New approach to the resolution of triangular fuzzy linear programs : MOMA-plus method

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Abstract

In this paper, we propose a new approach to the resolution of linear programs, whose coefficients are fuzzy triangular numbers. This new approach is an adaptation from the method MOMA-plus (***M**ulti-**O**bjective **M**etaheuristic based on **A**lineor method*) [1] to the resolution of fuzzy linear programs. First of all it consists in using of a new procedure proposed here to the converting of the fuzzy linear program into a deterministic multi-objective linear program, secondly of the using of the MOMA-plus procedure for resolution. Finally, three numerical examples are given to explain the procedures and highlight the performances of this new approach.

Keywords: Fuzzy linear programming; Fuzzy triangular numbers; MOMA-plus method.

1. Introduction

Since the appearance of Zadeh's article [18] on the fuzzy sets, several works were born. Some works are for the development of theory and others for applications. The concept of fuzzy decision was first proposed by Bellman and Zadeh [22]. The first application of fuzzy concepts to linear programming was introduced by Zimmerman [24]. Since then, several researches have been carried out for the resolution of linear programs with fuzzy triangular curves. The methods of resolution emanating from these researches can be classified into two categories. On the one hand, there are those which transform the fuzzy linear problem into a deterministic multi-objective linear problem. Among them, we can quote P. Pandian [5], P.A. Thakre and *al.* [8], G. Zhang and *al.* [15], Mr Zangiabadi and Mr H. Maleki. [20]. On the other hand, there are those that directly solve the problem using operations in fuzzy sets. For this category we can quote interactive methods proposed by Mariano Jiménez and *al.* [19], the primal-dual fuzzy simplex algorithm proposed by Nezam Mahdavi-Amiri and *al.* [10], the inner point method proposed by Yi-hua Zhong and *al.* [3], the use of the comparison index proposed by Yozo Nakahara and Mitsuo Gen [23], the use of membership functions proposed by Mr. Zarafat Angiz and al. [21] and the principle of comparison of fuzzy numbers proposed by H. R. Maleki and *al.* [17].

As the resolution of fuzzy linear problems generates difficulties as soon as we have a very high number of variables and constraints, we propose in the work a new approach to solve linear programs with fuzzy elements in the same perspective as our predecessors to overcome this difficulty. Sense it consists in converting a linear program to fuzzy triangular elements in a deterministic multi-objective linear program before applying MOMA which has proved its value in the deterministic linear programs [2].

The MOMA method(Multi-Objective Metaheuristic based on Alienor method), developed by Kounhinir Somé and *al.* [7], consists in transforming a multi-objective optimization problem into a global optimization problem of a single variable using an Alienor transformation(see [11, 13, 14, 16, 26]). In which desciption of the theoretical foundations with didactic examples of MOMA is presented [2]. Subsequently, in order to improve the performance of this method, another alternative named MOMA-plus is proposed by Kounhinir Somé and *al.* [1]. This version MOMA-plus right improves the speed, the good convergence and the good distribution of the solutions in relation to the true front of Pareto [1, 4]. One of the main challenges of this method MOMA-plus for the designers is its generalization to the resolution of all problems of optimization. As it is already done for the case of real optimization problems linear [2] and nonlinear [7] we will interest in the case of the problems of optimization in fuzzy numbers and particular to the cases of fuzzy triangular numbers.

In this work we propose two new elements: the new way of converting a problem with fuzzy elements into a real-world problem and the extension of the scope of MOMA, For the first time, to fuzzy optimization schemes. Thus, for a good presentation of this work, we will present it in 5 sections. Indeed, in the next section, we will describe the MOMA-plus method and in section 3 we will present some elements of the fuzzy triangular numbers. Section 4 will be devoted to the hybrid method. And end tis work with the section 5 in which we will make a didactic example and two other applications.

2. Fuzzy triangular numbers

This part presents some notions about the fuzzy triangular numbers we will need for the rest of the work. The essence of this section is taken from [5, 6, 8, 18, 22, 27].

2.1. Definitions

Definition 2.1. *Let X be a set, called universe, whose elements are denoted by x. A fuzzy subset $\widetilde{A}$ of X is defined using a $\mu_{\widetilde{A}}$ The interval $[0,1]$. $\widetilde{A}$ is therefore characterized by :*

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) | x \in X\}. \tag{1}$$

The membership function may, according to the situation, represent a degree of possibility or a degree of preference.

Definition 2.2. *A fuzzy number $\widetilde{a}$ is said to be triangular if it is in the form $\widetilde{a} = (a_1, a_2, a_3)$, where a_1, a_2 and a_3 are real, And has as a membership function:*

$$\mu_{\widetilde{a}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & if\ a_1 \le x \le a_2, \\ \frac{a_3 - x}{a_3 - a_2} & if\ a_2 \le x \le a_3, \\ 0 & if\ else \end{cases} \tag{2}$$

2.2. Operations

Let $\widetilde{a} = (a_1, a_2, a_3)$ and $\widetilde{b} = (b_1, b_2, b_3)$ two triangular fuzzy numbers and k a real. We have:

1. $(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$;
2. $(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$;
3. $k(a_1, a_2, a_3) = \begin{cases} (ka_1, ka_2, ka_3) & si\ k \ge 0, \\ (ka_3, ka_2, ka_1) & si\ k < 0. \end{cases}$
4. $\max\left((a_1, a_2, a_3), (b_1, b_2, b_3)\right) = \left(\max(a_1, b_1), \min(a_2, b_2), \max(a_3, b_3)\right)$
5. $\min\left((a_1, a_2, a_3), (b_1, b_2, b_3)\right) = \left(\min(a_1, b_1), \max(a_2, b_2), \min(a_3, b_3)\right)$.

in which min and max denote respectively minimum and maximum.

2.3. Comparison

Let $\widetilde{a} = (a_1, a_2, a_3)$ and $\widetilde{b} = (b_1, b_2, b_3)$ two fuzzy triangular numbers. We have:

1. $\widetilde{a} \approx \widetilde{b} \iff a_i = b_i, \forall i = 1, 2, 3$;
2. $\widetilde{a} \succeq \widetilde{b} \iff a_i \ge b_i, \forall i = 1, 2, 3$;
3. $\widetilde{a} \succ \widetilde{b} \iff a_i \ge b_i; i = 1, 2, 3$ et $a_r > b_r,\ \forall r \in \{1, 2, 3\}$;
4. $\widetilde{a} \preceq \widetilde{b} \iff \begin{cases} a_1 & \le & b_1 \\ a_1 - a_2 & \le & b_1 - b_2 \\ a_1 + a_3 & \le & b_1 + b_3. \end{cases}$

3. Linear program with fuzzy triangular elements

3.1. Definition and property

Definition 3.1. *A linear program with fuzzy triangular curves is any mathematical program of the form:*

$$\max \widetilde{Z} = \sum_{j=1}^{n} \widetilde{c}_j x_j$$

$$S.t: \begin{cases} \sum_{j=1}^{n} \widetilde{a}_{ij} x_j & \le \widetilde{b}_i & ; i = 1, \cdots, p \\ x_j & \ge 0 & ; j = 1, \cdots, n \end{cases} \tag{3}$$

Where the $\widetilde{c}_j, \widetilde{a}_{ij}$ and $\widetilde{b}_i$ are the fuzzy triangular elements of the problem and x_j of the positive reals.

By putting $\widetilde{c}_j = (c_j^m, c_j^l, c_j^u)$, $\widetilde{a_{ij}} = (a_{ij}^m, a_{ij}^l, a_{ij}^u)$, and $\widetilde{b}_i = (b_i^m, b_i^l, b_i^u)$ The fuzzy linear program can be written as follows:

$$\max \widetilde{Z} = \sum_{j=1}^{n} (c_j^m, c_j^l, c_j^u) x_j$$

$$S.t: \begin{cases} \sum_{j=1}^{n} (a_{ij}^m, a_{ij}^l, a_{ij}^u) x_j & \le (b_i^m, b_i^l, b_i^u) & ; i = 1, \cdots, p \\ x_j & \ge 0 & ; j = 1, \cdots, n. \end{cases} \tag{4}$$

Property 3.1. *:*

Let's note $X = \{x_j \in \mathbb{R}^{*+} \mid \sum_{j=1}^{n} (a_{ij}^m, a_{ij}^l, a_{ij}^u) x_j \le (b_i^m, b_i^l, b_i^u); i = 1, \cdots, p\}$. then:

$\diamond$ *A point $x^* \in X$ is called the optimal solution of the fuzzy PL if*

$$\sum_{j=1}^{n} (c_j^m, c_j^l, c_j^u) x_j^* \ge \sum_{j=1}^{n} (c_j^m, c_j^l, c_j^u) x_j, \forall x \in X$$

;

$\diamond$ *A point $x^* \in X$ is called the non-dominated solution of the fuzzy PL if there is no $x \in X$, such as*

$$\sum_{j=1}^{n} (c_j^m, c_j^l, c_j^u) x_j \ge \sum_{j=1}^{n} (c_j^m, c_j^l, c_j^u) x_j^*;$$

$\diamond$ *A point $x^* \in X$ is called a weakly dominated solution of the fuzzy PL there is no $x \in X$, such as*

$$\sum_{j=1}^{n} (c_j^m, c_j^l, c_j^u) x_j > \sum_{j=1}^{n} (c_j^m, c_j^l, c_j^u) x_j^*.$$

The steps of the new method are as follows:

3.2. Converting a fuzzy problem into a deterministic problem

This conversion will be done in two stages, namely the conversion of the fuzzy objective into deterministic objectives and fuzzy constraints into deterministic constraints.

First, by applying the operations on the fuzzy numbers to problem (4), we obtain:

$$\max \widetilde{Z} = (\sum_{j=1}^{n} c_j^m x_j, \sum_{j=1}^{n} c_j^l x_j, \sum_{j=1}^{n} c_j^u x_j)$$

$$S.t: \begin{cases} (\sum_{j=1}^{n} a_{ij}^m x_j, \sum_{j=1}^{n} a_{ij}^l x_j, \sum_{j=1}^{n} a_{ij}^u x_j) & \le (b_i^m, b_i^l, b_i^u) & ; \\ x_j & \ge 0 \end{cases} \tag{5}$$

where $i = 1, \cdots, p$ and $j = 1, \cdots, n$. Then by applying the techniques of comparison to problem (5), we deduce:

$$\max \widetilde{Z} = (\sum_{j=1}^{n} c_j^m x_j, \sum_{j=1}^{n} (c_j^m - c_j^l) x_j, \sum_{j=1}^{n} (c_j^m + c_j^u) x_j)$$

$$S.t: \begin{cases} \sum_{j=1}^{n} a_{ij}^m x_j & \le b_i^m & ; i = 1, \cdots, p \\ \sum_{j=1}^{n} (a_{ij}^m - a_{ij}^l) x_j & \le (b_i^m - b_i^l) & ; i = 1, \cdots, p \\ \sum_{j=1}^{n} (a_{ij}^m + a_{ij}^u) x_j & \le (b_i^m + b_i^u) & ; i = 1, \cdots, p \\ x_j & \ge 0 & ; j = 1, \cdots, n. \end{cases} \tag{6}$$

Finally, by using operations on the maxima and minima of fuzzy triangular numbers, the problem (6) is converted to a fully deterministic linear multi-objective problem as follows:

$$
\begin{aligned}
\max z_1 \quad &= \sum_{j=1}^{n} c_j^m x_j \\
\min z_2 \quad &= \sum_{j=1}^{n} (c_j^m - c_j^l) x_j \\
\max z_3 \quad &= \sum_{j=1}^{n} (c_j^m + c_j^u) x_j)
\end{aligned}
$$

$$
\text{S.t}: \begin{cases}
\sum_{j=1}^{n} a_{ij}^m x_j \le b_i^m & ; i = 1, \cdots, p \\
\sum_{j=1}^{n} (a_{ij}^m - a_{ij}^l) x_j \le (b_i^m - b_i^l) & ; i = 1, \cdots, p \\
\sum_{j=1}^{n} (a_{ij}^m + a_{ij}^u) x_j \le (b_i^m + b_i^u) & ; i = 1, \cdots, p \\
x_j \ge 0 & ; j = 1, \cdots, n
\end{cases}
\tag{7}
$$

For the resolution, we will use the MOMA-plus method.

4. MOMA-plus method

4.1. Basic Concept

Consider a constraint optimization problem that takes into account several objectives

$$
\begin{aligned}
\min \quad & f_1(x), \cdots, f_p(x) \\
\text{S.c}: \quad & \begin{cases} G_k(x) \le 0, & k = 1, \cdots, m \\ x \in \mathbb{R}^n. \end{cases}
\end{aligned}
\tag{8}
$$

The procedure of the MOMA-plus method for solving a multi-objective problem consists in transforming the problem into a global optimization problem without constraints to a single variable. The algorithm of this MOMA-plus method for the solving of a problem of type (8) proceeds in five essential steps that are [1]:

1. *the aggregation of objective functions*: consists in transforming a multiobjective problem into a mono-objective problem;
2. *penalization of the problem*: this makes it possible to reduce an optimization problem with constraints to a problem of optimization without constraint;
3. *reductive transformation*: this step makes it possible to reduce the optimization of a function of several variables to a function of a single variable using an Alienor transformation;
4. *search for the global optimum*: it consists in using the simplex algorithm of Nelder-Mead to determine the global optimum of the mono-objective function of a single variable obtained in the previous step;
5. *configuration of the solution*: it consists in bringing the obtained solution in dimension one into a solution of dimension n conform to the initial problem.

The algorithm of the MOMA-plus method is described in [1], the interested reader can consult it.

4.2. Description of Steps

4.2.1. Aggregate of Objective Functions

After transforming the problem (7) into a problem in which all the objectives to be minimized and by using the weighted sum, we transform the multiobjective problem (7) into a mono-objective

problem as follows:

$$
\min s(z, \lambda) \quad = -\lambda_1 z_1 + \lambda_2 z_2 - \lambda_3 z_3
$$

$$
\text{S.t}: \begin{cases}
\sum_{j=1}^{n} a_{ij}^m x_j \le b_i^m & ; i = 1, \cdots, p \\
\sum_{j=1}^{n} (a_{ij}^m - a_{ij}^l) x_j \le (b_i^m - b_i^l) & ; i = 1, \cdots, p \\
\sum_{j=1}^{n} (a_{ij}^m + a_{ij}^u) x_j \le (b_i^m + b_i^u) & ; i = 1, \cdots, p \\
x_j \ge 0 & ; j = 1, \cdots, n.
\end{cases}
\tag{9}
$$

With $\lambda_1 + \lambda_2 + \lambda_3 = 1$. Let $\mathscr{D}$ be the set of admissible solutions defined by the deterministic constraints of (9).

The obtaining of optimal Pareto solutions, relative to the method of weighting of the objective functions, is guaranteed by the following theorem:

Theorem 4.1. *[25]*
Either the parametric problem

$$
\min_{x \in \mathscr{D}} s(z, \lambda) \qquad (P)
$$

with $\lambda \in \Lambda = \left\{ \lambda_i \in [0,1]; \sum_i \lambda_i = 1 \right\}$.

- *If x is an optimal solution of (P), x is an efficient solution.*
- *If x is an efficient solution and $Z_{\mathscr{D}}$ (image set of D) is convex, if exists $\lambda \in \Lambda$ such as x is an optimal solution of (P).*

4.2.2. Penalization of the problem

The penalization of the problem (9) leads to a problem without constraints. The function of penalization we use is given in [2] and makes it possible to obtain the problem in the following form:

$$
\begin{cases}
Glob. \min L(X) \\
\quad X \in \mathscr{D}
\end{cases}
\tag{10}
$$

with

$$
L(X) = s(z, \lambda) + K \sum_{i=1}^{p} \Big(g_{1i}(X) + g_{2i}(X) + g_{3i}(X) + |g_{1i}(X)| + |g_{2i}(X)| + |g_{3i}(X)| \Big)
$$

and

$$
\mathscr{D} = \left\{ X \in \mathbb{R}^n; g_{ki}(X) \le 0, \ i = \overline{1, p}, \ k = \overline{1, 3} \right\}
$$

K is a positif real defined by :

$$
K \ge \frac{M - s(z, \lambda)}{\sum_{i=1}^{p} \sum_{k=1}^{3} g_{ki}(X)} \ \text{et} \ M = \max_{x \in \mathscr{D}} s(z, \lambda).
$$

Theorem 4.2. *[12]*
Let X^* be the global minimum of $L(X)$, then X^* is the global minimum of $\min_{x \in \mathscr{D}} s(z, \lambda)$.

4.2.3. Reductive transformation

The application of a transformation, Alienor, that of Konfé-Cherruault [11], in the form

$$
x_j = h_j(\theta) = \frac{1}{2}[(b_j - a_j)\cos(\omega_j \theta + \varphi_j) + b_j + a_j]; j = 1, \cdots, n \ \text{et} \ \theta \in [0, 2\pi]
$$

in which :

- $(\omega_j)_{j=1;n}$ et $(\varphi_j)_{j=1;n}$ are slowly increasing,
- a_j et b_j are the extreme values of x_j, in other words, $x_j \in [a_j; b_j]$,

to the problem (10) makes it possible to obtain the problem in the form :

$$Glob.\min L(\theta) = -\lambda_1 z_1(\theta) + \lambda_2 z_2(\theta) - \lambda_3 z_3(\theta) + K \sum_{i=1}^{p} \Big[g_{1i}(\theta) + g_{2i}(\theta) + g_{3i}(\theta)$$
$$+ |g_{1i}(\theta)| + |g_{2i}(\theta)| + |g_{3i}(\theta)|, \ \theta \in [0, 2\pi] \Big]$$

Theorem 4.3. *[2, 7]*
If θ^* is global minimum of $L(\theta)$ then $X^* = h(\theta^*)$ is global minimum of $L(X)$.

4.2.4. Search for the global optimum

As the problem (11) is the minimization of a function of a single variable without constraint, we use the simplex algorithm of Nelder Mead, known as *"fminsearch"* in MATLAB software, to determine its overall minimum.

4.2.5. Configuring the solution

The solutions of the Problem (7) are deduced by using the previous reductive transformation. It gives:

$$x_j^* = h(\theta^*).$$

5. Adaptation of MOMA-plus to fuzzy optimization

5.1. Principle

The extension of the MOMA-plus algorithm to the linear problem with fuzzy coefficients can be summarized as follows:

Step 1 *Converting the fuzzy linear problem into a deterministic multi-objective linear problem (the moving of the problem (5) to the problem (7));*
Step 2 *Using of MOMA-plus;*
Step 3 *Determination of the solution of the fuzzy linear problem:*
This step consists in deducing the solution of the initial problem (linear problem with fuzzy agents) from the solution obtained using MOMA-plus which is the solution of the associated deterministic linear program. Thus the optimal solution $\widetilde{Z}^*$ is obtained and solving the following problem:

$$\widetilde{Z}^* = \max \Big(\sum_{j=1}^{n} c_j^m x_j^*, \sum_{j=1}^{n} c_j^l x_j^*, \sum_{j=1}^{n} c_j^u x_j^* \Big).$$

in which x_j^* are the optimal solutions of the problem (7).

5.2. Numerical experiences

In this section we will apply the new method to deal with three examples.

Example 1

Let's consider the following fuzzy linear program:

$$
\max \widetilde{Z} = (7, 10, 14)x_1 + (20, 25, 35)x_2
$$
$$
\text{S.t}: \begin{cases} (3, 2, 1)x_1 + (6, 4, 1)x_2 & \leq (13, 5, 2) \\ (4, 1, 2)x_1 + (6, 5, 4)x_2 & \leq (7, 4, 2) \\ x_1; x_2 & \geq 0. \end{cases} \tag{11}
$$

Using Step 1.

The program (12) can be written as follows:

$$
\max \widetilde{Z} = (7x_1 + 20x_2, -3x_1 - 5x_2, 21x_1 + 55x_2)
$$
$$
\text{S.t}: \begin{cases} 3x_1 + 6x_2 & \leq 13 \\ x_1 + 2x_2 & \leq 8 \\ 4x_1 + 7x_2 & \leq 15 \\ 4x_1 + 6x_2 & \leq 7 \\ 3x_1 + x_2 & \leq 3 \\ 6x_1 + 10x_2 & \leq 9 \\ x_1; x_2 & \geq 0. \end{cases} \tag{12}
$$

Using Step 2.

Let us transform the problem (13) into a deterministic multi-objective linear problem as follows:

$$
\begin{aligned}
\max z_1 &= 7x_1 + 20x_2 \\
\min z_2 &= -3x_1 - 5x_2 \\
\max z_3 &= 21x_1 + 55x_2
\end{aligned}
$$
$$
\text{S.t}: \begin{cases} 3x_1 + 6x_2 & \leq 13 \\ x_1 + 2x_2 & \leq 8 \\ 4x_1 + 7x_2 & \leq 15 \\ 4x_1 + 6x_2 & \leq 7 \\ 3x_1 + x_2 & \leq 3 \\ 6x_1 + 10x_2 & \leq 9 \\ x_1; x_2 & \geq 0. \end{cases} \tag{13}
$$

Using Step 3.

$$
\min Z = -\lambda_1(7x_1 + 20x_2) + \lambda_2(-3x_1 - 5x_2) - \lambda_3(21x_1 + 55x_2)
$$
$$
\text{S.t}: \begin{cases} 3x_1 + 6x_2 & \leq 13 \\ x_1 + 2x_2 & \leq 8 \\ 4x_1 + 7x_2 & \leq 15 \\ 4x_1 + 6x_2 & \leq 7 \\ 3x_1 + x_2 & \leq 3 \\ 6x_1 + 10x_2 & \leq 9 \\ x_1; x_2 & \geq 0. \end{cases}
$$
$$\tag{14}$$

avec $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

Using Step 4

$$Glob.\min Z = -\lambda_1(7x_1 + 20x_2) + \lambda_2(-3x_1 - 5x_2) - \lambda_3(21x_1 + 55x_2) + K\Big[21x_1 + 32x_2$$
$$- 55 + |3x_1 + 6x_2 - 13| + |x_1 + 2x_2 - 8| + |4x_1 + 7x_2 - 15|$$
$$+ |4x_1 + 6x_2 - 7| + |3x_1 + x_2 - 3| + |6x_1 + 10x_2 - 9|\Big].$$

Using Step 5.

Let's use the reductive transformation, with $\omega_1 = 1500, \omega_2 = 1500.05, \varphi_1 = 0$ et $\varphi_2 = 0.005$. By using the eligible domain, we find $a_1 = 0, b_1 = 1, a_2 = 0$ et $b_2 = 0.9$.
So x_1 and x_2 can be rewritten as follows:

$$
\begin{cases} x_1 = h_1(\theta) &= \frac{1}{2}[\cos(1500\theta) + 1] \\ x_2 = h_2(\theta) &= \frac{1}{2}[0.9\cos(1500.05\theta + 0.005) + 0.9] \\ & \theta \in [0, 2\pi]. \end{cases} \tag{15}
$$

Using Step 6.

By applying the techniques of penalization with the coefficient of penalization $K = 10000$, we obtain :

$$L(\theta) = -\lambda_1 z_1(\theta) + \lambda_2 z_2(\theta) - \lambda_3 z_3(\theta) + K\Big[g_1(\theta) + g_2(\theta) + g_3(\theta) + g_4(\theta) + g_5(\theta)$$

$$+g_6(\theta)+|g_1(\theta)|+|g_2(\theta)|+|g_3(\theta)|+|g_4(\theta)|+|g_5(\theta)|+|g_6(\theta)|\big]$$

with $\theta \in [0, 2\pi]$.

Using Step 7.

With Nelder Mead's alogrithm, we obtain a permissible solution set based on the λ values and by applying the fuzzy triangular comparison techniques, the maximum value of the fuzzy objective function is reached for $(x_1^* x_2^*) = (0, 0.9)$. So the solution of the fuzzy linear program is $(18, 22.5, 31.5)$ whose matching function is :

$$\begin{cases} 0 & \text{si } x < 18 \\ \frac{x-18}{4.5} & \text{si } 18 \le x \le 22.5, \\ \frac{31.5-x}{9} & \text{si } 22.5 \le x \le 31.5, \\ 0 & \text{si } x > 31.5 \end{cases} \tag{16}$$

Example 2

Let's consider the following problem :

$$\begin{aligned} \max \widetilde{Z} \quad &= (19,20,21)x_1 + (29,30,31)x_2 \\ \text{S.t} : \quad &\begin{cases} (4.5,5,5.5)x_1 + (2.5,3,4)x_2 & \le (194,200,206) \\ (3,4,5)x_1 + (6.5,7,7.5)x_2 & \le (230,240,250) \\ x_1; x_2 & \ge 0 \end{cases} \end{aligned} \tag{17}$$

The determination of the domain of admissible solutions leads us to $x_1 \in [0, 40]$ and $x_2 \in [0, \frac{240}{7}]$ and keeping the values of the slowly rising sequences as in example 1, The solution of the problem (20) which is:

$$x_1^* = 27.9; x_2^* = 18.3 \text{ et } \widetilde{Z}^* = (1062, 1108.3, 1154.5) \tag{18}$$

And whose function of belonging is :

$$\begin{cases} 0 & \text{si } x < 1062 \\ \frac{x-1062}{46.3} & \text{si } 1062 \le x \le 1108.3, \\ \frac{1108.3-x}{46.2} & \text{si } 1108.3 \le x \le 1154.5, \\ 0 & \text{si } x > 1154.5 \end{cases} \tag{19}$$

Example 3

Let's consider this linear program below whose objective function coefficients are fuzzy triangular and non-fuzzy constraints :

$$\begin{aligned} \max \widetilde{Z} \quad &= (2,3,5)x_1 + 2x_2 \\ \text{S.t} : \quad &\begin{cases} 2x_1 - x_2 & \le 6 \\ x_1 + 5x_2 & \le 10 \\ x_1; x_2 & \ge 0 \end{cases} \end{aligned} \tag{20}$$

The coefficients of the constraints not being fuzzy, the transformations are carried out only in the objective function. The determination of the domain of admissible solutions leads us to $x_1 \in [0, 3.64]$ and $x_2 \in [0, 2]$, and keeping the values of the slowly rising sequences as in Example 1, Solution of the problem (23) is:

$$x_1^* = 3.64; x_2^* = 1.27 \text{ et } \widetilde{Z}^* = (9.824, 13.464, 20.744) \tag{21}$$

Whose membership function is:

$$\begin{cases} 0 & \text{si } x < 9.824 \\ \frac{x-9.824}{3.64} & \text{si } 9.8624 \le x \le 13.464, \\ \frac{13.464-x}{7.28} & \text{si } 13.464 \le x \le 20.744, \\ 0 & \text{si } x > 40.744. \end{cases} \tag{22}$$

6. Conclusion

In this work we proposed a new version of MOMA-plus in order to solve the fuzzy linear problems. In addition, a new way of defuzzification has been proposed in this new method. Numerical experiences during these works have proved the performance of this version of MOMA-plus to the resolution of the linear problems with fuzzy triangular numbers in the sense that it effectively treats the problems even in the case of a high number of variables.

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