ A\ if\ f - RMF \end{cases}$$

Proof

1 & 2. Following Marinová, by Theorem 2 in [11], if f – NNMF (or if f – SNNMF), ν_f is $\oplus_g -\ additive$ and continuous from bellow, where the proof of the continuity is realized in three steps (it is enough by continuity of g and the property of Lebesgue integral)

3. We will consider here the the normed generator $(\bar{g} = \bar{g}_{1,r} - normed\ generator), for\ r \geq 1, a = 1$. In case of $(\overline{\oplus}=\overline{\oplus}_{\bar{g}}=\overline{\oplus}_{\bar{g}_{1,r}} \neq \overline{\vee})$, the operation $\overline{\oplus}$ is generated by the normed generator $\bar{g}$, and for $A, B \in \mathcal{A}$ with $A \cap B = \phi$ are taken:

$$\nu_f(\mathrm{A})\,\overline{\oplus}\,\nu_f(\mathrm{B}) = \nu_f(B)\,\overline{\oplus}_{\bar{g}_{1,r}}\,\nu_f(B) = \overline{\oplus}_{\bar{g}_{1,r}}\left(\nu_f(A), \nu_f(B)\right) =$$

$$= \left(M, K - \int_A^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f - RMF)\,\overline{\oplus}_{\bar{g}_{1,r}}\left(M, K - \int_B^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f - RMF) =$$

$$= \int_A^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})} f\,\overline{\odot}_{\bar{g}_{1,r}}\,dm\,\overline{\oplus}_{\bar{g}_{1,r}} \int_B^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})} f\,\overline{\odot}_{\bar{g}_{1,r}}\,dm = \int_A^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)} f \cdot dm\,\overline{\oplus}_{\bar{g}_{1,r}} \int_B^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)} f \cdot dm =$$

$$= \bar{g}_{a,r}{}^{-1}\left[\bar{g}_{1,r}\left(\left(M, K - \int_A^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)}\right)(m,\ f)\right) + \bar{g}_{1,r}\left(\left(M, K - \int_B^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)}\right)(m,\ f)\right)\right] =$$

$$= \bar{g}_{1,r}{}^{-1}\left(\bar{g}_{1,r}\left(m_f(A)\right) + \bar{g}_{1,r}\left(m_f(B)\right)\right) = \bar{g}_{1,r}{}^{-1}\left[\bar{g}_{1,r}\left(\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)} f\,dm\right) + \bar{g}_{1,r}\left(\int_B^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)} f\,dm\right)\right] =$$

$$= \bar{g}_{1,r}{}^{-1}\left\{\bar{g}_{1,r}\left[\left(\bar{g}_{1,r}{}^{-1}\left(\int_A^{(+,\cdot)}(\bar{g}_{1,r} \circ f)d(\bar{g}_{1,r} \circ m)\right)\right)\right] + \bar{g}_{1,r}\left[\left(\bar{g}_{1,r}{}^{-1}\left(\int_B^{(+,\cdot)}(\bar{g}_{1,r} \circ f)d(\bar{g}_{1,r} \circ m)\right)\right)\right]\right\} =$$

$$= \bar{g}_{1,r}{}^{-1}\left\{\int_A^{(+,\cdot)}(\bar{g}_{1,r} \circ f)d(\bar{g}_{1,r} \circ m) + \int_B^{(+,\cdot)}(\bar{g}_{1,r} \circ f)d(\bar{g}_{1,r} \circ m)\right\} = \bar{g}_{1,r}{}^{-1}\left[\int_{A \cup B}^{(+,\cdot)}(\bar{g}_{1,r} \circ f)d(\bar{g}_{1,r} \circ m)\right] =$$

$$= \bar{g}_{1,r}{}^{-1}\left(\left(L - \int_X^{(+,\cdot)}\right)(\bar{g}_{1,r} \circ m, \bar{g}_{1,r} \circ f - RMF)\right) = \left(M, K - \int_{A \cup B}^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)}\right)(m,\ f - RMF) = \int_{A \cup B}^{(\overline{\oplus}_{\bar{g}_{1,r}}, \cdot)} f\,dm =$$

$$= \int_{A \cup B}^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})} f\,dm = \left(M, K - \int_{A \cup B}^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f - RMF) = \nu_f(A \cup B).$$

4. To prove that the set function ν_f is a finite σ-$\overline{\oplus}$- additive function on $\mathcal{A}$, it is enought to prove its continuity from bellow. Let $A_n \in \mathcal{A}$, n = 1,2,..., and let $A_1 \subset A_2 \subset \cdots \subset A_n ..., A_n \nearrow A$. By the continuity of the generator $\bar{g} = \bar{g}_{1,r}$ and properties of the Lebesgue integral are getting:

$$lim_{n\to\infty}\,\nu_f(A_n) = \lim_{n\to\infty}\left\{\left(M, K - \int_{A_n}^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f - RMF)\right\} = \lim_{n\to\infty}\int_{A_n}^{(\overline{\oplus}_{\bar{g}_{1,r}}, \overline{\odot}_{\bar{g}_{1,r}})} f\,dm =$$

$$= \lim_{n\to\infty}\bar{g}_{1,r}{}^{-1}\left(\int_{A_n}^{(+,\cdot)}(\bar{g}_{1,r} \circ f)d(\bar{g}_{1,r} \circ m)\right) = \bar{g}_{1,r}{}^{-1}\left(\lim_{n\to\infty}\left(\int_{A_n}^{(+,\cdot)}(\bar{g}_{1,r} \circ f)d(\bar{g}_{1,r} \circ m)\right)\right) =$$

$$= \bar{g}_{1,r}{}^{-1}\left(\int_A^{(+,\cdot)}(\bar{g}_{1,r}\circ f)d(\bar{g}_{1,r}\circ m)\right) = \bar{g}_{1,r}{}^{-1}\left(\lim_{n\to\infty}\left(\int_{A_n}^{(+,\cdot)}(\bar{g}_{1,r}\circ f)d(\bar{g}_{1,r}\circ m)\right)\right) =$$

$$=\bar{g}_{1,r}{}^{-1}\left(\int_A^{(+,\cdot)}(\bar{g}_{1,r}\circ f)d(\bar{g}_{1,r}\circ m)\right) = \bar{g}_{1,r}{}^{-1}\left(\bar{g}_{1,r}\left(\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)} f dm\right)\right) =$$

$$= \bar{g}_{1,r}{}^{-1}\left(\bar{g}_{1,r}\left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m,\ f)\right) = \bar{g}_{1,r}{}^{-1}\left(\bar{g}_{1,r}\left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f-RMF)\right) =$$

$$= \int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)} f dm = \int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\overline{\odot}_{\bar{g}_{1,r}})} f dm = \left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f-RMF) = \nu_f(A).$$

By the integrability of f, follow finitness of the set function ν_f.

5. The bi-pseudo-integral $\left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}},\cdot)}\right)$ is a $\overline{\odot}_{\bar{g}}=(\cdot)-homogeneous\ functional$ if f – RMF $(in\ case\ of\ \overline{\oplus}=\overline{\oplus}_{\bar{g}}\neq\overline{\vee})$, so

$$\nu_{c\,\overline{\odot}_{\bar{g}_{1,r}}f}(A) = \left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\overline{\odot}_{\bar{g}_{1,r}})}\right)(m, c\,\overline{\odot}_{\bar{g}_{1,r}}\,f-RMF) = \nu_{c\cdot f}(A) =$$

$$= \left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m, c\cdot f-RMF) = \int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)} c\cdot f\ dm = \bar{g}^{-1}\left[\int_A^{(+,\cdot)}(\bar{g}\circ c\cdot f)d(\bar{g}\circ m)\right] =$$

$$= c\cdot\left\{\bar{g}^{-1}\left[\int_A^{(+,\cdot)}(\bar{g}\circ f)d(\bar{g}\circ m)\right]\right\} = c\cdot\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)} f\ dm = c\cdot\left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m,\ f-RMF) =$$

$$= c\cdot\nu_f(A) = c\,\overline{\odot}_{\bar{g}_{1,r}}\left(M,K-\int_A^{(\overline{\oplus}_{\bar{g}_{1,r}},\overline{\odot}_{\bar{g}_{1,r}})}\right)(m,\ f-RMF) = c\,\overline{\odot}_{\bar{g}_{1,r}}\,\nu_f(A).$$

Using relationships that are presented between the bi-pseudo-integrals [10] of different types and classes can be written $(in\ case\ of\ \overline{\oplus}=\overline{\oplus}_{\bar{g}}\neq\overline{\vee})$:

$$\left(BP_{(\bar{g}-TR)}-\int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}_{a,r}},\overline{\odot}_{\bar{g}_{a,r}})}\right)(m_{\bar{g}},\ f_{\bar{g}}-RMF_{\bar{g}}) = \left(M,K-\int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}_{a,r}},\overline{\odot}_{\bar{g}_{a,r}})}\right)(m_{\bar{g}},\ f_{\bar{g}}-RMF_{\bar{g}}),$$

$$\left(BP_{(\bar{g}-TR)}-\int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}_{a,r}},\overline{\odot}_{\bar{g}_{a,r}})}\right)(m_{\bar{g}},\ f_{\bar{g}}-RMF_{\bar{g}}) =$$

$$= \bar{g}_{a,r}{}^{-1}\left(a^{-2/r}\odot_{g_{a,r}}\left(L-\int_{\bar{g}^{-1}(X)}^{(+,\cdot)}\right)(\bar{g}_{a,r}\circ m_{\bar{g}},\bar{g}_{a,r}\circ f_{\bar{g}}-RMF_{\bar{g}})\right).$$

In the same way as the above Lemma 4.1, can be formulated the statements for $\nu_{f_{\bar{g}}}$ on $\bar{g}^{-1}(A)$:

$$\nu_{f_{\bar{g}}}(\bar{g}^{-1}(A)) = \begin{cases} a\ finite\ \oplus_g-\ measure\ on\ g^{-1}(A)\ if\ f_g-SNNMF_g \\ a\ finite\ \oplus_g-\ measure\ on\ g^{-1}(A)\ if\ f_g-NNMF_g \\ a\ finite\ \overline{\oplus}_{\bar{g}}-\ measure\ on\ \bar{g}^{-1}(A)\ if\ f_{\bar{g}}-RMF_{\bar{g}} \\ a\ \sigma-\overline{\oplus}_{\bar{g}}-\ measure\ on\ \bar{g}^{-1}(A)\ if\ f_{\bar{g}}-RMF_{\bar{g}} \\ \overline{\oplus}_{\bar{g}}=(\cdot)-homogeneous\ set\ function\ on\ \bar{g}^{-1}(A)\ if\ f_{\bar{g}}-RMF_{\bar{g}} \end{cases}$$

Proposition 4.2 (The pseudo-linearity of Bi-pseudo-integral):
The extended bi-pseudo-integral for real measurable function [3], [7], [11], $(in\ case\ of\ \overline{\oplus}=\overline{\oplus}_{\bar{g}}\neq\overline{\vee})$

$\left(M,K-\int_X^{(\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}})}\right)(m,\ f-RMF) = \int_X^{(\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}})} f\ \overline{\odot}_{\bar{g}}\ dm$ is:

$$\left(M,K-\int_X^{(\overline{\oplus}_{\bar g},\overline{\odot}_{\bar g})}\right)(m,f)=\begin{cases}\oplus_g - functional\ on\ X\ if\ f-SNNMF\\ \oplus_g - functional\ on\ X\ if\ f-NNMF\\ \overline{\oplus}_{\bar g} - additive\ functional\ on\ X\ if\ f-RMF\\ \overline{\odot}_{\bar g}=(\cdot)-homogeneous\ functional\ on\ X\ if\ f-RMF\end{cases}$$

1& 2. For these cases can be used the same way as [3], [7], and [11].
3. In the conditions that the binary operation $\oplus$ (pseudo-addition) has been extended ($\overline{\oplus}=\overline{\oplus}_{\bar g}$) on the interval $[-\infty,+\infty]$, for functions $f_1, f_2 : X \to [-\infty,\infty]$ can get:

$$\left(f_1\,\overline{\oplus}_{\bar g}\,f_2\right)(x)=\overline{\oplus}_{\bar g}\left(f_1(x),f_2(x)\right)=\bar g^{-1}[(\bar g\circ f_1)(x)+(\bar g\circ f_2)(x)].$$

Here, are considered again the the normed generator ($\bar g=\bar g_{1,r}-normed\ generator$), $for\ r\geq 1,\ a=1$ as a general and important generator. In case of $\overline{\oplus}=\overline{\oplus}_{\bar g}=\overline{\oplus}_{\bar g_{1,r}}\neq\overline{\vee}$, the operation $\overline{\oplus}$ is generated by the normed generator $\bar g$, for all real integrable functions [3], [7], [11] (for which the expressions on both sides make sence) the bi-pseudo-integral $\left(M,K-\int^{(\overline{\oplus}_{\bar g},\overline{\odot}_{\bar g})}\right)$ is $\overline{\oplus}_{\bar g}-additive$ functional by:

$$\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}\right)(m,\ f_1-RMF)\,\overline{\oplus}_{\bar g_{1,r}}\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}\right)(m,\ f_2-RMF)=$$

$$=\bar g_{1,r}{}^{-1}\left[\bar g_{1,r}\left(\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}\right)(m,\ f_1)\right)+\bar g_{1,r}\left(\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}\right)(m,\ f_2)\right)\right]=$$

$$=\overline{\oplus}_{\bar g_{1,r}}\left(\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}f_1\,dm\,,\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}f_2\,dm\right)=\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}f_1\,dm\,\overline{\oplus}_{\bar g_{1,r}}\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}f_2\,dm=$$

$$=\bar g_{1,r}{}^{-1}\left[\bar g\left(\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}f_1\,dm\right)+\bar g_{1,r}\left(\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}f_2\,dm\right)\right]=$$

$$=\bar g_{1,r}{}^{-1}\left\{\bar g_{1,r}\left[\left(\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}\right)(m,\ f_1)\right)\right]+\bar g_{1,r}\left[\left(\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\cdot)}\right)(m,\ f_2)\right)\right]\right\}=$$

$$=\bar g_{1,r}{}^{-1}\left\{\bar g_{1,r}\left[\left(\bar g_{1,r}{}^{-1}\left(\int_X^{(+,\cdot)}(\bar g_{1,r}\circ f_1)d(\bar g_{1,r}\circ m)\right)\right)\right]+\bar g_{1,r}\left[\left(\bar g_{1,r}{}^{-1}\left(\int_X^{(+,\cdot)}(\bar g_{1,r}\circ f_2)d(\bar g_{1,r}\circ m)\right)\right)\right]\right\}=$$

$$=\bar g_{1,r}{}^{-1}\left\{\int_X^{(+,\cdot)}(\bar g_{1,r}\circ f_1)d(\bar g_{1,r}\circ m)+\int_X^{(+,\cdot)}(\bar g_{1,r}\circ f_2)d(\bar g_{1,r}\circ m)\right\}=$$

$$=\bar g_{1,r}{}^{-1}\left[\int_X^{(+,\cdot)}\left(\bar g_{1,r}\circ\left(f_1\,\overline{\oplus}_{\bar g_{1,r}}\,f_2\right)\right)d(\bar g_{1,r}\circ m)\right]=\left(L-\int_X^{(+,\cdot)}\right)(\bar g_{1,r}\circ m,f_1\,\overline{\oplus}_{\bar g_{1,r}}\,f_2-RMF)=$$

$$=\int_X^{(\overline{\oplus},\cdot)}(f_1\,\overline{\oplus}_{\bar g_{1,r}}\,f_2)\,dm=\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}\right)(m,\ f_1\,\overline{\oplus}_{\bar g_{1,r}}\,f_2-RMF).$$

4.The bi-pseudo-integral $\left(M,K-\int_A^{(\overline{\oplus}_{\bar g},\overline{\odot}_{\bar g})}\right)$ is a $\overline{\odot}_{\bar g}=(\cdot)-homogeneous\ functional\ (in\ case\ of\ \overline{\oplus}=\overline{\oplus}_{\bar g}=\overline{\oplus}_{\bar g_{1,r}}\neq\overline{\vee})$, if f – RMF, for $c\in(-\infty,+\infty)$ by theorem 1 in [7]:

$$\left(M,K-\int_X^{(\overline{\oplus}_{\bar g},\overline{\odot}_{\bar g})}\right)(m,\ c\,\overline{\odot}_{\bar g}\,f-RMF)=\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}\right)(m,\ c\cdot f-RMF)=$$

$$=\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}\right)(m,\ (c\cdot f)^+)\,\overline{\ominus}_{\bar g_{a,r}}\left(M,K-\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}\right)(m,\ (c\cdot f)^-)=$$

$$=\left(\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}(c\,\overline{\odot}_{\bar g_{1,r}}\,f)^+\,\overline{\odot}_{\bar g_{1,r}}\ dm\right)\overline{\ominus}_{\bar g_{a,r}}\left(\int_X^{(\overline{\oplus}_{\bar g_{1,r}},\overline{\odot}_{\bar g_{1,r}})}(c\,\overline{\odot}_{\bar g_{1,r}}\,f)^-\,\overline{\odot}_{\bar g_{1,r}}\ dm\right)=$$

$$=\left(\int_X^{(\overline{\oplus}_{\bar g},\overline{\odot}_{\bar g})}(c\cdot f)^+\,\overline{\odot}_{\bar g_{1,r}}\ dm\right)\overline{\ominus}_{\bar g_{a,r}}\left(\int_X^{(\overline{\oplus}_{\bar g},\overline{\odot}_{\bar g})}(c\cdot f)^-\,\overline{\odot}_{\bar g_{1,r}}\ dm\right)=$$

$$= \left(\int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)} (c \cdot f^+) \,\overline{\odot}_{\bar g_{1,r}}\ dm\right) \overline{\ominus}_{\bar g_{a,r}} \left(\int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)} (c \cdot f^-) \,\overline{\odot}_{\bar g_{1,r}}\ dm\right) =$$

$$= c \cdot \left(\int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)} f^+ \,\overline{\odot}_{\bar g_{1,r}}\ dm \,\overline{\ominus}_{\bar g_{a,r}} \int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)} f^- \,\overline{\odot}_{\bar g_{1,r}}\ dm\right) =$$

$$= c \,\overline{\odot}_{\bar g} \left(\int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)} f^+ \,\overline{\odot}_{\bar g_{1,r}}\ dm \,\overline{\ominus}_{\bar g_{a,r}} \int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)} f^- \,\overline{\odot}_{\bar g_{1,r}}\ dm\right) =$$

$$= c \,\overline{\odot}_{\bar g_{1,r}} \left(\left(M, K - \int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)}\right)(m,\ f^+) \,\overline{\ominus}_{\bar g_{a,r}} \left(M, K - \int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)}\right)(m,\ f^-)\right) =$$

$$= c \,\overline{\odot}_{\bar g_{1,r}} \left(\left(M, K - \int_X^{\left(\oplus_{\bar g_{1,r}}, \overline{\odot}_{\bar g_{1,r}}\right)}\right)(m,\ f - RMF)\right) = c \,\overline{\odot}_{\bar g} \left(\left(M, K - \int_X^{\left(\oplus_{\bar g}, \overline{\odot}_{\bar g}\right)}\right)(m,\ f - RMF)\right).$$

Using the definition of bi-pseudo-integrals presented in [10], are summarized (*in case of* $\overline{\oplus} = \overline{\oplus}_{\bar g} \neq \overline{\vee}$):

$$\left(BP_{(\bar g - TR)} - \int_{\bar g^{-1}(X)}^{\left(\oplus_{\bar g}, \overline{\odot}_{\bar g}\right)}\right)(m_{\bar g},\ f_{\bar g}) = \begin{cases} \oplus_g - functional\ on\ g^{-1}(X)\ if\ f_g - SNNMF_g \\ \oplus_g - functional\ on\ g^{-1}(X)\ if\ f_g - NNMF_g \\ \overline{\oplus}_{\bar g} - additive\ functional\ on\ \bar g^{-1}(X)\ \text{if}\ f_{\bar g} - RMF_{\bar g} \\ \overline{\odot}_{\bar g} = (\cdot) - homogeneous\ functional\ on\ \bar g^{-1}(X)\ if\ f_{\bar g} - RMF_{\bar g} \end{cases}$$

5. Conclusion

5.1. The first bi-pseudo-integral $\left(M, K - \int_X^{\left(\oplus_{\bar g}, \overline{\odot}_{\bar g}\right)}\right)(m, f)$ in case of the function $f -$ $SNNMF$; $NNMF$; RMF and relations with Lebesgue integral (depending on the generator $\bar g$) is summarized as follows

- $\left(M, K - \int_X^{\left(\oplus_{\bar g}, \overline{\odot}_{\bar g}\right)}\right)(m, f - SNNMF;\ NNMF; RMF) =$

$$= \begin{cases} \begin{cases} g_{a,r}{}^{-1}\left(\frac{1}{a}\right) \odot_{g_{a,r}} \left(g_{a,r}{}^{-1}\left(\left(L - \int_X^{(+,\cdot)}\right)(g_{a,r} \circ m, g_{a,r} \circ f - NNMF)\right)\right) for\ r \geq 1, a > 0 \\ a^{-2/r} \odot_{g_{a,r}} \left(g_{a,r}{}^{-1}\left(\left(L - \int_X^{(+,\cdot)}\right)(g_{1,r} \circ m, g_{1,r} \circ s - SNNMF) \quad for\ r \geq 1, a = 1\right)\right) \\ g_{1,r}{}^{-1}\left(\left(L - \int_X^{(+,\cdot)}\right)(g_{1,r} \circ m, g_{1,r} \circ s - SNNMF)\right) \quad for\ r \geq 1, a = 1 \end{cases} \\ \begin{cases} \bar g_{a,r}{}^{-1}\left(a^{-2/r} \odot_{g_{a,r}} \left(L - \int_X^{(+,\cdot)}\right)(\bar g_{a,r} \circ m, \bar g_{a,r} \circ f - RMF)\right) \quad for\ r \geq 1, a > 0 \\ \bar g_{a,r}{}^{-1}\left(a^{-2/r} \odot_{g_{a,r}} \left(L - \int_X^{(+,\cdot)}\right)(\bar g_{1,r} \circ m, \bar g_{1,r} \circ f - RMF) \quad for\ r \geq 1, a = 1\right) \\ \bar g_{1,r}{}^{-1}\left(\left(L - \int_X^{(+,\cdot)}\right)(\bar g_{1,r} \circ m, \bar g_{1,r} \circ f - RMF)\right) \quad for\ r \geq 1, a = 1 \end{cases} \end{cases}$$

5.2. The first Bi-Pseudo-Integral $\left(M, K - \int_X^{\left(\oplus_{\bar g}, \overline{\odot}_{\bar g}\right)}\right)(m, f)$ in case of the function $f - SNNMF$; $NNMF$; RMF and the generalizations depending on the generator $\bar g$, is summarized as follows

- $\left(\mathrm{M}, \mathrm{K} - \int_X^{\left(\oplus_{\bar g}, \overline{\odot}_{\bar g}\right)}\right)(m, f - SNNMF;\ NNMF; RMF) =$

$$= \begin{cases} \begin{cases} \left(M - \int_X^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m, s - SNNMF)\ for\ r \geq 1, a > 0 \\ a^{-2/r} \odot_{g_{a,r}} \left({g_{a,r}}^{-1}\left(\left(M - \int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m, s - SNNMF)\right)\ for\ r \geq 1, a = 1\right) \\ \left(M - \int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m, s - SNNMF)\ for\ r \geq 1, a = 1 \end{cases} \\ \begin{cases} \left(M - \int_X^{(\oplus_{g_{a,r}},\odot_{g_{a,r}})}\right)(m, f - NNMF)\ for\ r \geq 1, a > 0 \\ a^{-2/r} \odot_{g_{a,r}} \left({g_{a,r}}^{-1}\left(\left(M - \int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m, f - NNMF)\right)\ for\ r \geq 1, a = 1\right) \\ \left(M - \int_X^{(\oplus_{g_{1,r}},\cdot)}\right)(m, f - NNMF)\ for\ r \geq 1, a = 1 \end{cases} \\ \begin{cases} \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{a,r}},\overline{\odot}_{\bar{g}_{a,r}})}\right)(m, f - RMF)\ for\ r \geq 1, a > 0 \\ a^{-2/r} \odot_{\bar{g}_{a,r}} \left({\bar{g}_{a,r}}^{-1}\left(\left(M - \int_X^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m, f - RMF)\right)\ for\ r \geq 1, a = 1\right) \\ \left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}_{1,r}},\cdot)}\right)(m, f - RMF)\ for\ r \geq 1, a = 1 \end{cases} \end{cases}$$

5.3. The first bi-pseudo-integral (with properties of pseudo-linearity) on the reconstruction of pseudo-additive measures when $f - RMF$:

- $\nu_f(A) = \begin{cases} \sigma - \overline{\oplus}_{\bar{g}} - \ measure\ on\ A\ if\ f - RMF \\ \overline{\oplus}_{\bar{g}} = (\cdot) - homogeneous\ set\ function\ on\ A\ if\ f - RMF \end{cases}$
- $\left(M, K - \int_X^{(\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}})}\right)(m, f - RMF) = \begin{cases} \overline{\oplus}_{\bar{g}} - additive\ functional\ on\ X \\ \overline{\odot}_{\bar{g}} = (\cdot) - homogeneous\ functional\ on\ X \end{cases}$

5.4. The bi-pseudo-integral (with properties of the pseudo-linearity) on the reconstruction of pseudo-additive measures when $f_{\bar{g}} - RMF_{\bar{g}}$:

- $\nu_{f_{\bar{g}}}(\bar{g}^{-1}(A)) = \begin{cases} \sigma - \overline{\oplus}_{\bar{g}} - \ measure\ on\ \bar{g}^{-1}(A)\ if\ f_{\bar{g}} - RMF_{\bar{g}} \\ \overline{\oplus}_{\bar{g}} = (\cdot) - homogeneous\ set\ function\ on\ \bar{g}^{-1}(A)\ if\ f_{\bar{g}} - RMF_{\bar{g}} \end{cases}$
- $\left(BP_{(\bar{g}-TR)} - \int_{\bar{g}^{-1}(X)}^{(\overline{\oplus}_{\bar{g}},\overline{\odot}_{\bar{g}})}\right)(m, f_{\bar{g}} - RMF_g) = \begin{cases} \overline{\oplus}_{\bar{g}} - additive\ functional\ on\ \bar{g}^{-1}(X) \\ \overline{\odot}_{\bar{g}} = (\cdot) - homogeneous\ functional\ on\ \bar{g}^{-1}(X)\ . \end{cases}$

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