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Research paper

Pinwheel tiling fractal graph- a notion to edge cordial and cordial labeling

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Abstract

A fractal is a complex geometric figure that continues to display self-similarity when viewed on all scales. Tile substitution is the process of repeatedly subdividing shapes according to certain rules. These rules are also sometimes referred to as inflation and deflation rule. One notable example of a substitution tiling is the so-called Pinwheel tiling of the plane. Many examples of self-similar tiling are made of fractiles: tiles with fractal boundaries. . The pinwheel tiling was the first example of this sort. There are many as such as family of tiling fractal curves, but for my study, I have considered this Pinwheel and its two intriguing Pinwheel properties of tiling fractals. These fractals have been considered as a graph and the same has been viewed under the scope of cordial and edge cordial labeling to apply this concept for further study in Engineering and science applications.

Keywords: *Pinwheel Tiling; Fractals; Graph Labeling; Cordial and Edge Cordial.*

1. Introduction

Since a fractal curve is made of smaller copies of itself, it logically follows that a plane-filling fractal curve is a filled-in shape that is made of smaller filled-in shapes - identical to itself. This means that plane-filling fractal curves are tiling. Tile substitution is the process of subdividing shapes repeatedly according to certain rules. These rules are also sometimes referred to as inflation and deflation rules. With an infinite number of substitution steps, one can create a tiling that covers the entire plane. This is particularly interesting because tile substitution can also result in non-periodic tiling. One notable example of a substitution tiling is the so-called Pinwheel tiling of the plane. It is based on a construction by John Conway and described in detail in a thesis by Charlen Radin [3]. It shows how a substitution system can produce the Pinwheel tiling. One notable property of this tiling is that it consists of only copies of the same triangle, but in infinitely many rotations. Thus, it is a non-periodic tiling Simon Parzer [16]. Its construction and properties are discussed by many of researchers such as Natalie and Michael [12]. In this paper, our study is to enable the Cordial, total cordial, Edge cordial and total edge cordial labeling for the above mentioned tiling fractal curves. So in this study, the above mentioned tiling fractal curves have been considered as a Graph with number of vertices and edges. The construction of this graph is also part of the study of properties of the curve.
For our study, we have to make use of many definitions like graph, cordial labeling, edge cordial labeling, total cordial labeling and harmonious labeling etc. All these definitions are predefined by well-known researcher like Cahit I [2], Gallian, J. A [5], Harary, F [6].
Furthermore, those definitions which are redefined by Sathakathulla, [15] need to be adopted for a clear understanding of this paper. The present work is focused on cordial and edge cordial labeling of the above mentioned curves at every iteration.

2. Main results

The construction and its properties are already well investigated by many scholars. Here, we provide the Pattern of the pinwheel tiling fractal for the need of further study.

2.1. Cordial labeling for pinwheel tile fractal graph

Cordial labeling is the pattern of labeling graph vertices with 0's and 1's to hold the definition as described above. Here, the pattern of labeling of vertices is easily understandable by seeing the figure (fig. 2.2). It is clearly observed that all the edges having label 0's are denoted by a tick mark (√) and correspondingly all edges with 1's are denoted by a cross mark(x) for easy understanding. Hence it satisfies the existence of cordial labeling. In each iteration, the similar fashion of labeling is being tried to extend and is followed to check the existence of cordial labeling and total cordial labeling. Regarding the number of edges and vertices in each iteration the pattern of labeling is adjusted to hold good to the same fashion and to adhere the definition of cordial labeling. Therefore, the above fractal graph holds good for both cordial and total cordial labeling. The iteration wise number of edges and vertices are provided in the table 2.1. Every iteration values holds well for the previously mentioned definitions.

2.2. Edge cordial labeling for pinwheel tile fractal graph

Similar to the previous, the pattern of labeling of edges is easily understandable by seeing the below figure (Fig 2.3). This fractal graph is labeled with zeros and ones at the edges, by a tick mark (√), a cross mark(x) at vertices to distinguish. Furthermore, the same pattern has been tried to maintain in every iteration to fulfill the definition. Based on the number of increased edges and vertices, the pattern of labeling being adjusted to hold the same fashion as well to hold good for the definition of edge cordial labeling. The definition of edge cordial labeling is satisfying at every iteration. In addition to that, it also satisfies Total edge cordial labeling conditions. The results provided in the following table 2.1 hold well for all iterations.

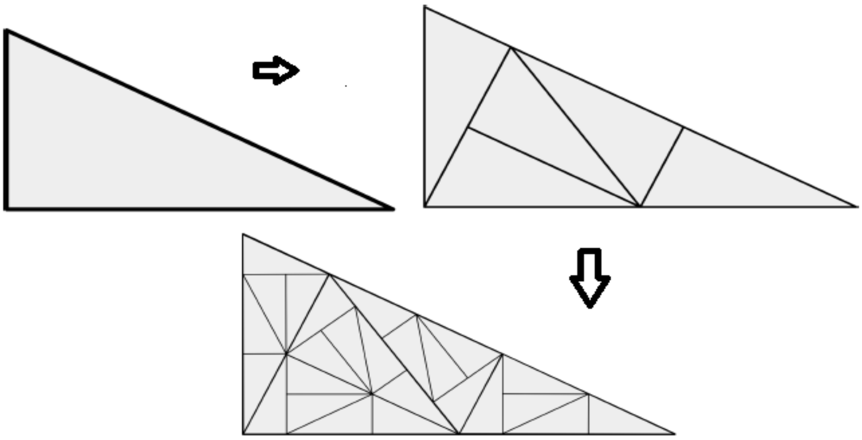

Fig .2.1: Construction of Pinwheel Tiling Fractal Graph by Inflate-and-Subdivide Rule.

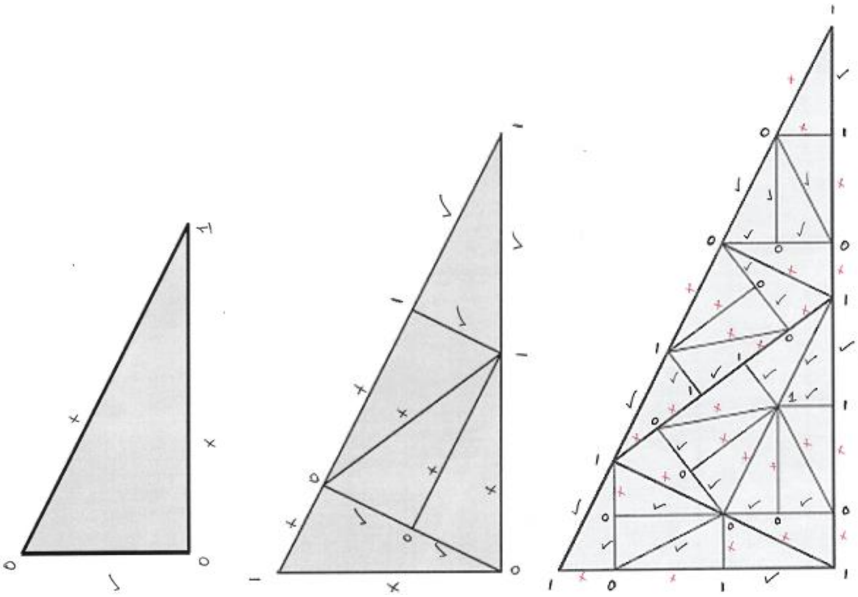

Fig. 2.2: Cordial Labeling of Pinwheel Tiling Fractal Graph.

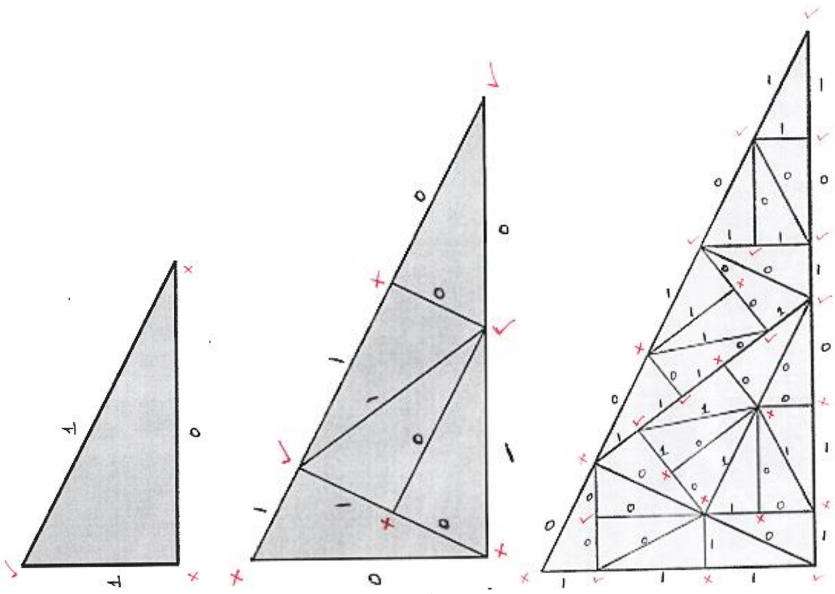

Fig. 2.3: Edge Cordial Labeling of Pinwheel Tiling Fractal Graph.

Table 2.1: Cordial/Edge Cordial Labeling

Iterations	No. of tiles	No. of Vertices	No. of Edges
1	1	|v| = 3 and |vf (0)| = 2 |vf(1)| =1	|e| = 3 and |ef (0)|= 1 |ef (1)|= 2
2	5	|v| = 7 and |vf (0)| =3 |vf(1)| =4	|e| = 11 and |ef (0)|= 5 |ef (1)|= 6
3	5^2	|v| = 25 and |vf (0)| =12 |vf(1)| =13	|e| = 49 and |ef (0)|= 24 |ef (1)|=25
4	5^3	|v| = 103 and |vf (0)| = 51 |vf(1)| = 52	|e| = 227 and |ef (0)|= 113 |ef (1)|= 114
5	5^4	|v| = 471 and |vf (0)| =235 |vf(1)| = 236	|e| = 1095 and |ef (0)|= 547 |ef (1)|=548
n	5^{n-1}	|v| = |e|- $(5^{n-1} - 1)$ and |vf (0)| = $\lfloor \|v\| /2 \rfloor$ |vf(1)|= $\lfloor \|v\| /2 \rfloor$ +1	By induction method |e| = 5|e_{n-1}|- $(R_{n-2}) + 10(n-3) + 2)$ Where |e_{n-1}| is number of edges in previous iteration and R_{n-2} is residue quotient from previous iteration. |ef (0)|= $\lfloor \|e\| /2 \rfloor$ |ef (1)|= $\lfloor \|e\| /2 \rfloor$ +1

2.3. Two intriguing pinwheel properties

Like many tiling spaces generated from inflate-and-subdivide rules, the pinwheel space has a sort of homogeneity. The aim of this work is to introduce a tiling substitution on fractal tiles that produces tiling that is locally derivable from the pinwheel tiling mutually. But first, following M. Baake, D. Frettlo¨ h, and U. Grimm, [10], Natalie and Michael [12] researchers have introduced a tiling substitution called the ‘‘kite-domino’’ pinwheel tiling. The pinwheel triangles in any pinwheel tiling meet up hypotenuse-to-hypotenuse to form either a kite or a domino.

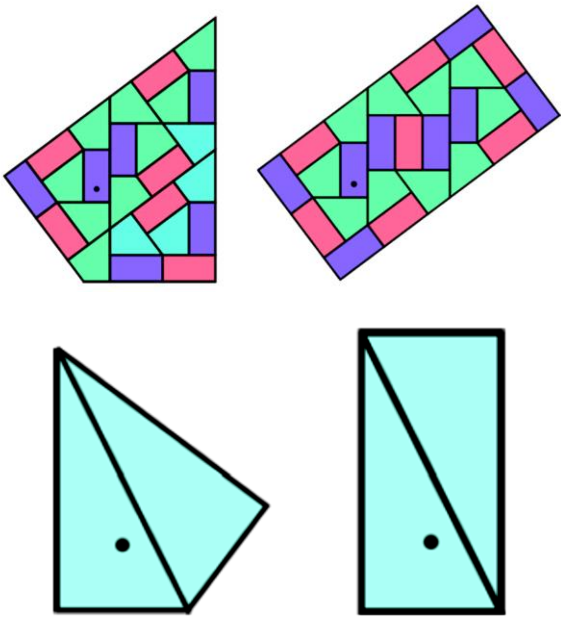

Fig. 2.4: The Kite and Domino.

2.4. Cordial and edge cordial labeling of kite and domino

This special property of pinwheel tile fractal graph is also tested with both cordial and edge cordial labeling as described before for each iteration and it holds well for the definitions of both. Further, it also suits for the total cordial and total edge cordial labeling. The pattern of the labeling of cordial and edge cordial can easily be understood from the given figures. To distinguish the labeling for vertices and edges, different notations like 0’s and 1’s followed by a tick mark (√), a cross mark(x) are used. The labeling fashion is described by the given example figures for cordial and edge cordial separately.

The kite-domino tiling are mutually locally derivable to the pinwheel tiling. The two prototiles are made of two pinwheel triangles, glued together at their long edge. There are two ways to do so; one gives a kite (a quadrilateral with edge lengths 1, 1, 2, 2) and a domino (a rectangle with edge lengths 1, 2, 1, 2). Then the substitution rule is obtained by considering two steps of the pinwheel substitution as one step. Thus, the inflation factor of the kite-domino tiling is five. These are the substitution rules which generate the Kite-Domino. As we can see, the algorithm is simple enough to be summarized in a single image. Any place which has a Kite prototile, replaces it with the designated configuration of kite and domino prototiles. Likewise, the domino prototile has its own substitution rule. After several iterations of substitution, it can turn a single domino into an extremely complicated pattern (Robin Wilson, [14]).

Hence, the labeling fashion for cordial and edge cordial is clearly depicted from the below figures. Further, it satisfies the definition of cordial and edge cordial well. Additionally it holds well for total cordial and total edge cordial too. Therefore, it is believed that these conditions will prevail in all its iteration to hold well the definition. The number of edges and vertices for every iteration which holds good for the mentioned definitions.

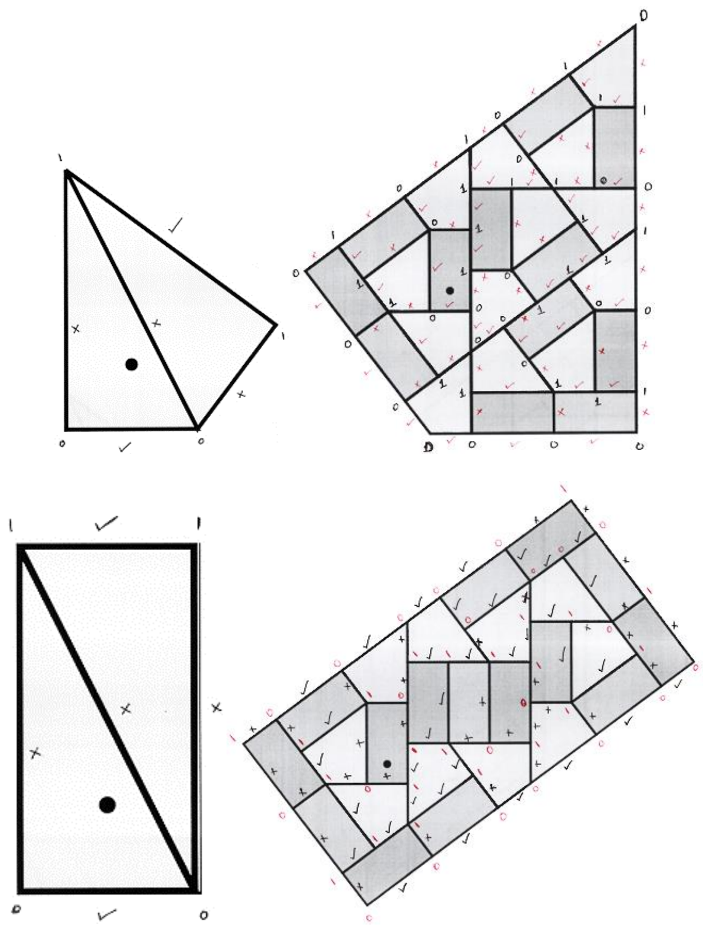

Fig. 2.5: Cordial Labeling of Kite and Domino.

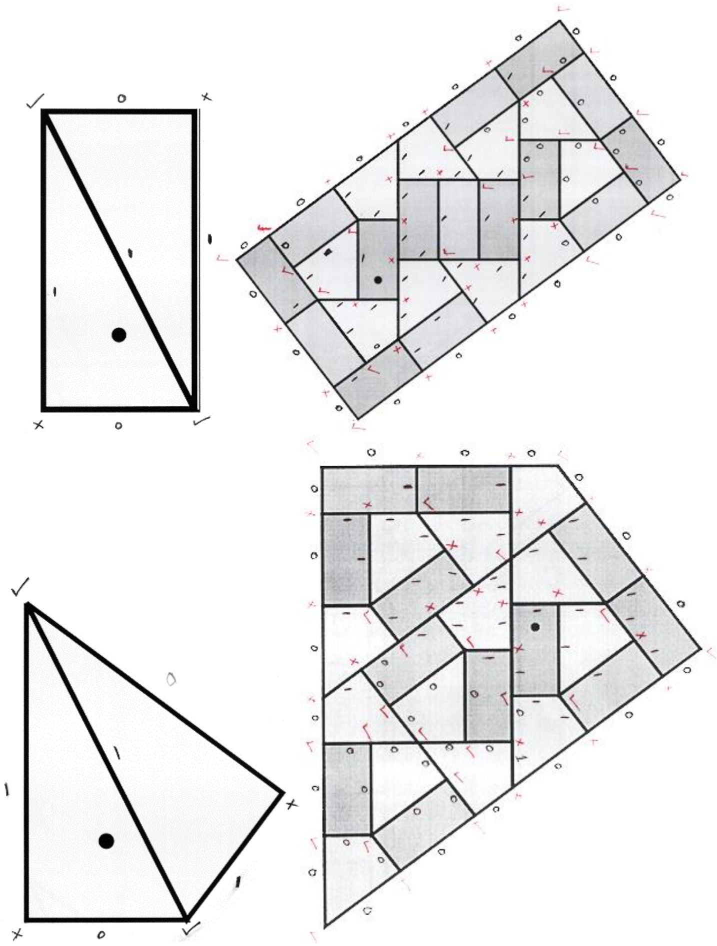

Fig. 2.6: Edge Cordial Labeling of Kite and Domino.

3. Conclusion

In this paper, Pinwheel tiling fractal is considered for our study. For instance, the core pinwheel tiling and its two intriguing properties which are called kite and domino have been viewed in the form of a fractal graph with the concept of labeling, particularly cordial, edge cordial, and total cordial, and total edge cordial concepts. All the examples on each one of tiling is held good for the study of cordial and edge cordial labeling. It seems that all this kind of this tiling graphs are satisfying the conditions of cordial and edge cordial. Some of the iteration form pinwheel and kite and domino are proven with the cordial and edge cordial to with figures provided to authenticate that it holds well to the definitions. It is strongly believed that this view of labeling will lead to further study of this pinwheel tiling fractal graph in to other types of labeling in graph theory and emerging application of this tiling graph curve in the field of Engineering and science. The existence of the above mentioned labeling is proved and the results are provided in details in tables. Hence, it is concluded that the pinwheel tiling fractal graph is cordial, total cordial, edge cordial and total edge cordial.

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