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A theoretical foundation metaheuristic method to solve some multiobjectif optimization problems

Kounhinir Somé*[1], Berthold Ulungu[2], Wenddabo Olivier Sawadogo[1], Blaise Somé[1]

[1]*Laboratory, LANIBIO, Ouagadougou, Burkina-Faso*
[2]*Laboratory, MATHRO,University of Mons, Belgium*
**Corresponding author E-mail: kounhinir.some@univ-ouaga.bf*

Abstract

In the literature, many metaheuristics are available to find a good approximation of efficient solutions of optimization problems. But most of these methods don't have a theoretical foundation. In this work, we propose the theoretical foundation of MOMA(**M**ulti-**O**bjectif **A**lienor **M**etaheuristic) method and moreover its efficiency to solve linear optimization problems. This method is the combination of multiobjectif concepts and the Alienor transformation, which allows to transform a multiobjectif optimization problem in optimization of a single variable function. We solve two didactic examples in order to allow the best presentation of the MOMA method and besides the quality of obtained solutions is proved.

Keywords: *metaheuristics, Alienor transformation, linear multiobjectif optimization, weighted Tchebychev metric, Pareto optimality.*

1 Introduction

In the past two decades, a lot of efforts have been devoted to study and develop general heuristics [1, 2, 3, 4, 5, 10, 14, 15, 8, 9]. These methods, called "metaheuristics", offer many advantages, but the choice of a suitable method for a particular problem can be difficult. unfortunately, in addition these methods have not axiomatic foundations. In general, they are based on observation of a natural phenomenon, and the well known and used are the heuristic of simulated annealing, the tabu, the genetic algorithms or specifically the evolutionary algorithms, the method of the anthill, ...

New methods regularly appears and their evolution offers many interests like MOMA method [1, 7, 6]. Our MOMA method is a new method that consist to combine the multiobjectif concepts with the Alienor transformation. MOMA is a metaheuristic method efficiently which can be applied to all kind of multiobjectif optimization problems. In a preceding work, we have obtained good results by solving with MOMA method non linear optimization problems with many variables and any kind of Pareto front [1]. This work comes to confirm the possibilities to widen the application of this method at all kind of optimization problems. Otherwise, the difficulty of choice of metaheuristic is excluded and computer time is acceptable and obtained solutions are good.

In this work, we are only interested by linear problems. Firstly, we present the background of MOMA method, then we apply the method on two didactic examples, and finally we study the quality of obtained solutions. Here, it will be about studying the quality of obtained solutions by MOMA method of didactic examples.

2 MOMA method

We recall that the mathematical modeling of a multiobjectif optimization problem, gives the following mathematical program :

$$(P_1)\begin{cases} "\min" Z_k(x), \quad k=\overline{1,K} \\ x\in D \end{cases}$$

where $D = \{x \in \mathbb{R}^n_+ / \ g_i(x) \leq 0, \ i = \overline{1,n}\}$ is a set defined by the problem constraints and $Z_k(x)$, $k = \overline{1,K}$, $K \geq 2$, define the objectif functions of the problem. The steps for solve this problem by MOMA method can be summarized in fives steps described below.

2.1 Problem Scalarizing

The scalarizing is a multiobjectif concept, which consists of the transformation of multiobjectif problem in mono-objectif. That implies the using of a scalarizing function [1, 7, 6, 14, 15, 10].
In this work, we only consider the weighted distance of Tchebychev, defined by:

$$S(Z(x), \lambda, \overline{Z}(x)) = \max_{\overline{k=1,K};\ x \in D} \left(\lambda_k \left|Z_k(x) - \overline{Z}_k(x)\right|\right), \tag{1}$$

with $(\overline{Z}_k(x))$ the reference point of the problem or also a target point. To determinate the maximum of the functions in equation (1), we use the following relations [6, 1, 7]:

$$\begin{aligned} max(\alpha, \beta) &= \frac{1}{2}\left[\alpha + \beta + \mid \alpha - \beta \mid \right], \\ max(\alpha, \beta, \gamma) &= max\left[max(\alpha, \beta), \gamma\right]. \end{aligned} \tag{2}$$

Thus, the problem (P_1) becomes:

$$(P_2) \begin{cases} \min \ S(Z(x), \lambda, \overline{Z}(x)) \\ x \in D. \end{cases}$$

There existe a theorem, called Bowman theorem [], that show that the unique optimal solution of P_2 is efficient or Pareto optimal solution of P_1. With precision the following theorem has been demonstrated :

Theorem 2.1 *If x is a unique optimal solution of (P_2), then x is an efficient solution of (P_1).*

2.2 Penalization

This step aim is to transform the problem (P_2) to an optimization of a function without constraint. For that, a penalization function is needed [1,2]. This penalization function is defined by :

$$L(x) = S(Z(x), \lambda, \overline{Z(x)}) + \omega \sum_{i=1}^{m} (g_i(x) + |g_i(x)|) \tag{3}$$

where ω is a positive real such that :

$$\omega \geq \frac{M - S(Z(x), \lambda, \overline{Z}(x))}{\sum_{i=1}^{m} g_i(x)} \ \text{ et } M = \max_{x \epsilon D} S(Z(x), \lambda, \overline{Z}(x)).$$

The problem (P_2) is transformed in :

$$(P_3) \begin{cases} Glob.\min L(x) \\ x \in D. \end{cases}$$

2.3 Dimensional reduction

The aim of this section is the transformation of the function of the problem (P_3) to a single variable function. It is possible by using the Alienor transformation or one of its variants [6, 7, 11, 12, 16]. Here, we use the Konfé-Cherruault transformation define by :

$$x_i = h_i(\theta) = \frac{1}{2}\left[(b_i - a_i)\cos(\omega_i \theta + \varphi_i) + (b_i + a_i)\right], \tag{4}$$

where $(\omega_i)_{i=\overline{1,n}}$ and $(\varphi_i)_{i=\overline{1,n}}$ are two sequences chosen, slowly growing, a_i and b_i are the bounded value of $x_i, i = \overline{1,n}$, otherwise $x_i \in [a_i, b_i]$. Where (P_3) becomes :

$$(P_4)\begin{cases} Glob.\min L^*(\theta) \\ \theta \in [0, \theta_{\max}] \end{cases}$$

where $L^*(\theta) = L(h_1(\theta), h_2(\theta), ...h_n(\theta))$,

$$\theta_{\max} = \frac{(b-a)\theta^1_{\max} + (b-a)}{2},$$

and $\theta^1_{\max} = \dfrac{2\pi - \varphi_1}{\omega_1}$.

It is demonstrated that there is a equivalence between the two laster above problem. Consequently the following theorem [11, 16] :

Theorem 2.2 *$(P_3) \Longleftrightarrow (P_4)$*

2.4 Resolution

According the above presentation, the use of Alienor transformation allows to get a function $L^*(\theta)$. To search a global optimum, we use an operator preserving optimum (OPO). This OPO is proposed first by Mora [11, 16] allowing to get a global optimum. We use here OPO defined by

$$T_{L^*}(\theta) = \frac{1}{2}[L^*(\theta) - L^*(\theta_0) + |L^*(\theta) - L^*(\theta_0)|], \tag{5}$$

where θ_0 is an arbitrary real of $[0, \theta_{\max}]$, and θ_{max} is defined in above section, and L^* is the new function to optimize.
Cherruault and Mora [11, 12] have shown that if the unique solution of the problem $\min_{\theta\in[0,2\pi]} L^*(\theta)$ exits, it is the solution of $T_{L^*}(\theta) = 0$. Precisely the following theorem has been demonstrated:

Theorem 2.3 *:*

- *T_{L*} and f at least have the same global minimum.*
- *Let be S the set of solutions of $T_{L*} = 0$. Then if $S = \{x^*\}$, then x^* is the global minimum.*

2.5 Solution configuration

The obtained solution in θ is 1-dimensional. To obtain the solution in x, we simply use formulas of variable change $x_i = h_i(\theta)$.

2.6 Method summary

- Step 1: Problem scalarizing
  To use one of the scalarizing function to aggregate all objectifs of the problem. We have the choice among sum weight, weighted Tchebychev metric, weighted augmented Tchebychev metric... But in this work we use weighted Tchebychev metric because it is efficient for linear or non linear problem than contrary to weighted sum which is efficient for linear case [10, 14, 15].
- Step 2: Penalization
  To use the penalization function to transform constraints optimization problem at the unconstraint optimization problem.

- Step 3: Dimensional reduction
  To use the basic Alienor method or one of its variants to transform the problem in a single variable problem.
- Step 4: Resolution
  To use an OPO which allows to obtain uniquely a global optimum.
- Step 5: Solution configuration
  To use the transformation of the third step to transform the obtained solution at the fourth step to the solution of the initial problem, which is a compromise.

Now, we are going to apply the MOMA method on two examples.

3 Didactic examples

Let us recall that one of the aim of this work is the presentation retailed of the MOMA method. Thus, we consider the following bi-objectif optimization problems. The problem linear denoted $PL1$ is extract of Steuer and Choo book [17] and $PL2$ is extract to Teghem book [18] :

Didactic examples				
$(PL1)$	$maximize\ \ z_1(x)$	$=$	x_1	
	$maximize\ \ z_2(x)$	$=$	$-x_1 + 2x_2$	
	$-8x_1 + 6x_2$	$\leq$	$0,$	(a);
	$7x_1 - 18x_2$	$\leq$	$0,$	(b);
	$11x_1 + 30x_2$	$\leq$	$102,$	(c);
	x_1, x_2	$\geq$	$0,$	.
$(PL2)$	$minimize\ \ z_1(x)$	$=$	$3x_1 - x_2$	
	$minimize\ \ z_2(x)$	$=$	$-x_1 + 3x_2$	
	$4x_1 + 3x_2$	$\geq$	$12,$	(a);
	$x_1 + 3x_2$	$\geq$	$6,$	(b);
	$x_1 - x_2$	$\leq$	$2,$	(c);
	$-x_1 + x_2$	$\leq$	$3,$	(d);
	$x_1 + x_2$	$\leq$	$8,$	(e).
	x_1, x_2	$\geq$	$0,$	.

3.1 Problem $PL1$

For the reference point we have consider the ideal point which is $\overline{Z} = (5, 4)$, obtained by individual optimization of the two objective functions. The representation in the decision space is given by :

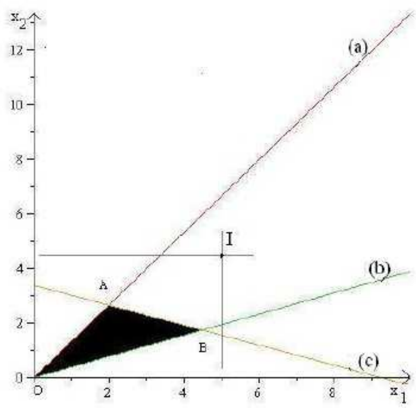

Figure 1: Decision space of PL1

In this figure, it appears that the area of admissible solutions is $D \subset [0, \frac{9}{2}] \times [0, \frac{8}{3}]$. We denoted by I the point that this coordinates realize the reference point. For this example it is easy to see that $E(P)$, the set of Pareto optimal solutions consists of the segment $[AB]$ because the non-negative orthant placed anywhere in this segment, contains no element of D.

3.1.1 Step 1

Let us transforme our maximization problem to a minimization problem. As for all fonction f we have $maximum(f) = -minimum(f)$. Then the last optimization problem is equivalent to :

$$\begin{cases} minimize \;\; z'_1(x) &= -x_1 \\ minimize \;\; z'_2(x) &= x_1 - 2x_2 \\ 8x_1 - 6x_2 &\geq 0; \\ -7x_1 + 18x_2 &\geq 0; \\ -11x_1 - 30x_2 &\geq -102; \\ x &= (x_1, x_2) \in [0, \frac{9}{2}] \times [0, \frac{8}{3}]. \end{cases} \tag{6}$$

The reference point becomes $\overline{z}' = (-5, -4)$. The using of weighted Tchebychev metric to aggregate the objectif lead us to obtain :

$$\begin{aligned} S(z', \lambda, \overline{z}) &= \max_{k=1,2} \Big[\lambda_k |z'_k - \overline{z}_k| \Big] \\ &= \max \Big[\lambda_1 |z'_1 - \overline{z}_1|, \lambda_2 |z'_2 - \overline{z}_2| \Big] \end{aligned}$$

Moreover as the reference point is a minimum, we have $z'_k \geq \overline{z}_k$ from where :

$$S(z', \lambda, \overline{z}) = \max \Big[\lambda_1 (z'_1 - \overline{z}_1), \lambda_2 (z'_2 - \overline{z}_2) \Big]$$

and by using the relation (2), we have :

$$\begin{aligned} S(z', \lambda, \overline{z}) &= \max \Big[\lambda_1 (z'_1 + 5), \lambda_2 (z'_2 + 4) \Big] \\ &= \frac{1}{2} \Big[\lambda_1 (z'_1 + 5) + \lambda_2 (z'_2 + 4) + |\lambda_1 (z'_1 + 5) - \lambda_2 (z'_2 + 4)| \Big] \\ &= \frac{1}{2} \Big[\lambda_1 (-x_1 + 5) + \lambda_2 (x_1 - 2x_2 + 4) + |\lambda_1 (-x_1 + 5) - \\ &\quad \lambda_2 (x_1 - 2x_2 + 4)| \Big]. \end{aligned}$$

As, $\lambda_1 + \lambda_2 = 1$, setting $\lambda_1 = \lambda$ i.e $\lambda_2 = 1 - \lambda$ with $0 \leq \lambda \leq 1$ and in the follow we obtain :

$$S(z', \lambda, \overline{z}) = \frac{1}{2} \Big[(1 - 2\lambda) x_1 - 2(1 - \lambda) x_2 + \lambda + 4 + | - x_1 + 2(1 - \lambda) x_2 + 9\lambda - 4| \Big]$$

Thus, the single objective optimization problem is :

$$\begin{cases} minimize \;\; S_2(z', \lambda, \overline{z}) &= \frac{1}{2} \Big[(1 - 2\lambda) x_1 - 2(1 - \lambda) x_2 + \lambda + 4 + \\ & \quad | - x_1 + 2(1 - \lambda) x_2 + 9\lambda - 4| \Big] \\ 8x_1 - 6x_2 &\geq 0; \\ -7x_1 + 18x_2 &\geq 0; \\ -11x_1 - 30x_2 &\geq -102; \\ x &= (x_1, x_2) \in [0, \frac{9}{2}] \times [0, \frac{8}{3}]. \end{cases}$$

3.1.2 Step 2

As announced, we are going to use the Konfé-Cherruault transformation defined by (4) and as $D \subset [0, \frac{9}{2}] \times [0, \frac{8}{3}]$, we can put $a_1 = 0, b_1 = \frac{9}{2}, a_2 = 0$ et $b_2 = \frac{8}{3}$, from where :

$$\begin{cases} x_1 = h_1(\theta) = \frac{1}{2}[\frac{9}{2}cos(\omega_i\theta + \varphi_i) + \frac{9}{2}] \\ x_2 = h_2(\theta) = \frac{1}{2}[\frac{8}{3}cos(\omega_i\theta + \varphi_i) + \frac{8}{3}] \end{cases}$$

and the :

$$\begin{cases} x_1 = h_1(\theta) = \frac{9}{4}[cos(\omega_1\theta + \varphi_1) + 1] \\ x_2 = h_2(\theta) = \frac{4}{3}[cos(\omega_2\theta + \varphi_2) + 1] \end{cases}$$

The choice of ω_i et α_i such that they are slowly growing, thus by convenience we take

$$\begin{cases} \omega_1 = 1500 & \omega_2 = 1500 + 0.05 \\ \varphi_1 = 0.005 & \varphi_2 = 0.005. \end{cases}$$

We obtain

$$\begin{cases} x_1 = h_1(\theta) = \frac{9}{4}[cos(1500\theta) + 1] \\ x_2 = h_2(\theta) = \frac{4}{3}[cos\big((1500 + 0.05)\theta + 0.005\big) + 1] \end{cases}$$

and $S_2(z', \lambda, \overline{z})$ becomes :

$$\begin{aligned} S_2(z', \lambda, \overline{z}) = & \frac{1}{24}\Big[27(1-2\lambda)\cos(1500\theta) - 32(1-\lambda)\cos((1500+0.05)\theta + 0.005)\theta \\ & - 10\lambda + 43 + |-27\cos(1500\theta) + 32(1-\lambda)\cos\big((1500+0.05)\theta + 0.005\big) \\ & + 76\lambda - 43|\Big]. \end{aligned}$$

In the follow we obtain :

$$\begin{cases} minimize\ S_2(z', \lambda, \overline{z}) \\ \quad g_1(\theta) = -9\cos(1500\theta) + 4\cos((1500+0.05)\theta + 0.005) + 5 \leq 0 \\ \quad g_2(\theta) = 21\cos(1500\theta) - 32\cos((1500+0.05)\theta + 0.005) - 11 \leq 0 \\ \quad g_3(\theta) = 99\cos(1500\theta) + 160\cos((1500+0.05)\theta + 0.005) - 149 \leq 0 \\ \quad \theta\epsilon\,[0, 2\pi]\,. \end{cases}$$

3.1.3 Step 3

We use the penalization function define by equation (3) and we obtain :

$$\begin{aligned} L(\theta) = & \frac{1}{24}\Big[27(1-2\lambda)cos(1500\theta) - 32(1-\lambda)cos((1500+0.05)\theta + 0.005) - 10\lambda + 43 \\ & + |-27cos(1500\theta) + 32(1-\lambda)cos((1500+0.05)\theta + 0.005) + 76\lambda - 43|\Big] \\ & + W\Big[111cos(1500\theta) + 132cos((1500+0.05)\theta + 0.005) - 155 \\ & + |-9cos(1500\theta) + 4cos((1500+0.05)\theta + 0.005) + 5| \\ & + |21cos(1500\theta) - 32cos((1500+0.05)\theta + 0.005) - 11| \\ & + |99cos(\theta) + 160cos((1500+0.05)\theta + 0.005) - 149|\Big] \end{aligned}$$

Now, we use the operator preserving optimality in order to optimize $L(\theta)$.

3.1.4 Step 4

As $L(\theta)$ is the function to optimize, from the equation (5), Moreover again, by convenience we choose $W = 10000$, $\theta_0 = \frac{\pi}{2}$ and $\lambda = 1$ (i.e the value corresponding the optimal value of Z_1) we have :

$$\begin{aligned} L(\theta) =& \frac{1}{24}[-27cos(1500\theta) + 33 + |-27cos(1500\theta) + 33|] + 10000\Big[111cos(1500\theta) \\ &+ 132cos((1500 + 0.05)\theta + 0.005) - 155 + |-9cos(1500\theta) \\ &+ 4cos((1500 + 0.05)\theta + 0.005) + 5| + |21cos(1500\theta) \\ &- 32cos((1500 + 0.05)\theta + 0.005) - 11| + |99cos(1500\theta) \\ &+ 160cos((1500 + 0.05)\theta + 0.005) - 149|\Big] \end{aligned}$$

$$\begin{aligned} L(\theta_0) &= -\frac{1999999}{2} + 3200000\cos((1500 + 0.05)\frac{\pi}{2} + 0.005) \\ &= 2.188840753 * 10^6 \end{aligned}$$

from where :

$$\begin{aligned} T_L(\theta) =& \frac{1}{2}\Big[\frac{1}{24}[-27cos(\theta) + 33 + |-27cos(\theta) + 33|] + 10000\Big[111cos(\theta) \\ &+132cos(\sqrt{2}\theta) - 155 + |-9cos(\theta) + 4cos(\sqrt{2}\theta) + 5| + |21cos(\theta) \\ &-32cos(\sqrt{2}\theta) - 11| + |99cos(\theta) + 160cos(\sqrt{2}\theta) - 149|\Big] - 2.188840753 * 10^6 \\ &+|\frac{1}{24}[-27cos(\theta) + 33 + |-27cos(\theta) + 33|] + 10000\Big[111cos(\theta) \\ &+132cos(\sqrt{2}\theta) - 155 + |-9cos(\theta) + 4cos(\sqrt{2}\theta) + 5| + |21cos(\theta) \\ &-32cos(\sqrt{2}\theta) - 11| + |99cos(\theta) + 160cos(\sqrt{2}\theta) - 149|\Big] - 2.188840753 * 10^6 \ |\Big] \end{aligned}$$

We have now a single variable function. To calculate the value for wich $T_L(\theta) = 0$ we have used the Maple software.

3.1.5 Step 5

In the follow the using of Alienor transformation defined in (4) which allows to obtain the following solution :

$$x_1 = 4.402832722 \text{ and } x_2 = 1.783562399.$$

Remark 3.1 *:*

- *It is very important to precise that with MOMA method, the solutions are obtained at more three seconds.*
- *The algorithm of the MOMA method has already been programmed with the Maple software.*

Thus, in the same manner, from various values of the weight, we have obtained the following results (see the table below) :

Table 1: The obtained solution by MOMA method for problem PL1

	λ values	Obtained solutions with MOMA
S_1	1.0	$(4.402832722, 1.783562399)$
S_2	0.9	$(4.402832722, 1.783562399)$
S_3	0.8	$(3.994843068, 1.911648460)$
S_4	0.7	$(3.487884303, 2.119724502)$
S_5	0.6	$(3.096876918, 2.178054281)$
S_6	0.5	$(2.781380087, 2.312742112)$
S_7	0.4	$(2.454345494, 2.428452458)$
S_8	0.3	$(2.211292490, 2.478074215)$
S_9	0.2	$(2.034622016, 2.549530163)$
S_{10}	0.1	$(2.034622016, 2.549530163)$
S_{11}	0.0	$(2.034622016, 2.549530163)$

The graphic representation in the decision space, gives the following figure :

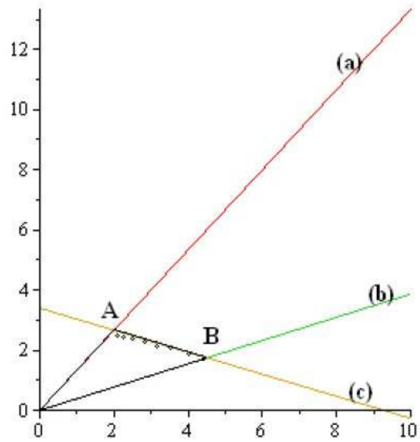

Figure 2: The graphic representation of obtained solutions of PL1

3.2 Problem PL2

In this example, the set of efficient solutions $E(P) = \overline{AB} \cup \overline{BC}$ (see figure (4)).
By applying the MOMA method, we obtained the following results :

Table 2: The obtained solution by MOMA method for problem PL2

	λ values	Obtained solutions with MOMA
S_1	1.0	$(0.4642819583, 3.429174288)$
S_2	0.9	$(0.4642819583, 3.429174288)$
S_3	0.8	$(0.7958084160, 3.270626816)$
S_4	0.7	$(0.9785732930, 2.746266433)$
S_5	0.6	$(1.189137244, 2.521551950)$
S_6	0.5	$(1.512787663, 2.086677156)$
S_7	0.4	$(1.689624698, 1.871638439)$
S_8	0.3	$(1.968535406, 1.520636181)$
S_9	0.2	$(2.162853486, 1.380302605)$
S_{10}	0.1	$(2.663090313, 1.165855305)$
S_{11}	0.0	$(2.966393972, 1.095417146)$

The graphic representation in the decision space, gives the following figure :

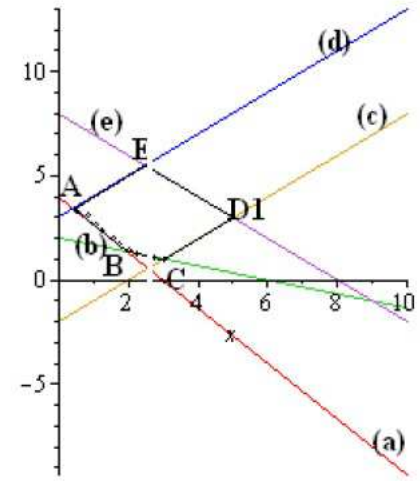

Figure 3: The graphic representation of obtained solutions of PL2

Remark 3.2 *On the two figures, the points $S_i : i = \overline{1, n}$ are described from left to right.*

4 Quality of obtained solutions

The fowolling measures of quality are examined in this work :

- the adherence of the obtained solutions in decision space;
- the distribution [14, 15] of obtained solutions by report Pareto optimal solutions $E(P)$;
- the convergence [14, 15] of obtained solution by report Pareto optimal solutions $E(P)$.
  For this measure we have calculate the distance to the Pareto optimal solutions $E(P)$ for each solution and

the average distance. This distances are calculated based on the usual formula. Recall that the distance of a point A to a straight $(\Delta) : ax + by + c = 0$ is define by [7] :

$$d(A, \Delta) = \mid \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \mid$$

and the average distance [13] by :

$$e_m = \left(\frac{1}{\left|\widehat{E}(P)\right|} \sum_{x \in \widehat{E}(P)} d(x, E(P)) \right) \times 100,$$

where $\widehat{E}(P)$ is the approached solutions obtained by MOMA method. The e_m value will be given in percentages.

4.1 Problem PL1

1. According the figure 3 we see that the obtained solutions par MOMA method are all in the decision space.

2. The solutions are "very good" distribution on Pareto front because all of obtained solutions are regularly spacing and cover all the straight of $E(P)$.

3. For this example the equation of the straight of $E(P)$ is $[AB] : 11x + 30y - 102 = 0$. For the each obtained solution the distances between it and $E(P)$ straight are calculated and united in the following table : The

Table 3: Distance between obtained solutions and $E(P)$

	Distances to $E(P)$
S_1	0.001939346048
S_2	0.001939346048
S_3	0.022134710180
S_4	0.001299955629
S_5	0.081141617930
S_6	0.063297654180
S_7	0.067243130110
S_8	0.104326564200
S_9	0.098057898940
S_{10}	0.098057898940
S_{11}	0.098057898940

average distance is $e_m = 5.79\% = 5.79 \times 10^{-2}$. Thus we conclude that the convergence of obtained solutions to Pareto front is "very good".

4.2 Problem PL2

1. According the figure 4 we see that the obtained solutions par MOMA method are all in the decision space as the last example.

2. As the last example the solutions are "very good" distribution on Pareto optimal straight $E(P)$.

3. The using the same formulas that in previous example, we obtain for the each obtained solution the distances between it and $E(P)$ straight are calculated and united in the following table : The average difference is $e_m = 1.02\% = 1.02 \times 10^{-2}$. Thus we conclude that, also here, the convergence of obtained solutions to Pareto front is "very good".

Table 4: Distance between the obtained solutions and $E(P)$

	Distances to $E(P)$
S_1	0.004526970231
S_2	0.004526970231
S_3	0.03114296899
S_4	0.004791163142
S_5	0.01005238692
S_6	0.009738717414
S_7	0.01168632213
S_8	0.01364657257
S_9	0.009506476373
S_{10}	0.005027877584
S_{11}	0.007906759727

5 Conclusion

During this work we have tried to highlight the different septs of MOMA method. That allow us to describ the theoretical fundation of MOMA method through the didactic examples. The numerical results obtained shows that MOMA method is a "best" alternative to solve the linear problems with mathematics theoretical foundations. Also, contrary to stochastic methods, it always gives the same solution, whatever the implementation with the same parameters. This work complete the previous results of MOMA on the non linear optimization problems. For the future research of MOMA method we will concentrate on :

- The resolution of multiobjectif combinatorial optimization problems,
- The comparaison with other metaheuristics method on large kind of benchmarks.

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