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***Research paper***

# Probabilistic approach to performance evaluation of a series-parallel system

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**Abstract**

The paper deals with modeling and performance evaluation of a series-parallel system using Markov Birth-Death process and probabilistic approach. The system consists of four subsystems arranged in series-parallel with three possible states, working with full capacity, reduced capacity and failed. Through the transition diagrams, systems of differential equations are developed and solved recursively via probabilistic approach. Explicit expressions for steady-state availability are de-rived. Availability matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. The results from this paper will enhance the system performance and useful for timely execution of proper maintenance improvement, decision, planning and optimization.

***Keywords***: *Series; Parallel; Probabilistic; Performance; Markov.*

## 1. Introduction

The industrial and manufacturing systems comprise of large complex engineering systems arranged in series, parallel, or a combination of both. Some of these systems are feeding, crushing, refining, steam generation, evaporation, crystallization, fertilizer plant, crystallization unit of a sugar plant, piston manufacturing plant, etc. The reliability, availability and profit are the most important factors in any successful industries and manufacturing settings. Profit of system may be enhancing using highly reliable structural design of the system or subsystem of higher reliability. Improving the reliability and availability of the system, the production and associated profit will also increase. Availability and profit of an industrial system may be enhancing using highly reliable structural design into the system or subsystem of higher reliability. Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. Improving the reliability and availability of system/subsystem, the production and associated profit will also increase. Increase in production lead to the increase of profit. This can be achieve be maintaining reliability and availability at the highest order.

A large volume of literature exists over the issue of predicting performance evaluation of various systems. Kumar et al [1] discussed the reliability analysis of the Feeding system in the paper industry, Kumar el al.[2] discussed the availability analysis of the washing system in the paper industry, Kumar el al. [3] deal with reliability, availability and operational behavior analysis for different systems in paper plant. Kumar el al. [4] discussed the behavior analysis of Urea decomposition in the fertilizer industry under the general repair policy. Kumar et al.[5] studied the design and cost analysis of a refining system in a Sugar industry. Srinath [6] has explained a Markov model to determine the availability expression for a simple system consisting of only one component. Gupta el al. [7] has evaluated the reliability parameters of butter manufacturing system in a diary plant considering exponentially distributed failure rates of various components. Gupta et al. [8] studied the behavior of Cement manufacturing plant. Arora and Kumar [9] studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg [10] perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates.

In the present paper, we study a series-parallel system consisting of four different subsystems arranged in series. Through the transition diagram obtained in this study, systems of differential equations are developed and solved recursively via probabilistic approach. Availability matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated.

## 2. System descriptions

The System consists of four dissimilar subsystems which are:

1) Subsystem A: Single units in series whose failure cause complete failure of the entire system.
2) Subsystem B: consisting of four units in which two are in operation while the remaining two on standby. Failure of the system occurs when all the four units have failed.
3) Subsystem C: Single units in series whose failure cause complete failure of the entire system.
4) Subsystem D: A single unit in series whose failure cause complete failure of the entire system.

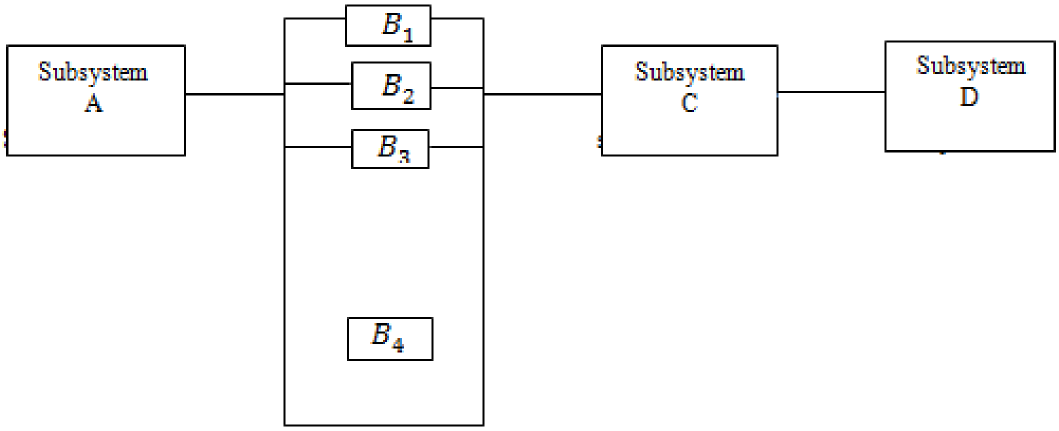


**Fig. 1:** Reliability Block Diagram of the System.

## 3. Assumptions and notations

The assumptions used in models development are as follows:

1) At any given time, the system is either in operating state or in failed state.
2) Subsystems/units do not fail simultaneously
3) Standby units in the same subsystem are of the same nature and capacity as the active units.

A, B, C, D, represent full working state of subsystem a, b, c,d represent failed state of subsystem $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ represent failure rates of subsystems A, B,C,D $\mu_1,\mu_2,\mu_3,\mu_4$: represent repair rates of subsystems A,B,C,D $P_0(t)$ , $P_1(t)$ , $P_2(t)$ : Probability of the system working with full capacity at time t $P_3(t)$, $P_4(t)$ , $P_{12}(t)$: Probability of the system in failed state $P_i'(t), i=0,1,2,...,12$ : represents the derivatives with respect to time t Av : Steady state availability of the system

## 4. Performance modelling of the system

The following system linear differential equations associated with the transition diagram (Figure 2) are derived:

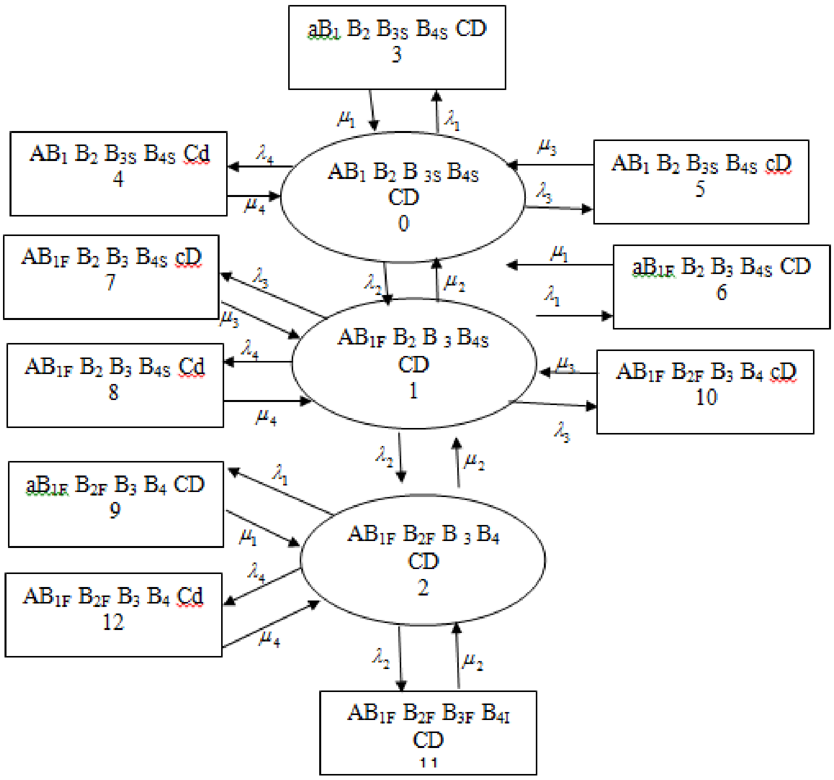


**Fig. 2:** Transition Diagram of the System.

$$\left(\frac{d}{dt}+\sum_{i=1}^{4}\lambda_i\right)P_0(t)=\mu_2P_1(t)+\mu_1P_3(t)+\mu_3P_4(t)+\mu_4P_5(t) \quad (1)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{4}\lambda_i+\mu_2\right)P_1(t)=\lambda_2P_0(t)+\mu_2P_2(t)+\mu_1P_6(t)+\mu_3P_7(t)+\mu_4P_8(t) \quad (2)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{4}\lambda_i+\mu_2\right)P_2(t)=\lambda_2P_1(t)+$$
$$\mu_1P_9(t)+\mu_3P_{10}(t)+\mu_2P_{11}(t)+\mu_4P_{12}(t) \quad (3)$$

$$\left(\frac{d}{dt}+\mu_1\right)P_3(t)=\lambda_1P_0(t) \quad (4)$$

$$\left(\frac{d}{dt}+\mu_3\right)P_4(t)=\lambda_3P_0(t) \quad (5)$$

$$\left(\frac{d}{dt}+\mu_4\right)P_5(t)=\lambda_4P_0(t) \quad (6) \quad \left(\frac{d}{dt}+\mu_1\right)P_6(t)=\lambda_1P_1(t) \quad (7)$$

$$\left(\frac{d}{dt}+\mu_3\right)P_7(t)=\lambda_3P_1(t) \quad (8)$$

$$\left(\frac{d}{dt}+\mu_4\right)P_8(t)=\lambda_4P_1(t) \quad (9)$$

$$\left(\frac{d}{dt}+\mu_1\right)P_9(t)=\lambda_1P_2(t) \quad (10)$$

$$\left(\frac{d}{dt}+\mu_3\right)P_{10}(t)=\lambda_3P_2(t) \quad (11)$$

$$\left(\frac{d}{dt}+\mu_2\right)P_{11}(t)=\lambda_2P_2(t) \quad (12)$$

$$\left(\frac{d}{dt}+\mu_4\right)P_{12}(t)=\lambda_4P_2(t) \quad (13)$$

With initial condition $P_i(t)=\begin{cases}1, & i=0\\ 0, & i\neq 0\end{cases}$ (14)

Setting $\frac{d}{dt}=0$ as $t\to\infty$ in equations (1 – 13) and solving them recursively we obtained the steady state probabilities given below:

$$P_1=X_2P_0 \quad (15)$$

$$P_2=X_2^2P_0 \quad (16)$$

$$P_3=X_1P_0 \quad (17)$$

$$P_4=X_3P_0 \quad (18)$$

$$P_5=X_4P_0 \quad (19)$$

$$P_6=X_1X_2P_0 \quad (20)$$

$$P_7=X_2X_3P_0 \quad (21)$$

$$P_8=X_2X_4P_0 \quad (22)$$

$$P_9=X_1X_2^2P_0 \quad (23)$$

$$P_{10}=X_3X_2^2P_0 \quad (24)$$

$$P_{11}=X_2^3P_0 \quad (25)$$

$$P_{12}=X_4X_2^2P_0 \quad (26)$$

Where $X_J=\frac{\lambda_J}{\mu_J}$, $J=1,2,3,4$

The probability of full working capacity, namely, $P_0(t)$, $P_1(t)$ and $P_2(t)$ determined by using normalizing condition:

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+...+P_{12}(\infty)=1 \quad (27)$$

Substituting (15) to (26) in (27) we have

$$P_0\left(1+X_2+X_2^2+X_1+X_3+X_4+...X_2^3+X_4X_2^2\right)=1 \quad (28)$$

$$P_0(\infty)=\frac{1}{\left(1+X_2+X_2^2+X_1+X_3+X_4+...X_2^3+X_4X_2^2\right)} \quad (29)$$

The steady state availability $A_v$ is summation of all working and reduced capacity states probabilities. Thus

$$A_v=P_0(\infty)+P_1(\infty)+P_2(\infty) \quad (30)$$

$$=\frac{1+X_2+X_2^2}{\left(1+X_2+X_2^2+X_1+X_3+X_4+...X_2^3+X_4X_2^2\right)} \quad (31)$$

## 5. Numerical example

Table 1 and Figure 3 reveal the effect of failure and repair rates of subsystem A on the availability of the system. It is observed that for some known values of failure / repair rates of other four sub-systems, as failure rate of first subsystem increases from 0.004 to 0.007, the subsystem availability decreases. Similarly as repair rate of first subsystem increases from 0.35 to 0.5, the subsystem availability increases.

Table 2 and Figure 4 reveal the effect of failure and repair rates of subsystem B on the availability of the system. It is observed that for some known values of failure / repair rates of other four sub-systems, as failure rate of first subsystem increases from 0.004 to 0.007, the subsystem availability decreases. Similarly as repair rate of first subsystem increases from 0.005 to 0.2, the subsystem availability increases.

Table 3 and Figure 5 reveal the effect of failure and repair rates of subsystem C on the availability of the system. It is observed that for some known values of failure / repair rates of other four sub-systems, as failure rate of first subsystem increases from 0.0005 to 0.002, the subsystem availability decreases. Similarly as repair rate of first subsystem increases from 0.45 to 0.6, the subsystem availability increases.

**Table 1:** Availability Matrix of the Subsystem A of Series-Parallel System

| $\mu_1$ $\lambda_1$ | 0.35 | 0.4 | 0.45 | 0.5 | $\mu_2 = 0.1$ |
|---|---|---|---|---|---|
| 0.004 | 0.9675 | 0.9689 | 0.9699 | 0.9708 | $\lambda_2 = 0.005$ |
| 0.005 | 0.9649 | 0.9665 | 0.9678 | 0.9689 | $\mu_3 = 0.5$ |
| 0.006 | 0.9622 | 0.9642 | 0.9658 | 0.9670 | $\lambda_3 = 0.001$ |
| 0.007 | 0.9596 | 0.9619 | 0.9637 | 0.9651 | $\mu_4 = 0.1$ <br> $\lambda_4 = 0.002$ |

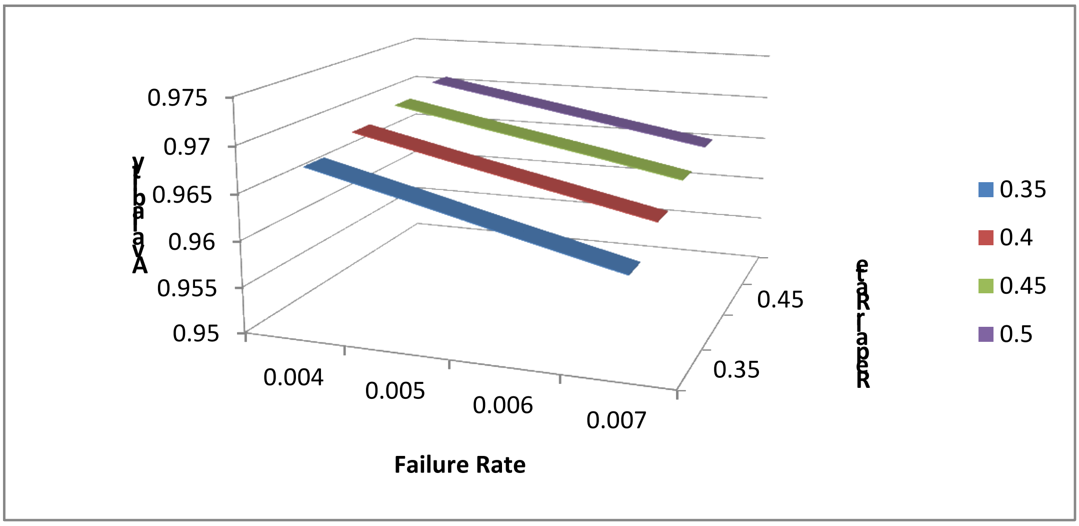


**Fig. 3:** Impact of Failure $\lambda_1$ and Repair $\mu_1$ Rates of Subsystem A on Availability.

**Table 2:** Availability Matrix of the Subsystem B of Series-Parallel System

| $\mu_2$ $\lambda_2$ | 0.05 | 0.1 | 0.15 | 0.2 | $\mu_1 = 0.4$ |
|---|---|---|---|---|---|
| 0.004 | 0.9662 | 0.9666 | 0.9666 | 0.9666 | $\lambda_1 = 0.005$ |
| 0.005 | 0.9658 | 0.9665 | 0.9666 | 0.9666 | $\mu_3 = 0.5$ |
| 0.006 | 0.9652 | 0.9665 | 0.9666 | 0.9666 | $\lambda_3 = 0.001$ |
| 0.007 | 0.9644 | 0.9664 | 0.9666 | 0.9666 | $\mu_4 = 0.1$ <br> $\lambda_4 = 0.002$ |

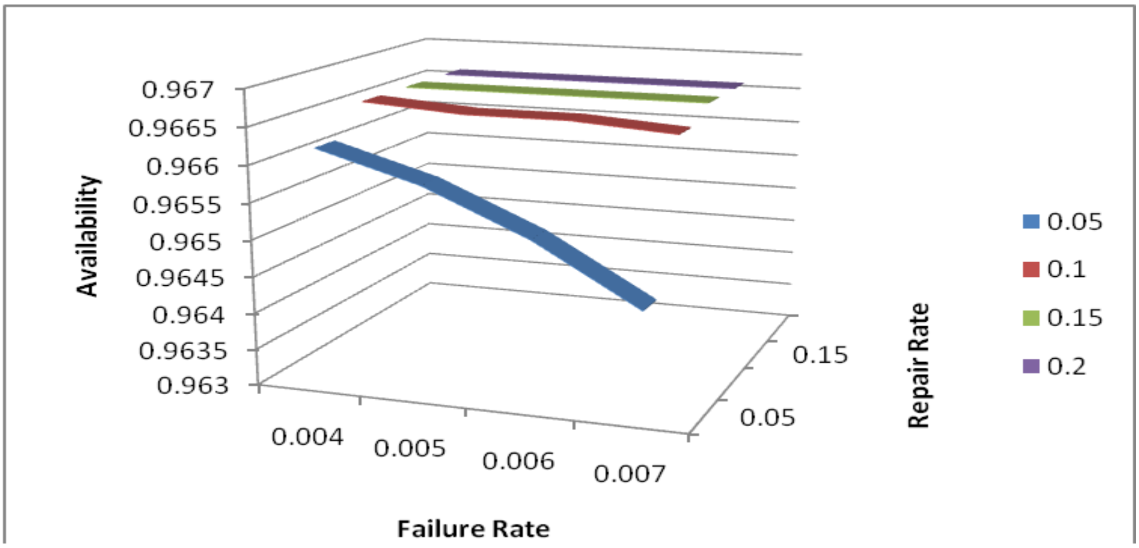


**Fig. 4:** Impact of Failure $\lambda_2$ and Repair $\mu_2$ Rates of Subsystem A on Availability.

**Table 3:** Availability Matrix of the Subsystem C of Series-Parallel System

| $\mu_3$ $\lambda_3$ | 0.45 | 0.5 | 0.55 | 0.6 | $\mu_1 = 0.5$ |
|---|---|---|---|---|---|
| 0.0005 | 0.9697 | 0.9698 | 0.9699 | 0.9700 | $\lambda_1 = 0.005$ |
| 0.001 | 0.9687 | 0.9689 | 0.9691 | 0.9692 | $\mu_2 = 0.1$ |
| 0.0015 | 0.9676 | 0.9679 | 0.9682 | 0.9684 | $\lambda_2 = 0.005$ |
| 0.002 | 0.9666 | 0.9670 | 0.9673 | 0.9676 | $\mu_4 = 0.1$ <br> $\lambda_4 = 0.002$ |

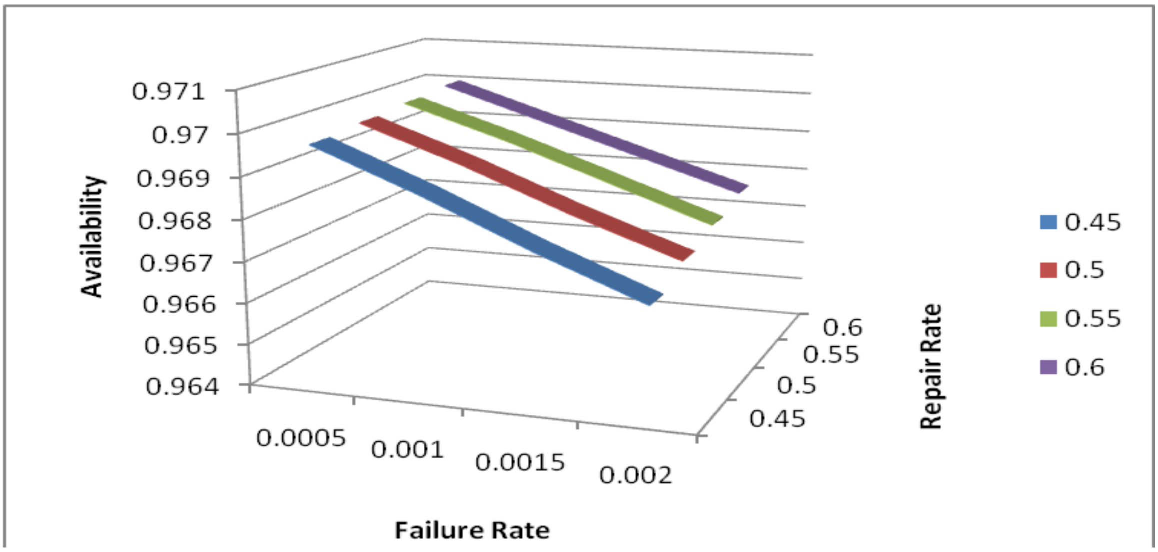


**Fig. 5:** Impact of Failure $\lambda_3$ and Repair $\mu_3$ Rates of Subsystem A on Availability.

**Table 4:** Availability Matrix of the Subsystem D of Series-Parallel System

| $\mu_4$ $\lambda_4$ | 0.05 | 0.1 | 0.15 | 0.2 | $\mu_1 = 0.4$ |
|---|---|---|---|---|---|
| 0.001 | 0.9665 | 0.9760 | 0.9792 | 0.9808 | $\lambda_1 = 0.005$ |
| 0.002 | 0.9482 | 0.9665 | 0.9728 | 0.9760 | $\mu_2 = 0.1$ |
| 0.003 | 0.9306 | 0.9573 | 0.9665 | 0.9712 | $\lambda_2 = 0.005$ |
| 0.004 | 0.9136 | 0.9482 | 0.9604 | 0.9665 | $\mu_3 = 0.5$ $\lambda_3 = 0.001$ |

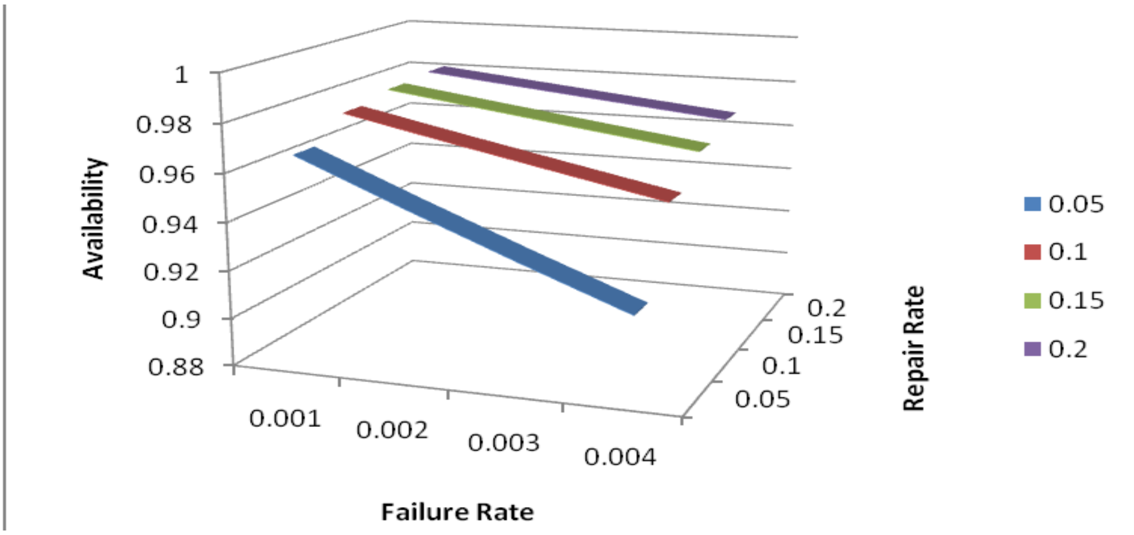


**Fig. 6:** Impact of Failure $\lambda_4$ and Repair $\mu_4$ Rates of Subsystem A on Availability

Table 4 and Figure 6 reveal the effect of failure and repair rates of subsystem D on the availability of the system. It is observed that for some known values of failure / repair rates of other four subsystems, as failure rate of first subsystem increases from 0.001 to 0.004, the subsystem availability decreases. Similarly as repair rate of first subsystem increases from 0.05 to 0.2, the subsystem availability increases.

## 6. Conclusion

Explicit expression in the availability model is developed and used for the evaluation of performance of different subsystems of the series-parallel system in this study. Using the model, tables 1-4 are constructed to show the relationship between failure and repair rates on system availability. The availability decreases as the failure rate increases. Similarly as availability increases so also the repair rates. Models presented within this paper are important to engineers, maintenance managers and plant management for proper maintenance analysis, decision and safety of the system as a whole. The models will also assist engineers, decision-makers and plant management to avoid an incorrect reliability assessment and consequent erroneous decision making.

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## References

[1] kumar, D., Singh Jai and Pandey PC. Reliability analysis of the feeding system in the paper industry. Microelectron Reliability, vol. 28, no. 2, 1988. pp 213-215. https://doi.org/10.1016/0026-2714(88)90353-8.

[2] Kumar, D., Singh Jai and Pandey PC. Availability analysis of the washing system in the paper industry. Microelectron Reliability, vol.29, 1989, pp 775-778. https://doi.org/10.1016/0026-2714(89)90177-7.

[3] Kumar, D., Singh Jai and Pandey PC. Operational behavior and profit function for a bleaching and screening in the paper industry. Microelectron Reliability, vol. 33, 1993, pp 1101-1105. https://doi.org/10.1016/0026-2714(93)90338-Y.

[4] Kumar, D., Singh Jai and Pandey PC. Behavior analysis of urea decomposition in the fertilizer industry under general repair policy. Microelectron Reliability, vol. 31, no. 5, 1991, pp 851-854. https://doi.org/10.1016/0026-2714(91)90023-Z.

[5] Kumar, D., Singh Jai and Pandey PC. Design and cost analysis of a refining system in a Sugar industry. Microelectron Reliability, vol. 30, no. 6, 1990, pp 1025-1028. https://doi.org/10.1016/0026-2714(90)90274-Q.

[6] Srinath, L.S. Reliability Engineering 3rd edition, East west press Pvt Ltd. New Delhi, India, 1994.

[7] P. Gupta, A. Lal, R. Sharma and J. Singh. Numerical analysis of reliability and availability of series processes in butter oil processing plant. International Journal of Quality and Reliability Management, vol. 22, no. 3 2005, pp 303-316. https://doi.org/10.1108/02656710510582507.

[8] P. Gupta, A. Lal, R. Sharma and J. Singh. Behavioral study of the Cement Manufacturing plant. A numerical approach. Journal of Mathematics and systems sciences. Vol.1, no. 1, 2005, pp 50-69.

[9] Arora, N., and Kumar, D. System analysis and maintenance management for coal handling system in a paper plant. International Journal of Management and Systems, 2000.

[10] Singh, J., and Garg, S. Availability analysis of core veneer manufacturing system in plywood industry. International Conference on Reliability and safety engineering, India institute of Technology, Kharagpur, 2005,pp 497-508.