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Adaptic Synchronization of two coupled Newton-Leipnik System with Uncertain Parameter

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Abstract

In this letter, an adaptive synchronization and parameters identification scheme is proposed for two coupled chaotic Newton-Leipnik systems. Based on Lyapunov stability theory an adaptive controller is designed to make the states of two identical chaotic Newton-Leipnik system with unknown system parameters asymptotically synchronized. Especially when some unknown parameters are positive, we can make the controller more simple, beside the controller is independent of those positive uncertain parameters. Numerical simulation results are presented to visualize the effectiveness and feasibility of the developed approaches.

Keywords: *Chaotic systems, Chaos control, Chaos synchronization, Uncertain parameters, Newton-Leipnik system.*

1 Introduction

Since the pioneer works by Ott, Grebogi and Yorke(1990) [1] and Pecorra and Carroll (1990) [2], chaos control and synchronization has received increasing attention due to its theoretical challenges and its potential application to various disciplines. Synchronization in biological systems is one of the fascinating area that has attracted a lots of renewed attention. Various modern synchronization methods, such as adaptive control [3,4], backstepping design [5], active control [6], nonlinear feedback control [7] , impulsive control [8] has been successfully applied to obtain chaos synchronization in recent years. Basically, chaos synchronization problem can be formulated as follows: given a chaotic system, which is considered as the master system, and we have to formulate another system which is considered as the slave system which synchronizes to the master system. Tarai et.al. studied synchronization of bidirectionally coupled chaotic Chen's system with delay in 2007 [9] and they also observed synchronization of generalised linearly bidirectionally coupled unified chaotic system in 2009 [10]. Synchronization via adaptive control has been successfully tested in variety of non-linear dynamical systems including Unified chaotic system, Lu system and Chen system [11,12,13]. In 1994 J.K.John and R.E.Amritkar have studied synchronization of unstable orbits using adaptive control [14]. Adaptive synchronization of Lu system with uncertain parameters was studied by Ellabasy et.al. [15]. Dai et.al. have investigated chaos synchronization by using intermittent parametric adaptive control method in 2001 [16]. Chen et.al. have studied parameters identifications and synchronizations of chaotic systems based upon adaptive control [17]. Adaptive control for the synchronization of Chen system via a single variable was studied by Wang et.al [18]. In 2004, adaptive control of uncertain Lu system was studied by Wu et.al. [19]. Aim of this work is to study chaos synchronization of two coupled Newton-Leipnik system with former case where few parameters are unknown and the latter case where all parameters are unknown. In this paper, an adaptive controller is derived based on Lyapunov stability theory where both master and slave Newton-Leipnik system have unknown parameters. We introduce the parameters update law into the design of the adaptive synchronization controller based on Lyapunov stability theorem. Especially, when some unknown uncertain parameters are positive, we can make the controller more simple, besides the controller is independent of those positive uncertain parameters.

2 Newton-Leipnik System

The Newton-Leipnik system is charaterized by the following differential equation

$$\begin{aligned} \dot{x} &= -ax + y + 10yz \\ \dot{y} &= -x - 0.4y + 5xz \\ \dot{z} &= bz - 5xy \end{aligned} \tag{1}$$

where x, y and z are state variables and a, b are unknown positive constant parameters. The Newton-Leipnik system exhibits a chaotic attractor at the parameter values $a = 0.4$ and $b = 0.175$ shown in Fig.1(a)

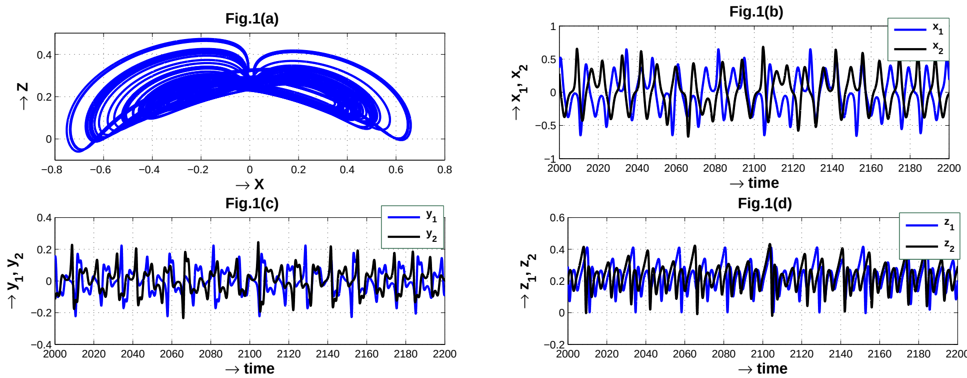

Figure 1: Fig.1(a). Chaotic attractor of Newton-Leipnik system at $a = 0.4$ and $b = 0.175$, Fig.1(b). Time evolution of x_1 and x_2 states, Fig.1(c). Time evolution of y_1 and y_2 states and Fig.1(d). Time evolution of z_1 and z_2 states when the control functions are deactivated.

3 Formulation of the Problem

In order to observe the synchronization behavior in the Newton-Leipnik system, we have two Newton-Leipnik systems where the drive system with three state variables denoted by the subscript 1 and the response system having identical equations denoted by the subscript 2. The initial conditions for the drive system are different from that of the response system. The drive and response systems are defined as follows.

$$\begin{aligned} \dot{x_1} &= -ax_1 + y_1 + 10y_1z_1 \\ \dot{y_1} &= -x_1 - 0.4y_1 + 5x_1z_1 \\ \dot{z_1} &= bz_1 - 5x_1y_1 \end{aligned} \tag{2}$$

and

$$\begin{aligned} \dot{x_2} &= -ax_2 + y_2 + 10y_2z_2 + u_1(t) \\ \dot{y_2} &= -x_2 - 0.4y_2 + 5x_2z_2 + u_2(t) \\ \dot{z_2} &= bz_2 - 5x_2y_2 + u_3(t) \end{aligned} \tag{3}$$

where $U = [u_1(t), u_2(t), u_3(t)]^T$ is the controller function introduced in the response system. The controller is determined for the purpose of synchronizing of two coupled identical Newton-Leipnik systems with the same unknown parameters a and b but with different in initial conditions.

Substracting equation (2) from equation (3) yields the error dynamical system as

$$\dot{e_1} = -ae_1 + e_2 + 10(e_2e_3 + e_2z_1 + e_3y_1) + u_1(t)$$

$$\begin{aligned}
\dot{e_2} &= -e_1 - 0.4e_2 + 5(e_1e_3 + e_1z_1 + e_3x_1) + u_2(t) \\
\dot{e_3} &= be_3 - 5(e_1e_2 + e_2x_1 + e_1y_1) + u_3(t)
\end{aligned} \tag{4}$$

where $e_1 = x_2 - x_1, e_2 = y_2 - y_1$ and $e_3 = z_2 - z_1$. Our goal is to find a controller $U = [u_1(t), u_2(t), u_3(t)]^T$ and a parameter update law for equation (4) such that the states of the response system (3) and the states of the drive system (2) are globally synchronized asympotically i.e.

$$\lim_{t\to\infty} \parallel e(t) \parallel = 0 \quad \text{for all} \quad a \in R \quad and \quad b > 0$$
$$\text{where } e(t) = [e_1, e_2, e_3]^T$$

4 Adaptive Synchronization of Newton-Leipnik System with Single Parameter Estimation

$$V(e, \tilde{a}) = \frac{1}{2}e^T e + \frac{1}{2}\tilde{a}^2 = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) + \frac{1}{2}\tilde{a}^2 \tag{5}$$

where $\tilde{a} = a - \hat{a}$, $\hat{a}$ is an estimated value of the unknown parameter a. Now

$$\dot{V} \leq -w(e) \tag{6}$$

where $w(e) = e_1^2 + 0.4e_2^2 + e_3^2$
We therefore need to find a controller U and a parameter estimation and update law $\hat{a}$ to guarantee that for all $e \in R^3$ the equality (6) holds. There are many possible choices for the controller U. We choose

$$\begin{aligned}
u_1 &= (a-1)e_1 \\
u_2 &= -(\hat{a}e_2 + 15e_1z_1) \\
u_3 &= -be_3 - 10e_1e_2 - 5e_1y_1 - e_3
\end{aligned} \tag{7}$$

and the parameter estimation update law $\dot{\hat{a}}$.

$$\dot{\hat{a}} = e_2^2 \tag{8}$$

By the choice of the controller (7) the error dynamical systems becomes

$$\begin{aligned}
\dot{e_1} &= -e_1 + e_2 + 10(e_2e_3 + e_2z_1 + e_3y_1) \\
\dot{e_2} &= -e_1 - 0.4e_2 + 5(e_1e_3 - 2e_1z_1 + e_3x_1) - \hat{a}e_2 \\
\dot{e_3} &= -e_3 - 5(3e_1e_2 + e_2x_1 + 2e_1y_1)
\end{aligned} \tag{9}$$

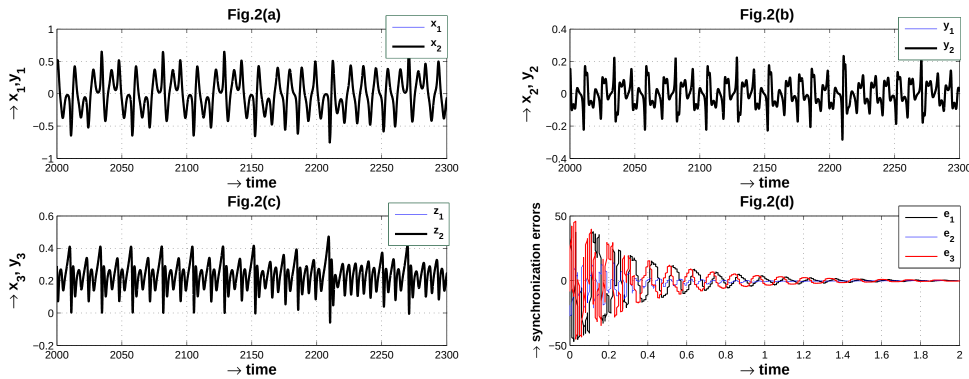

Figure 2: Fig.2(a). Time evolution of x_1 and x_2 states, Fig.2(b). Time evolution of y_1 and y_2 states, Fig.2(c). Time evolution of z_1 and z_2 states and Fig.2(d). Display the error systems tends to zero when the control functions are activated with the parameter estimation update law $\dot{\hat{a}} = e_2^2$.

Under with this choice

$$
\begin{aligned}
\dot{V} &= e_1\dot{e_1} + e_2\dot{e_2} + e_3\dot{e_3} - (a - \hat{a})\dot{\hat{a}} \\
&= -e_1^2 - 0.4e_2^2 - e_3^2
\end{aligned}
$$

This leads to

$$
\lim_{t\to\infty} \| e(t) \| = 0 \quad \text{for all} \;\; a \in R \;\; and \;\; b(>0) \in R
$$
$$
\text{where } e(t) = [e_1, e_2, e_3]^T
$$

It is clear that (4) is independent of unknown uncertain parameter b, if $b > 0$ Therefore the synchronization of two Newton-Leipnik systems are achieved under the controller (7) and a parameter estimation and update law equation (8).

5 Adaptive Synchronization of Newton-Leipnik System with all Parameters Estimation

Let the unknown uncertain parameter $b > 0$ is cancelled, we take a Lyapunov function $V(e, \tilde{a}, \tilde{b})$for equation (4)

$$
V(e, \tilde{a}, \tilde{b}) = \tfrac{1}{2}ee^T + \tfrac{1}{2}(\tilde{a}^2 + \tilde{b}^2)
$$

where $\tilde{a} = a - \hat{a}$, $\tilde{b} = b - \hat{b}$, $\hat{a}$ and $\hat{b}$ are estimate values of the unknown parameters a and b respectively.

We have the time derivative $V(e, \tilde{a}, \tilde{b})$ along the solution of the equation (4)

$$\begin{aligned}\dot{V} = -ae_1^2 - 0.4e_2^2 + 10e_1e_2e_3 + 15e_1e_2z_1 + 5e_1e_3y_1 + be_3^2 \\ +e_1u_1 + e_2u_2 + e_3u_3 - a\dot{\hat{a}} + \hat{a}\dot{\hat{a}} - b\dot{\hat{b}} + \hat{b}\dot{\hat{b}}\end{aligned} \tag{10}$$

There are many possible choices for the controller U but we choose as follows

$$\begin{aligned} u_1 &= (\hat{a} - 1)e_1 \\ u_2 &= -(10e_3 + 15z_1)e_1 \\ u_3 &= -(5e_1y_1 + (\hat{b} + 1)e_3) \end{aligned} \tag{11}$$

and the parameter estimation update laws $\dot{\hat{a}}$ and $\dot{\hat{b}}$.

$$\begin{aligned} \dot{\hat{a}} = e_1^2 \\ \dot{\hat{b}} = e_3^2 \end{aligned} \tag{12}$$

By the choice of the controller (11) the error dynamical system becomes

$$\begin{aligned} \dot{e_1} &= -e_1 + (\hat{a} - a)e_1 + e_2 + 10(e_2e_3 + e_2z_1 + e_3y_1) \\ \dot{e_2} &= -e_1 - 0.4e_2 - 5(e_1e_3 + 2e_1z_1 - e_3x_1) \\ \dot{e_3} &= -e_3 + (b - \hat{b})e_3 - 5(e_1e_2 + e_2x_1 + 2e_1y_1) \end{aligned} \tag{13}$$

Under this choice we have

$$\dot{V} = -e_1^2 - 0.4e_2^2 - e_3^2$$

This leads to

$$\begin{gathered}\lim_{t\to\infty} \parallel e(t) \parallel = 0 \quad \text{for all } \ a \in R \ \ and \ \ b \in R \\ \text{where } e(t) = [e_1, e_2, e_3]^T\end{gathered}$$

Therefore the synchronization of two Newton-Leipnik systems are achieved under the controller (11) and a parameter estimation and update law equation (12).

6 Results and Discussions

Fourth order Runge-Kutta method is used to solve the systems of differential equations (2), (3) and (4). Time step of size .001 is employed. The initial states of the drive system are taken as $x_1(0) = -5$, $y_1(0) = 8$ and $z_1(0) = 10$ and the initial states of the response system are $x_2(0) = 40$, $y_2(0) = -4$ and $z_2(0) = -5$. Hence the error system has the initial values $e_1(0) = 45$, $e_2(0) = -12$ and $e_3(0) = -15$. The parameters are chosen to be $a = 0.4$ and

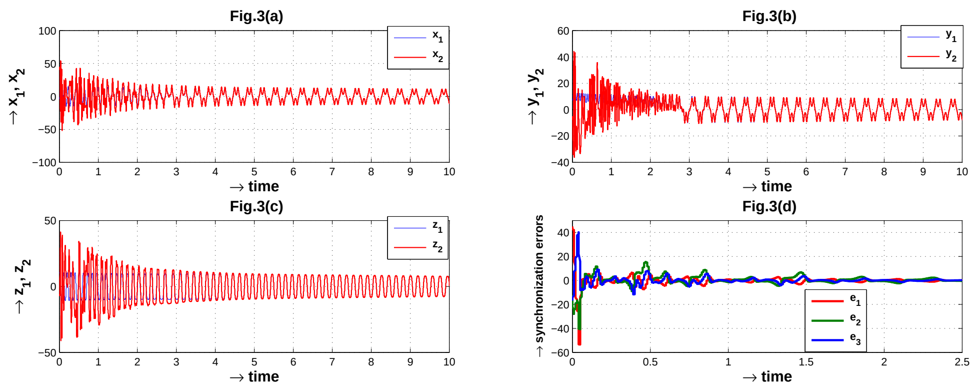

Figure 3: Fig.3(a). Time evolution of x_1 and x_2 states, Fig.3(b). Time evolution of y_1 and y_2 states, Fig.3(c). Time evolution of z_1 and z_2 states and Fig.3(d). Display the error systems tends to zero when the control functions are activated with the parameter estimation update laws $\dot{\hat{a}} = -e_1^2$ and $\dot{\hat{b}} = e_3^2$.

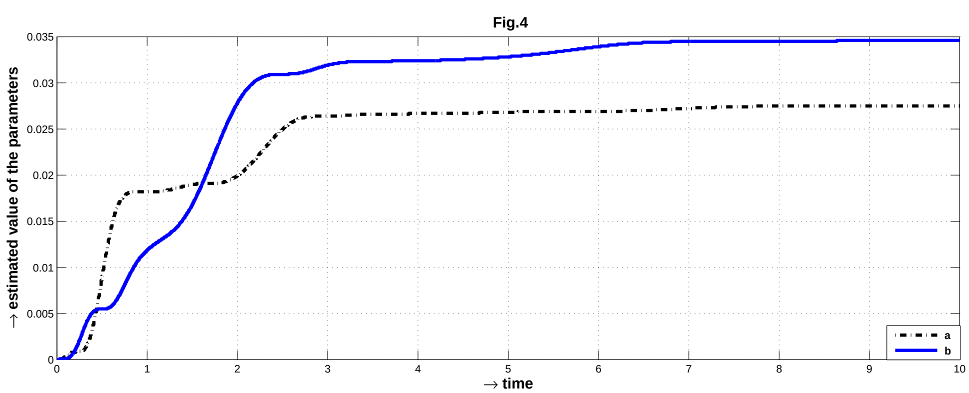

Figure 4: Estimated value of the unknown parameters **a** and **b**.

$b = 0.175$ in all simulations so that the Newton-Leipnik system exhibits the chaotic behavior (Fig.1(a)). In this paper we discuss adaptive synchronization scheme between two coupled chaotic Newton-Leipnik systems. Lyapunov direct method is used to prove the stability of the synchronized state. Secure communication utilizing the synchronization of chaos has been an intersting issue in recent years. Adaptive synchronization of Newton-Leipnik system can also be used for chaotic masking, chaotic modulation and chaotic shift. Numerical results are presented to show the effectiveness of the proposed scheme.

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