**Dual Series Method for Solving Helmholtz Equation**

**with Mixed Boundary Conditions of the Third Kind**

**Dr. N. A. Hoshan**

**Department of Mathematics**

**E mail:** [**dr\_naserh@yahoo.com**](mailto:dr_naserh@yahoo.com) **or**

[**drnaserh@ttu.edu.jo**](mailto:drnaserh@ttu.edu.jo)

**Tafila Technical University**

**Tafila – Jordan**

**P.O. Box 179 , Tel: 962 3 2250236**

## Abstract

The paper involves with the application of the dual series equation to the problem of Helmholtz equation of cylindrical coordinates subject to inhomogeneous mixed boundary conditions of the third kind located on the surface of the cylinder of bounded radius and infinite high. By choosing the Hankel integral transform, the dual series equations were reduced to a Fredholm integral equation of the second kind which is solved conveniently by using numerical techniques.

**Key Words**: Helmholtz equation, dual series equations, mixed boundary conditions

**Introduction**

It is well known that heat and wave equations with different coordinate systems and various initial and boundary conditions can be reduced to Helmholtz equation, for example temperature function  is transformed to , such transformation simplify the solution of the required partial differential equation. In this paper we presented a dual series equations related to Helmholtz equation in cylindrical coordinates subject to inhomogeneous mixed boundary conditions of the third kind located on a level surface of the cylinder of the bounded radius and infinite high. The radius of the surface cylinder  is divided into two intervals such that, inside the disk  located a third kind boundary conditions which is different with the boundary conditions of the third kind located outside the disk . No boundary conditions on the line of discontinuity . Application of separation of variables to the Helmholtz partial differential equation, we obtain

a general solution with unknown coefficients, next step the use of the boundary conditions leads to dual series equations with Bessel function as a kernel for determination unknown coefficients.

Problems of the heat and Helmholtz equations with mixed boundary conditions of the first, the second and of the third kind in axial cylindrical coordinates have previously been treated and founded in the scientific or technical monographs on this subject [1-6,8]. Much attention was received to investigate mixed problems with partly infinite boundaries can often be reduced to study dual integral equations[1-6],while for the finite regions the problems can often reduced to study dual series equations. Dual series equations related to Laplace equation in cylindrical, spherical and other coordinates involving potential theory, diffraction theory, elasticity theory and other applications can be found in [10-12] and other monographs. In this paper , we treated the dual series equations by defining Hankel integral transform of a function related to the unknown function satisfying the dual series equations , then the inverse transformation produces ultimately the function sought for. In [9-11] Hankel integral transform method is used for investigation solutions for dual integral equations involving a Bessel function of the first kind of order zero as a kernel. The analysis of the dual series equations led to a Fredholm integral equation of the second kind which is solved numerically.

**Formulation and Solution of the Problem**

Find the solution of Helmholtz mixed boundary value problem in a axial symmetrical solid cylinder

, (1)

 (2)

 ( 3)

where , , ,

 are the corresponding cylindrical coordinates,  is a constant, , is the radius of the cylinder

 are constants,  known continuous functions, further we assume that as , .

Separation variables in (1), under the assumption that the conditions bounded at infinity and zero , we obtain the general solution of the problem

 (4)

 unknown coefficients , is the root of Bessel function of the first kind order zero .

Use a mixed conditions (2) and (3) for (4), we obtain the following dual series equations to determine the unknown coefficients 

 (5)

 (6)

If and some of the coefficients is zero, the dual equations (5) and (6) reduced to known solutions[10,11].

Now to solve (5) and (6), let us to introduce the substitution

, 

Dual equations (5) and (6) accept the form

 (7)

 (8)

To solve (7),(8), let us to write down the weight function as

=-q+q=

Where q is constant,  is continuous function

 (9)

Dual series equations(7),(8) by meaning (9) will be

 (10)

Series (8) is written over the interval  such that



 is continuous unknown function defined over the interval  appears is the nature of a mixed boundary conditions. Use an inversion formula for determination of  in (8), we have

 (11)

Putting (11) into (10), we have



 (12)

First sum in (12), is equal to  , simplifying the second sum and integration yields a Fredholm integral equation of the second kind

 (13)

With kernel and free term respectively





#### The known function and the kernel satisfy the inequalities [12]

 , . 

And 

**Numerical solution for integral equation**

It can be seen that the problem are reduced to determine the solution of an integral equation for the unknown function as presented in (13).This equation can be approximated by a sum discrete values of  and leading to the system of linear equations[8]

 (14)

is the weight function depending on the numerical integration . Both  are equally from 0 to ,  is the discretized kernel integral equation,  discretized values for the right-hand side function and auxiliary function respectively,  is the error resulted by replacing integral by series. In expression (14) neglecting the error ,then eplacing the integral by a set of simultaneous algebraic equations, we have

 . (15)

where ,the values of , , 

Approximation of the unknown function  by using (3.1) and (3.2) is

 (16)

Where





To solve equation (15), Simpson rule is used for this purpose to set up the simultaneous equations [10] , the weight function is







In view of equation (15), one expands to the linear n equations in the matrix





  ,  (17)

It is noted that for simplification the infinite series  and free term  appeared in (13) can be treated numerically by approximated  such that



Without loss of generality if we consider  in the equality

(17) calculation will be more simple moreover, the functional series  converges for any values  and . Notice that as , the dual series (5), (6) reduced to dual integral equations of the form



The solution of the above dual integral equation discussed with details in[4].

The obtained results the conclusion can be drown that the paper aim an analytical method the mixed boundary value problem with boundary conditions of the third kind will lead to study

dual series equations. The use of a Hankel integral transform always reduces to inhomogeneous Fredholm integral equation of the second kind which is treated numerically by using simple numerical method (Simpson's rule of integration). It can be revealed that there is no approximation before the numerical evaluation of the integral equation solution. The present results can be served other investigations mixed problems, in particular for mixed boundary condition of the third kind with various applications and coordinate systems.

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