*International Journal of Applied Mathematical Research*, *6 (3) (2017) 67-73*

**International Journal of Applied Mathematical Research**

*Website: www.sciencepubco.com/index.php/IJAMR*
*doi: 10.14419/ijamr.v6i3.7813*
***Research paper***

# Markov chain approach to availability and profit evaluation of a series-parallel system with minor and major failures

**Saminu I. Bala [1], Ibrahim Yusuf [1] *, Yasin Ali Nuhu [2]**

[1] *Department of Mathematical Sciences, Bayero University, Kano, Nigeria*
[2] *Audu Bako College of Agriculture, Dambatta, Kano, Nigeria*
**Corresponding author E-mail: iyusuf.mth@buk.edu.ng*

**Abstract**

The paper deals with modeling and performance evaluation of a series-parallel with independent failures using Markov Birth-Death process and probabilistic approach. The system consists of three subsystems arranged in series and parallel configurations with three possible states, working, reduced capacity and failed. Through the transition diagram, systems of differential equations are developed and solved recursively via probabilistic approach. Failure and repair rates for all the subsystems are assumed constant. Availability and profit matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated. The present study will help the plant management in understanding the optimum system availability and profit to be achieved, and the maintenance efforts needed to maintain an appropriate level of availability and profit and useful for timely execution of proper maintenance improvement, decision, planning and optimization.

***Keywords***: *Availability; Optimization; Profit; Markov Chain.*

## 1. Introduction

The importance in promoting, sustaining industries, manufacturing systems and economy through reliability measurement has become an area of interest. The reliability, availability and profit are the most important factors in any successful industries and manufacturing settings. Profit of system may be enhancing using highly reliable structural design into the system or subsystem of higher reliability. Improving the reliability and availability of system, the production and associated profit will also increase. Availability and profit of an industrial system may be enhancing using highly reliable structural design into the system or subsystem of higher reliability. Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. Improving the reliability and availability of system/subsystem, the production and associated profit will also increase. Increase in production lead to the increase of profit. This can be achieved to be maintaining reliability and availability at the highest order. To achieve high production and profit, the system should remain operative for maximum possible duration.

Large volume of literature exists on the issue of predicting performance evaluation of various systems. Kumar et al [1] discussed the reliability analysis of the Feeding system in the paper industry, Kumar el al.[2] discussed the availability analysis of the washing system in the paper industry, Kumar el al. [3] deal with reliability, availability and operational behavior analysis for different systems in paper plant. Kumar el al. [4] discussed the behavior analysis of Urea decomposition in the fertilizer industry under the general repair policy. Kumar et al.[5] studied the design and cost analysis of a refining system in a Sugar industry. Srinath [6] has explained a Markov model to determine the availability expression for a simple system consisting of only one component. Gupta el al. [7] has evaluated the reliability parameters of butter manufacturing system in a diary plant considering exponentially distributed failure rates of various components. Gupta et al. [8] studied the behavior of Cement manufacturing plant. Arora and Kumar [9] studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg [10] perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates.

In the present paper, we study a series-parallel system consisting of three different subsystems arranged in series. Through the transition diagram obtained in this study, systems of differential equations are developed and solved recursively via probabilistic approach. Availability and profit matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated.

## 2. System description

The System consists of three dissimilar subsystems arranged in series-parallel as follows:

1) Subsystem A: Consists of two active parallel units. Failure of one unit, the system to work in reduced capacity. Complete failure occurs when both units failed.
2) Subsystem B: Consists of three active parallel units. Failure of one unit, the system to work in reduced capacity. Complete failure occurs when both units failed.
3) Subsystem C: A single unit in series whose failure causes complete failure of the entire system.

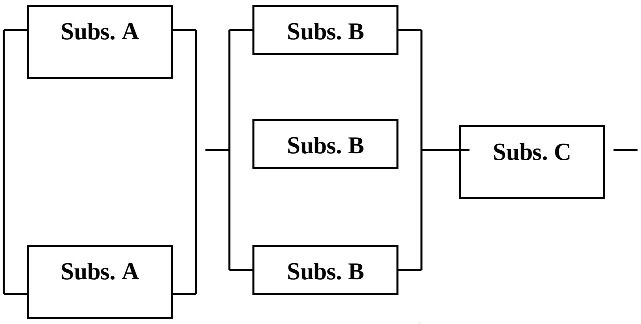


**Fig. 1:** Reliability Block Diagram of the System.

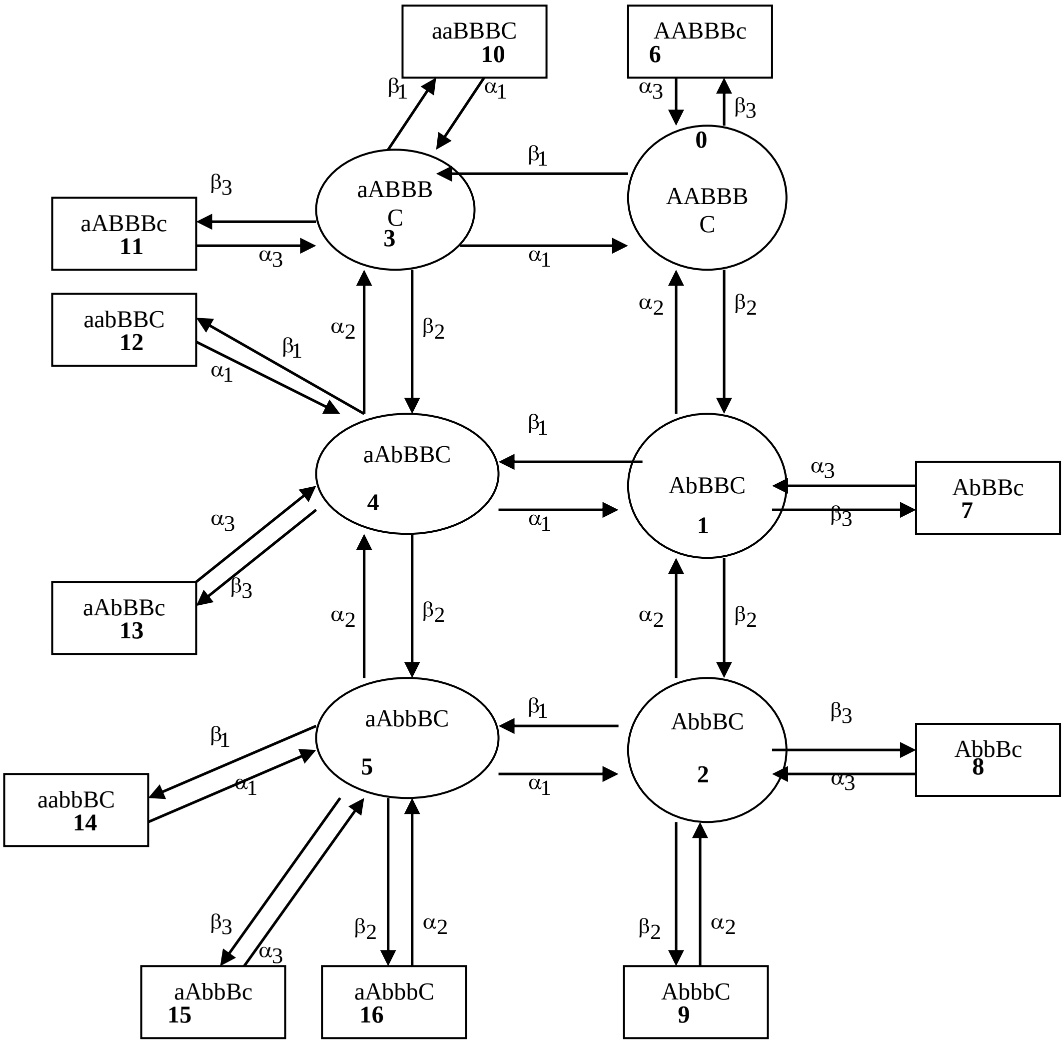


**Fig. 2:** Transition Diagram of the System.

a) Assumptions and Notations

The assumptions used in model development are as follows:

1) Failure and repair rates are constant over time and are statistically independent
2) At any given time, the system is either in operating state, reduced capacity or in failed state.
3) System failure/repair follows exponential distribution
4) System work in a reduced capacity
5) Repair is as good as new
6) Subsystems/units do not fail simultaneously
7) The system is exposed to two types of failures minor and major failure.
8) Minor failure forces the system to work in reduced capacity states (there is no system failure) whereas major bring about system failure.
9) The system is attended by two repairmen.

Indicate the system is in full working state
Indicate the system is in failed state
Indicate the system in reduced capacity state
A, B, C represent full working state of subsystem
a, b, c represent failed state of subsystem
$\beta_1, \beta_2, \beta_3$ Represent failure rates of subsystems A, B, C
$\alpha_1, \alpha_2, \alpha_3$ : represent repair rates of subsystems A, B, C
$P_i(t),\ i = 0,1,2,...,16$ : Probability that the system is in state $S_i$ at time t
$P_0(t)$ : Probability of the system working in full capacity at time $t$
$P_0(t) - P_5(t)$ : Probability of the system working at time $t$
$P_1(t) - P_5(t)$ : Probability of the system working in reduced capacity at time $t$

$P_6(t)-P_{16}(t)$ : Probability of the system in failed state at time $t$

$B_{P1}(\infty)$ : Steady state busy period of repairman due to minor failure

$B_{P2}(\infty)$ : Steady state busy period of repairman due to major failure

$C_0$ : Total revenue generated from system using

$C_1$ : Cost incurred due to minor failure

$C_2$ : Cost incurred due to major failure

$A_V(\infty)$ : Steady state availability of the system

$P_F(\infty)$ : Profit

## 3. Model formulations

The following differential equations associated with the transition diagram (Fig. 2) of the system are formed using Markov birth-death process:

$$\left(\frac{d}{dt}+\sum_{i=1}^{3}\beta_i\right)P_0(t)=\alpha_2P_1(t)+\alpha_1P_3(t)+\alpha_3P_6(t) \quad (1)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{3}\beta_i+\alpha_2\right)P_1(t)=\beta_2P_0(t)+\alpha_2P_2(t)+\alpha_1P_4(t)+\alpha_3P_7(t) \quad (2)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{3}\beta_i+\alpha_2\right)P_2(t)=\beta_2P_1(t)+\alpha_1P_5(t)+\alpha_3P_8(t)+\alpha_2P_9(t) \quad (3)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{3}\beta_i+\alpha_1\right)P_3(t)=\beta_1P_0(t)+\alpha_2P_4(t)+\alpha_1P_{10}(t)+\alpha_3P_{11}(t) \quad (4)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{3}\beta_i+\alpha_1+\alpha_2\right)P_4(t)=\beta_1P_1(t)+\beta_2P_3(t)+\alpha_2P_5(t)+\alpha_1P_{12}(t)+\alpha_3P_{13}(t) \quad (5)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{3}\beta_i+\alpha_1+\alpha_2\right)P_5(t)=\beta_3P_2(t)+\beta_2P_4(t)+\alpha_1P_{14}(t)+\alpha_3P_{15}(t)+\alpha_2P_{16}(t) \quad (6)$$

$$\left(\frac{d}{dt}+\alpha_m\right)P_i(t)=\beta_mP_j(t) \quad (7)$$

$m=1,2,3$ , $i=6,7,8,...,16$ , $j=0,1,2,3,4,5$

With initial conditions

$$P_i(t)=\begin{cases}1 & i=0\\ 0 & i>0\end{cases} \quad (8)$$

The steady state availability of the system is obtained by putting $\frac{d}{dt}=0$ as $t\to\infty$ in (1) to (7), the steady state probabilities are given in the table below by solving (1) to (7)

**Table 1:** Computed Steady States Probabilities

| | |
|---|---|
| $P_1=X_2P_0$ | $P_9=X_2^3P_0$ |
| $P_2=X_2^2P_0$ | $P_{10}=X_1^2P_0$ |
| $P_3=X_1P_0$ | $P_{11}=X_1X_3P_0$ |
| $P_4=X_1X_2P_0$ | $P_{12}=X_1^2X_2P_0$ |
| $P_5=X_1X_2^2P_0$ | $P_{13}=X_1X_2X_3P_0$ |
| $P_6=X_3P_0$ | $P_{14}=X_1^2X_2^2P_0$ |
| $P_7=X_2X_3P_0$ | $P_{15}=X_1X_3X_2^2P_0$ |
| $P_8=X_3X_2^2P_0$ | $P_{16}=X_1X_2^3P_0$ |

Where

$X_1=\frac{\beta_1}{\alpha_1}$ , $X_2=\frac{\beta_2}{\alpha_2}$ , $X_3=\frac{\beta_3}{\alpha_3}$ .

The probability of full working state, namely, $P_0(t)$ is determined by using normalizing condition i.e. sum of the probabilities of all working states, reduced capacity and failed state is equal to 1.

$P_0$ (The probability of full working state) is determine using the condition (normalizing):

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+...+P_{16}(\infty)=1 \quad (9)$$

$$P_0D_0=1 \quad (10)$$

$$P_0(\infty)=\frac{1}{D_0} \quad (11)$$

The steady state availability $A_V(\infty)$ is summation of probabilities of working and reduced capacity states probabilities and busy period $B_{Pi}(\infty)$, $i=1,2$ . Thus

$$A_V(\infty)=\sum_{k=0}^{5}P_k(\infty)=\frac{N_0}{D_0} \quad (12)$$

The steady state availability in (12) represents the performance model of system which is used for performance analysis. It is also utilized for the performance optimization of the system.

The steady state busy period $B_{P1}(\infty)$ of repairman due to minor failure is the summation of state probabilities of reduced capacity states .Thus,

$$B_{P1}(\infty)=\sum_{k=1}^{5}P_k(\infty)=\frac{N_1}{D_0} \quad (13)$$

The steady state busy period $B_{P2}(\infty)$ of repairman due to major failure is the summation of state probabilities of failed states .Thus,

$$B_{P2}(\infty)=\sum_{k=6}^{16}P_k(\infty)=\frac{N_2}{D_0} \quad (14)$$

Profit=total revenue generated – cost incurred by repairman due to minor failure – cost incurred by repairman due to major failure. Thus,

$$P_F(\infty)=C_0A_V(\infty)-C_1B_{P1}(\infty)-C_2B_{P2}(\infty) \quad (15)$$

Where

$$N_0 = (1+X_1)\left(1+X_2+X_2^2\right)$$

$$N_1 = X_1 + X_2 + X_2^2 + X_1X_2 + X_1X_2^2$$

$$N_2 = \left(1+X_2+X_2^2\right)\left(X_1^2+X_3+X_1X_3\right)+X_2^3(1+X_1)$$

$$D_0 = \begin{pmatrix} \left(1+X_2+X_2^2\right)\left(1+X_1+X_1^2+X_3\right)+ \\ X_3(1+X_1+X_1X_2)+X_2^3(1+X_1) \end{pmatrix}$$

## 4. Results and discussions

In this section, we numerically obtained the results for availability and profit for the developed models. For Table 2 the following set of parameter values were fixed:

$C_0 = 100,000$ ,

$C_1 = 10,000$ , $C_2 = 15,000$ , $\alpha_2 = 0.1$ , $\beta_2 = 0.005$ ,

$\alpha_3 = 0.5$ , $\beta_3 = 0.001$

Table2 and Fig 3a and Fig 3b: shows the effect of failure and repair rates of subsystem A on the Availability and Profit simultaneously. It is obvious that for some values fo failure/repair rates of the remaining two subsystems, as failure rate of subsystem A increases from 0.004 to 0.007 , the Availability and Profit of the subsystem is decreasing by 0.02 percentage and 627 respectively. Similarly as repair rate increases from 0.35 to 0.5 the Availability remain constant but Profit increases by 37.

For Table 3 the following set of parameter values were fixed:

$C_0 = 100,000$ , $C_1 = 10,000$

$C_2 = 15,000$ , $\alpha_1 = 0.4$ , $\beta_1 = 0.005$ , $\alpha_3 = 0.5$ , $\beta_3 = 0.001$

Table 3 and Fig 4a and Fig 42bshows the effect of failure and repair rates of subsystem B on the Availability and Profit. It is obvious that for some values of failure / repair rates of the remaining two subsystems, as failure rate of subsystem increases from 0.004 to 0.007 the Availability and Profit decreases by 0.18 percentage and 773 respectively. Similarly s repair rate increases from 0.05 to 0.2, the Availability and Profit increase by 0.03 percentage and 627 reaspectively.

For Table 4 the following set of parameter values were fixed:

$C_0 = 100,000$ , $C_1 = 10,000$ ,

$C_2 = 15,000$ , $\alpha_1 = 0.4$ , $\beta_1 = 0.005$ , $\alpha_2 = 0.1$ , $\beta_2 = 0.005$

Table 4 and Fig 5a and Fig 5b shows the effect of failure and repair rates of subsystem C on the Availability and Profit. Likewise it is obvious that for some values of failure / repair rates of the remaining two subsystems, as Failure rate increases from 0.0005 to 0.002, the Availability and Profit decreases by 0.64 percentage. Similarly as repair rate increases from 0.45 to 0.6, the Availability and Profit of the subsystem increase by 0.05 percentage and 57 respectively.

From above we can easily see that failure rate is inversely proportional to Availability & Profit. Similarly repair rate is Directly Proportional to Availability & Profit.

**Table 2:** Availability and Profit Matrix for Subsystem A

| $\alpha_1$ $\beta_1$ | Availability | | | | Profit | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.35 | 0.4 | 0.45 | 0.5 | 0.35 | 0.4 | 0.45 | 0.5 |
| 0.004 | 0.9305 | 0.9305 | 0.9305 | 0.9305 | 98952 | 98968 | 98981 | 98990 |
| 0.005 | 0.9304 | 0.9305 | 0.9305 | 0.9305 | 98918 | 98939 | 98955 | 98968 |
| 0.006 | 0.9304 | 0.9304 | 0.9305 | 0.9305 | 98882 | 98909 | 98929 | 98945 |
| 0.007 | 0.9303 | 0.9304 | 0.9304 | 0.9305 | 98845 | 98878 | 98902 | 98921 |

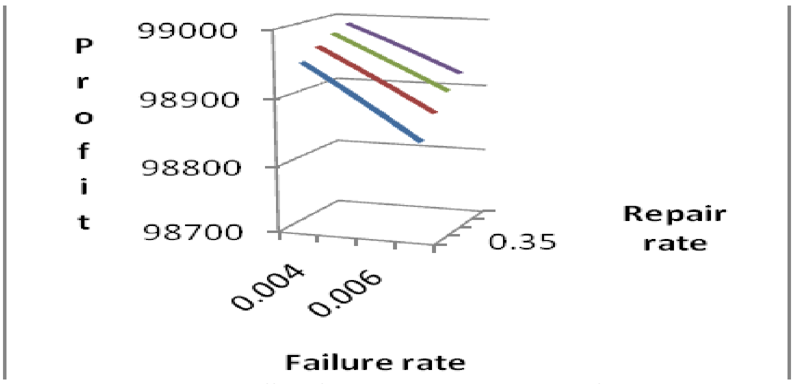


**Fig. 3a:** Effect of Failure and Repair Rate on Profit.

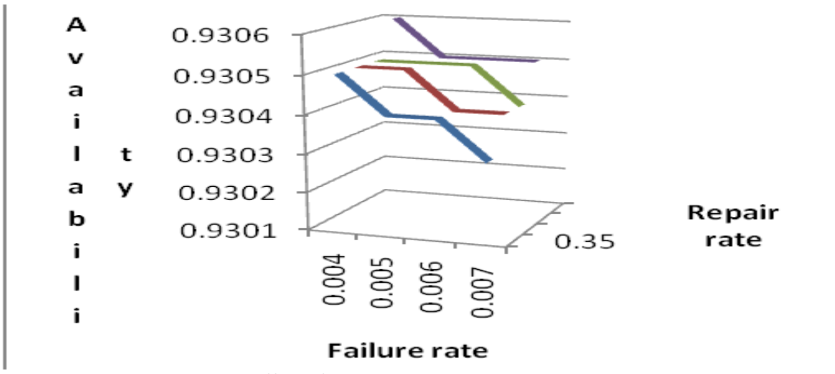


**Fig. 3 b:** Effect of Failure and Repair Rate on Availability.

| $\alpha_2$ $\beta_2$ | Availability | | | | Profit | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.05 | 0.1 | 0.15 | 0.2 | 0.05 | 0.1 | 0.15 | 0.2 |
| 0.004 | 0.9956 | 0.9959 | 0.9959 | 0.9959 | 98613 | 99042 | 99175 | 99240 |
| 0.005 | 0.9952 | 0.9959 | 0.9959 | 0.9959 | 98376 | 98939 | 99109 | 99191 |
| 0.006 | 0.9946 | 0.9958 | 0.9959 | 0.9959 | 98120 | 98834 | 99042 | 99142 |
| 0.007 | 0.9938 | 0.9957 | 0.9959 | 0.9959 | 97840 | 98726 | 98974 | 99092 |

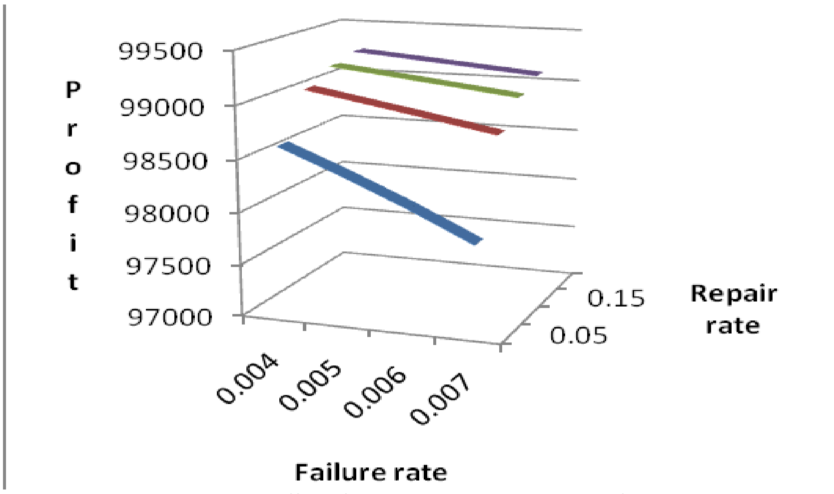


**Fig. 4a:** Effect of Failure and Repair Rate on Profit.

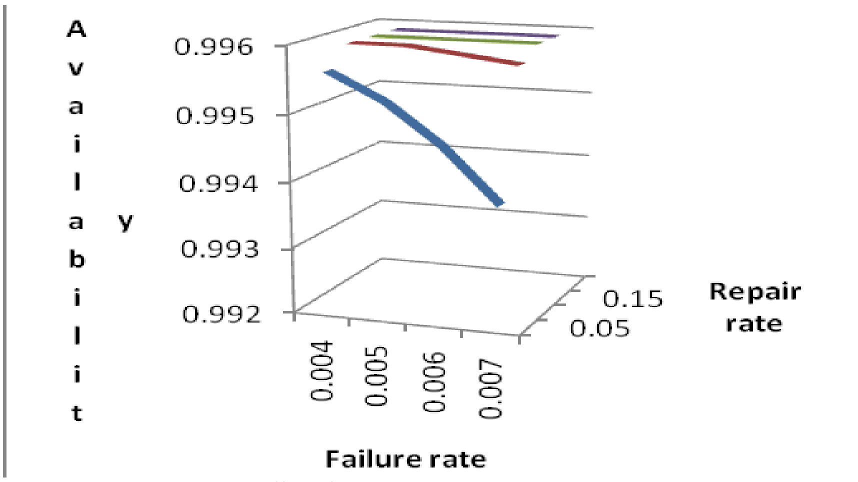


**Fig. 4b:** Effect of Failure and Repair Rate on Availability.

| $\alpha_3$ $\beta_3$ | Availability | | | | Profit | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.45 | 0.5 | 0.55 | 0.6 | 0.45 | 0.5 | 0.55 | 0.6 |
| 0.0005 | 0.9976 | 0.9978 | 0.9980 | 0.9981 | 99123 | 99146 | 99164 | 99180 |
| 0.001 | 0.9954 | 0.9959 | 0.9962 | 0.9965 | 98893 | 98939 | 98977 | 99008 |
| 0.0015 | 0.9933 | 0.9939 | 0.9945 | 0.9949 | 98665 | 98734 | 98790 | 98836 |
| 0.002 | 0.9912 | 0.9920 | 0.9927 | 0.9933 | 98438 | 98529 | 98603 | 98665 |

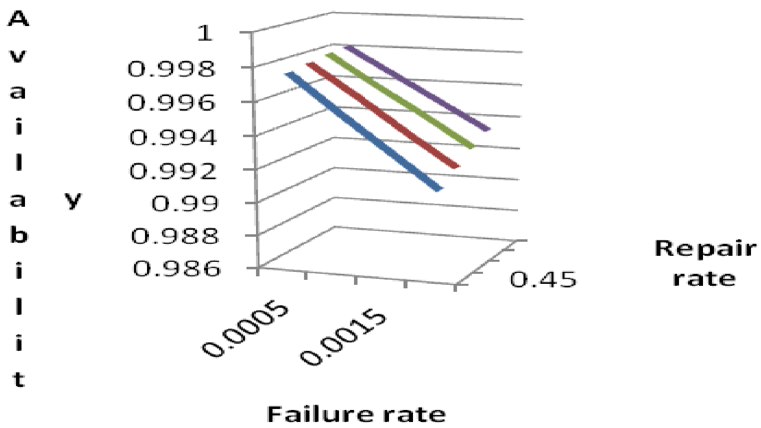


**Fig. 5a:** Effect of Failure and Repair Rate on Profit.

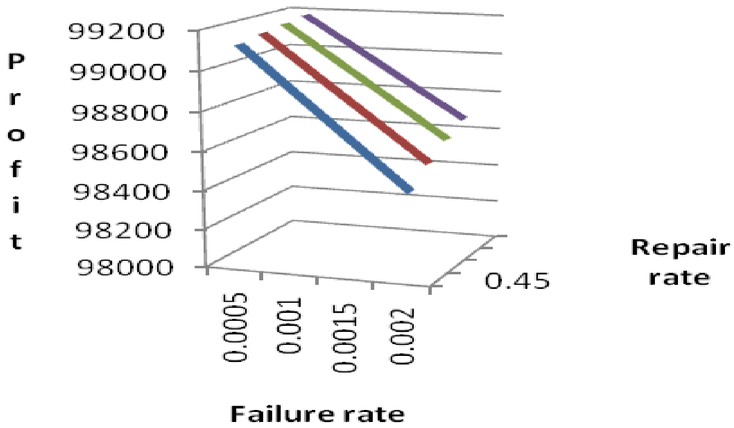


**Fig. 5b:** Effect of Failure and Repair Rate on Availability.

**Table 5:** Optimal Values of Failure/Repair Rates of Subsystems Availability and Profit

| S/N | Subsystem | Failure | Repair | Maximum level Availability | Profit |
|---|---|---|---|---|---|
| 1 | A | 0.04 | 0.5 | 0.9305 | 98990 |
| 2 | B | 0.004 | 0.2 | 0.9959 | 99240 |
| 3 | C | 0.0005 | 0.6 | 0.9981 | 99180 |

Table 5 helps in determining the subsystem with maximum availability and profit. It is observed that subsystem B is having maximum availability (0.9959) and profit (99240). Shown in the Table 6 are the optimum values of failure and repair rates for maximum availability and profit for each subsystem. From Table 6, it is observed that the most critical subsystem as far as maintenance is concerned and required immediate attention is subsystem B.

## 5. Conclusion

Explicit expressions for availability and profit are developed and used for the evaluation of performance of different subsystems of the series-parallel system for this study. Using the model, tables 2-6 are constructed to show the relationship between failure and repair rates on system availability and profit. Both availability and profit decrease as the failure rate increases. Similarly as also the repair rates increases so are the availability and profit increases. The model will assist maintenance engineers and managers for proper maintenance utilization. The results of this study will be beneficial to the plant management for the availability and profit analysis of series-parallel system.

## Acknowledgement

The authors are grateful to the anonymous reviewers and handling editor for their constructive comments, which have helped to improve the manuscript.

## References

[1] kumar, D., Singh Jai and Pandey PC. Reliability analysis of the feeding system in the paper industry. Microelectron Reliability, vol. 28, no. 2, 1988. pp 213-215. https://doi.org/10.1016/0026-2714(88)90353-8.

[2] Kumar, D., Singh Jai and Pandey PC. Availability analysis of the washing system in the paper industry. Microelectron Reliability, vol.29, 1989, pp 775-778. https://doi.org/10.1016/0026-2714(89)90177-7.

[3] Kumar, D., Singh Jai and Pandey PC. Operational behavior and profit function for a bleaching and screening in the paper industry. Microelectron Reliability, vol. 33, 1993, pp 1101-1105. https://doi.org/10.1016/0026-2714(93)90338-Y.

[4] Kumar, D., Singh Jai and Pandey PC. Behavior analysis of urea decomposition in the fertilizer industry under general repair policy. Microelectron Reliability, vol. 31, no. 5, 1991, pp 851-854. https://doi.org/10.1016/0026-2714(91)90023-Z.

[5] Kumar, D., Singh Jai and Pandey PC. Design and cost analysis of a refining system in a Sugar industry. Microelectron Reliability, vol. 30, no. 6, 1990, pp 1025-1028. https://doi.org/10.1016/0026-2714(90)90274-Q.

[6] Srinath, L.S. Reliability Engineering 3rd edition, east west press Pvt Ltd. New Delhi, India, 1994.

[7] P. Gupta, A. Lal, R. Sharma and J. Singh. Numerical analysis of reliability and availability of series processes in butter oil processing plant. International Journal of Quality and Reliability Management, vol. 22, no. 3 2005, pp 303-316. https://doi.org/10.1108/02656710510582507.

[8] P. Gupta, A. Lal, R. Sharma and J. Singh. Behavioral study of the Cement Manufacturing plant. A numerical approach. Journal of Mathematics and systems sciences. Vol.1, no. 1, 2005, pp 50-69.

[9] Arora, N., and Kumar, D. System analysis and maintenance management for coal handling system in a paper plant. International Journal of Management and Systems, 2000.
[10] Singh, J., and Garg, S. Availability analysis of core veneer manufacturing system in plywood industry. International Conference on Reliability and safety engineering, India institute of Technology, Kharagpur, 2005, pp 497-508.